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Abstract

Adopting an asset-market view of international risk sharing, we identify various sources of macroeconomic risk faced by international investors using a structural Vector Autoregression model. We find that most of the risk of exogenous financial market shocks are shared by international investors through the existing asset markets. However, other macroeconomic risks such as those associated with exogenous shocks to consumption growth, inflation and monetary policies are not fully shared across countries. This finding helps us understand the apparently contradicting pictures of international risk sharing painted by asset market returns and by aggregate consumption growths across countries.

JEL Classification: E44, F31, G12
Key Words: macroeconomic shocks; nonlinear VAR; risk sharing; stochastic discount factor.

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1 Introduction

If markets were complete and if there were no transport cost, the standard economic theory predicts that we would have perfect risk sharing across national borders. In such a case, the growth rates of marginal utility would be equal across countries as international investors are able to pool all of the idiosyncratic risk they face. In reality, however, a significant part of consumption goods is nontradables, and transport costs vary by good and country-pair. Moreover, international financial markets are far from being complete. There is no shortage of examples of market frictions such as liquidity and short-sale constraints as well as government restrictions on holdings of foreign assets. These imperfections are likely to introduce a wedge between the marginal utilities of consumption in different countries. The degree of international risk sharing has been the focus of a large and growing literature in economics.

Most empirical studies based on aggregate consumption have concluded that international risk sharing is poor[1]. In fact, correlations between consumption growth rates across countries are even weaker than correlations between output growth rates (Backus, Kehoe and Kydland, 1992). This observation supports the view that individuals have not done a good job of hedging risks across countries (Lewis, 1999) and, therefore, gains from international risk sharing could be very large (Van Wincoop, 1999).

The marginal utility of consumption is, however, not observable. To assess the degree of international risk sharing based on aggregate consumption, one needs to make specific assumptions about individual's utility function. For example, under the conventional assumption of power utility function, equality of two countries' marginal utility growth rates implies equal consumption growth rates for the two countries. It is well known that such a utility function fails to reconcile the observed high equity premium with consumption data even in a single-country setting [see, for example, Kocherlakota (1996) for an excellent survey of the literature on equity premium puzzle]. Moreover, the distinction between traded and non-traded goods and their measurement are at best approximations. In addition, limited participation in asset markets is yet another challenge to the empirical studies of international risk sharing relationship based upon aggregate consumption.[2]

[1] See Canova and Ravn (1996), Crucini (1999), Lewis (1999), Pakko (1998) among many others

[2] For example, Davis, Nalewaik and Willen (2000) find that limited participation goes

A recent study by Brandt, Cochrane and Santa-Clara (2001) obtains a very different result about the degree of international risk sharing. Recognizing that the real exchange rate moves by the domestic-foreign marginal utility growth differential, they derive the information about the marginal utility growth directly from asset returns. They find that investors face a considerable amount of risks on one hand, as measured by the large volatility of the marginal utility growth. On the other hand, they find that the correlation of the marginal utility growth rates between two countries is very high, implying that these risks are shared surprisingly well by international investors using the existing asset markets.

Using the above asset-pricing approach to international risk sharing, this paper attempts to address the following empirical questions: What macroeconomic risks are shared and what not shared by investors who participate in the international asset markets? How well do international investors insure against different kinds of idiosyncratic shocks to the economy using the existing asset markets? To this end, we incorporate the asset-market view of international risk sharing into a non-linear structural Vector Autoregression (VAR) model that identifies various sources of macroeconomic risk, including exogenous shocks to the domestic and foreign consumption growth, inflation and monetary policies as well as exogenous shocks to asset markets. We then examine the dynamic effect of these shocks on the domestic-foreign marginal utility growth differential, which is approximated by the real depreciation of the domestic currency against the foreign. If one macroeconomic risk is fully shared by international investors, then domestic and foreign marginal utility growth rates would move together in response to the shock. Hence, their *difference* would not be affected by this shock, and the shock should account for little of the volatility of the change in the real exchange rate. Our main finding is that international investors are actually doing a very good job of hedging against the risk of exogenous shocks to asset markets, although they are not able to fully share other macroeconomic risks, probably due to market incompleteness and transport costs.

There has been an increasingly large body of literature on international macroeconomics stressing the importance of fluctuations of the real exchange rate in accounting for the cross-country co-movement of consumption and output.[3] While the real exchange rate plays a crucial role in our approach,

a long way toward addressing the international risk sharing puzzle.

[3] See, for example, Chari, Kehoe and McGrattan (1998), Obstfeld and Rogoff (1999), Betts and Devereux (2000), Ravn (2001) among others

the goal of the current paper is to document some empirical facts about international risk sharing from a new perspective, rather than to examine the deep structural relationship between the real exchange rate and consumption. The rest of the paper is organized as follows. Section 2 lays out the empirical model used in the paper. Section 3 discusses the main results and section 4 provides a summary.

2 The Model

In this section, we first outline the theoretical framework that motivates our empirical specification. We then incorporate this asset-market view of international risk sharing into a structural VAR analysis with various sources of macroeconomic risk.

2.1 The framework

The key economic relationship underlying our empirical analysis is that, under the assumption of absence of arbitrage in international financial markets, variations in the real exchange rate can be directly linked to the difference between the growth rates of marginal utility of domestic and foreign investors, i.e.

$$\log S_{t+1} - \log S_t = -(\log M_{t+1} - \log M^*_{t+1}), \tag{1}$$

where S_t is the real exchange rate (in units of domestic goods/foreign goods) between the two countries, and $\log M_{t+1}$ and $\log M^*_{t+1}$ are the growth rates of the domestic and foreign marginal utility, respectively (see Appendix A for details of derivation of this and other relations in this section).[4]

Under perfect risk sharing, M_{t+1} and M^*_{t+1} would be equal and hence, the real exchange rate would remain constant. If risk sharing is poor, $\Delta \log S_{t+1}$ will fluctuate as $\log M_{t+1}$ and $\log M^*_{t+1}$ move differently in response to various economic shocks. The main advantage of this asset-market view of international risk sharing is that one does not have to make assumptions about the functional form of (or the inputs into) the utility function of international investors.[5]

[4]See Brandt et al. (2001). Other studies that also exploit this relationship include Backus, Foresi and Telmer (2001), Brandt and Santa-Clara (2001), Hollifield and Yaron (2000) and Nielsen and Saa-Requejo (1993) among others.

[5]However, see section 3.4 for discussions of the major caveats of this approach.

To get a useful empirical specification, we assume that M_{t+1} and M^*_{t+1} both follow the log-normal distribution. More specifically, it is assumed that

$$\log M_{t+1} = \mu_t - \lambda_t^{'} \varepsilon_{t+1} \quad \text{and} \quad \log M^*_{t+1} = \mu^*_t - \lambda_t^{*'} \varepsilon_{t+1} \tag{2}$$

where $\mu_t = E_t(\log M_{t+1})$, $\mu^*_t = E_t(\log M^*_{t+1})$ and the term ε_t represents a vector of fundamental economic shocks (to be described below) distributed as $\mathcal{N}(\mathbf{0}, \mathbf{I})$. Parameters λ_t and λ^*_t are referred to in the literature as the market prices of risk. It follows from (1) that

$$\triangle \log S_{t+1} - (i_t - i^*_t) = \frac{1}{2}(\lambda_t^{'}\lambda_t - \lambda_t^{*'}\lambda^*_t) + (\lambda_t - \lambda^*_t)^{'}\varepsilon_{t+1}. \tag{3}$$

where i_t and i^*_t are the one-period risk-free real interest rates in the home and foreign country respectively.

It is easy to see from (3) that the conventional uncovered interest rate parity (UIP) does not hold in general, or

$$\phi_t \equiv E_t \triangle \log S_{t+1} - (i_t - i^*_t) \neq 0. \tag{4}$$

Note the UIP deviation ϕ_t can be decomposed as $\phi_t = u_t + v_t$, where $u_t = (\lambda_t - \lambda^*_t)'\lambda_t$ and $v_t = -\frac{1}{2}(\lambda_t - \lambda^*_t)'(\lambda_t - \lambda^*_t)$. Using equations (2) and (3), u_t can be expressed as

$$u_t = Cov_t[\Delta \log S_{t+1} - (i_t - i^*_t), \;\; -\log M_{t+1}]. \tag{5}$$

In other words, u_t is the conditional covariance between the excess return on the foreign exchange and the log of the stochastic discount factor, or, in short, the risk premium from investing in the foreign exchange. By (2), we can write u_t as

$$u_t = \sum_{i=1}^{N} \lambda_{i,t} \cdot \mathrm{Cov}_t[\Delta \log S_{t+1} - (i_t - i^*_t), \;\varepsilon_{i,t+1}] \tag{6}$$

which explains why λ_t is called the market price of risk. The ith component λ_{it} of λ_t prices the covariance between the foreign exchange return and the ith fundamental economic shock.[6]

[6] For example, if $\varepsilon_{i,t+1}$ is an exogenous shock to monetary policy in the home country, then the risk associated with the policy when investing in the foreign exchange is characterized by the conditional covariance between the foreign exchange return and the policy shock, and $\lambda_{i,t}$ is the expected excess rate of return per unit of such covariance. Note that similar results hold for the foreign country as well. The foreign exchange risk premium for foreign investors can be expressed as $u^*_t = \sum_{i=1}^{N+1} \lambda^*_{i,t} \cdot \mathrm{Cov}_t[-\triangle \log S_{t+1} - (i^*_t - i_t), \;\varepsilon_{i,t+1}]$, and the similar interpretation applies to $\lambda^*_{i,t}$.

Let r_t and r_t^* be the domestic and foreign real stock returns. Under the no-arbitrage condition and log-normal assumption, we similarly obtain

$$r_{t+1} - i_t = -\frac{1}{2}\sigma'\sigma + \sigma'\lambda_t + \sigma'\varepsilon_{t+1} \tag{7}$$

$$r_{t+1}^* - i_t^* = -\frac{1}{2}\sigma^{*'}\sigma^* + \sigma^{*'}\lambda_t^* + \sigma^{*'}\varepsilon_{t+1}. \tag{8}$$

where σ and σ^* are the volatilities of the stock returns.

Equation (3) together with (7) and (8) can be used to find a link between the excess returns and macroeconomic shocks. In what follows, we will model λ_t and λ_t^* as functions of observable macroeconomic variables,[7] which are in turn driven by the fundamental macroeconomic shocks.

2.2 A nonlinear VAR model

We postulate two types of shocks in our analysis. One includes exogenous innovations to consumption growth, inflation and monetary policies in the home and foreign countries. The other is the exogenous financial market shocks orthogonal to those macroeconomic shocks. More specifically, we assume that the ε_t has 9 components[8]: $\varepsilon_t = (\varepsilon'_{Y,t}, \varepsilon'_{\Pi,t}, \varepsilon'_{M,t}, \varepsilon'_{S,t})'$, where $\varepsilon_{Y,t} = (\varepsilon_{y,t}, \varepsilon^*_{y,t})'$ and $\varepsilon_{\Pi,t} = (\varepsilon_{\pi,t}, \varepsilon^*_{\pi,t})'$ can be thought of as the home and foreign countries' aggregate supply and demand shocks, respectively, while $\varepsilon_{M,t} = (\varepsilon_{m,t}, \varepsilon^*_{m,t})'$ includes exogenous shocks to monetary policies in the two countries and $\varepsilon_{S,t} = (\varepsilon_{s,t}, \varepsilon_{r,t}, \varepsilon^*_{r,t})'$ represents exogenous shocks to international financial markets, including the exogenous innovations to the foreign exchange rate as well as the domestic and foreign stock market returns.

Let $\mathbf{z}_t$ be a 9×1 vector of macroeconomic variables that summarizes the current state of the economy. We include in $\mathbf{z}_t$ the home and foreign consumption growth rates (y_t and y_t^*), inflation rates (π_t and π_t^*) as well as real short-term interest rates (i_t and i_t^*) in the two countries. The last three

[7] In the finance literature, there have been several trials to use observable economic variables as priced risk factors in asset pricing models, including classic studies by Chen, Roll and Ross (1986), Chan, Chen and Hsieh (1985) and Ferson and Harvey (1991). More recently, Ang and Piazzesi (2001) incorporates macroeconomic variables in a VAR analysis of the term structure of interest rates.

[8] The model can be generalized include more economic shocks. We focus on consumption, inflation, monetary policy and exogenous financial market shocks because they are the main economic shocks considered in most international macro models.

components of $\mathbf{z}_t$ are the change in the real exchange rate ($\Delta \log S_t$) and the domestic and foreign real stock returns (r_t and r_t^*).

We assume that the market prices of risk are linear functions of $\mathbf{z}_t$[9]

$$\lambda_t = \mathbf{\Gamma}\mathbf{z}_t \quad \text{and} \quad \lambda_t^* = \mathbf{\Gamma}^*\mathbf{z}_t, \tag{9}$$

where $\mathbf{\Gamma}$ and $\mathbf{\Gamma}^*$ are 9×9 matrices. The dynamics of the first 6 components of $\mathbf{z}_t$ (denoted by $\mathbf{z}_t^+$) is assumed to be described by the reduced-form equation

$$\mathbf{z}_t^+ = \mu + \mathbf{B}_1^+\mathbf{z}_{t-1} + \cdots + \mathbf{B}_p^+\mathbf{z}_{t-p} + \mathbf{u}_t^+ \tag{10}$$

where $\mathbf{z}_t = (\mathbf{z}_t^{+\prime}, \Delta \log S_t, r_t, r_t^*)'$, $\mathbf{B}_1^+, \ldots, \mathbf{B}_p^+$ are 6×9 matrices and μ are a 6×1 vector of constants. The $\mathbf{u}_t^+$ stands for a vector of one-step-ahead forecast errors and it is assumed that $\mathbf{u}_t^+ \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$, where $\mathbf{\Sigma}$ is a symmetric positive definite matrix. The error term $\mathbf{u}_t^+$ is related to the structural shocks according to $\mathbf{u}_t^+ = \mathbf{C}\varepsilon_t$, where $\mathbf{C}$ is a 6×9 matrix. Using (3), (7) and (8) together with (9), the last three components of $\mathbf{z}_t$ may be written as

$$\Delta \log S_t = (i_{t-1} - i_{t-1}^*) + \frac{1}{2}\mathbf{z}_{t-1}'(\mathbf{\Gamma}'\mathbf{\Gamma} - \mathbf{\Gamma}^{*\prime}\mathbf{\Gamma}^*)\mathbf{z}_{t-1} + \mathbf{z}_{t-1}'(\mathbf{\Gamma} - \mathbf{\Gamma}^*)'\varepsilon_t \tag{11}$$

$$r_t = -\frac{1}{2}\sigma'\sigma + \sigma'\mathbf{\Gamma}\mathbf{z}_{t-1} + i_{t-1} + \sigma'\varepsilon_t \tag{12}$$

$$r_t^* = -\frac{1}{2}\sigma^{*\prime}\sigma^* + \sigma^{*\prime}\mathbf{\Gamma}^*\mathbf{z}_{t-1} + i_{t-1}^* + \sigma^{*\prime}\varepsilon_t \tag{13}$$

Equations (10), (11), (12) and (13) therefore constitute a constrained nonlinear VAR, on which our empirical analysis will be based.

2.3 Identification

In conventional VAR models, the identification problem reduces to the restrictions on matrix $\mathbf{C}$. In the nonlinear VAR model above, we need to identify the pricing matrices $\mathbf{\Gamma}$ and $\mathbf{\Gamma}^*$ as well. We discuss these two identification conditions in turn.

We impose the following restrictions to identify the macroeconomic shocks. First, since it is widely believed that monetary policy actions only affect the

[9] Similar parameterizations of the market price of risk have been widely used in the literature where $\mathbf{z}_t$ is treated as a latent state variable, including Constantinides (1992), Ahn et al. (2000) and Dai and Singleton (2002) among many others.

real economy as well as inflation with a delay, we assume that consumption growth and inflation do not respond contemporaneously to shocks to monetary policies in both countries. We also assume that exogenous shocks to the exchange rate and the stock returns have no immediate impact on consumption growth and inflation. This kind of recursive identification assumption is commonly used in the monetary VAR literature (Christiano et al. 1999). Second, we assume that, when setting its policy instrument, the monetary authority in one country does not respond contemporaneously to the other country's aggregate supply and demand shocks as well as the monetary policy shocks. The main reason is that the exact information about a foreign country's output, price and monetary policy stance may not be available immediately to the domestic central bank. Another reason is that the countries included in the current study are all large economies. Foreign shocks are hence less likely to produce severe impact on the domestic economy that requires systematic monetary policy responses. Finally, we allow monetary authorities to respond contemporaneously to the exogenous shocks to the exchange rate, but not to the innovations to the stock returns in our model. This is mainly because that while stability in foreign exchange markets has been one of the major policy goals of all central banks, very rarely do monetary policies respond to the development in stock markets.

To identify $\mathbf{\Gamma}$ and $\mathbf{\Gamma}^*$, we make the following three assumptions (see Appendix B for more details). First, we assume that the domestic (the U.S.) stock returns do not respond contemporaneously to an exogenous shock to the foreign stock returns, but not vice versa. Second, consistent with the usual representative-agent approach in macroeconomics, we assume that home investors and foreign investors price the risk factors in a symmetrical fashion as detailed in Appendix B. Third, we assume that the contribution of y_t^* to the market price of home consumption risk is equal in size to the contribution of y_t to the market price of foreign consumption risk. This assumption of symmetry is obviously a simplification.

The above set of identification assumptions corresponds to our baseline model.[10] We check the robustness of the results associated with these restrictions by estimating the model under alternative identification schemes in section 3.5.

[10]Note that this set of restrictions is the minimum required for identification. In other words, the model is just identified with the above assumptions.

3 Empirical Results

3.1 The Data and Estimation Results

The data used in this study are based on quarterly observations on aggregate consumption, consumer price indices (CPI), the short-term interest rates, stock market returns, the foreign exchange rates and total population for Germany, Britain, Japan and the United States over the period between 1973 and 1996. The data on international stock market returns are from Morgan Stanley Capital International Indices. The data on other macroeconomic variables are from International Financial Statistics of the IMF.[11] All variables except total population are in nominal terms. We obtain the real per capita consumption growth rate by subtracting the CPI inflation rate and population growth rate from the aggregate consumption growth rate. Similarly, the real interest rates, real stock returns and the change in the real exchange rates are obtained by adjusting for the CPI inflation rates.[12]

Using the maximum likelihood method, we estimate the 9-variable VAR described in section 2.2 separately for three pairs of countries: US/Germany, US/UK and US/Japan. In each case, the variables included in $\mathbf{z}_t$ are the growth rates of the U.S. and the foreign real per capita consumption (y_t, y_t^*), the U.S. and the foreign rates of inflation (π_t, π_t^*), the U.S. and the foreign real interest rates (i_t, i_t^*), the depreciation of the real exchange rate $(\Delta \log S_t)$ and the U.S. and foreign real stock market returns (r_t, r_t^*). Given the large dimension of the model and limited data, we only allow one lag in our VAR model in the current paper.[13]

Since our primary interest is in the properties of the real exchange rate

[11]The data are generously provided to us by John Campbell of Harvard University. It is the same data set used in "Asset Price, Consumption and the Business Cycle" by John Campbell (1999) in the Handbook of Macroeconomics. More detailed information about the data can be found in Campbell (1999).

[12]Note that we use the ex-post real interest rates, instead of the ex-ante real interest rate, in our empirical exercise. The impact of this choice on the estimation results is minimal because the dependent variables in the regression are the excess rates of returns, see equations (3), (7) and (8).

[13]Even with one lag, our model explains about 30% of the variation of the exchange rate change $\Delta log S_t$ in all three cases. For the US/Germany exchange rate, the R^2 is 27.7%, and for the US/Japan and US/UK exchange rates the R^2 are 29.2% and 26.3% respectively. The one-lag VAR system seems to offer a good approximation of the system dynamics.

movement under the exogenous macroeconomic shocks, in the following discussion we focus on the exchange rate equation given in (11). Appendix B shows that the equation can be written as

$$\triangle \log S_t = \mathbf{z}_{t-1}^{'} \mathbf{A}_1^{'} \mathbf{B}_S \mathbf{A}_2 \mathbf{z}_{t-1} + \mathbf{b}^{'} \mathbf{z}_{t-1} + (\mathbf{C}_S \mathbf{z}_{t-1})^{'} \varepsilon_t \tag{14}$$

where $\mathbf{B}_S$ and $\mathbf{C}_S$ are, respectively, 5×4 and 9×9 matrices whose elements are to be estimated, $\mathbf{b} = (0, 0, 0, 0, 1, -1, 0, 0, 0)'$, and $\mathbf{A}_1$ and $\mathbf{A}_2$ are some constant matrices defined in Appendix B. Under the symmetry assumption, the matrix $\mathbf{C}_S$ has a simple structure with only 7 unknown coefficients. The estimates of the parameters in $\mathbf{B}_S$ and $\mathbf{C}_S$ are reported, respectively, in Tables 1 and 2.

Two points are worth to mention. First, it appears that stochastic volatility is an important characteristic of the exchange rate movements. Recall the conditional variance of $\triangle \log S_t$ given $\mathbf{z}_{t-1}$ is determined by $\mathbf{z}_{t-1}' \mathbf{C}_S' \mathbf{C}_S \mathbf{z}_{t-1}$. Most estimates of the parameters in $\mathbf{C}_S$ reported in Table 1 are highly significant. Many previous VAR studies on the exchange rate usually assume homoskedasticity. Our result suggests that it is important to take into account the stochastic volatility in order to understand the dynamic behavior of the exchange rate. It is interesting to note that the most significant estimates of the elements in matrix $\mathbf{C_S}$ are C_{55}, C_{37} and C_{57} (see Table 1), where C_{55} is the coefficient on the interest rates i_{t-1} and i_{t-1}^*, and both C_{37} and C_{57} are coefficients on the exchange rate $\Delta \log S_{t-1}$ (see Appendix B for the definition of matrix $\mathbf{C_S}$ and section 2.2 for definition of $\mathbf{z}_t$). This suggests that the lagged interest rates and the lagged exchange rate are the most important economic variables affecting the stochastic volatility of the exchange rate.

Second, consistent with the findings in the large literature on the forward premium puzzle, we find that there is a substantial deviation from the uncovered interest rate parity and that the time-varying risk premiums are an important component of the exchange rate movements. In particular, given the exchange rate equation (14), the ex-ante UIP deviation can be expressed as

$$\begin{aligned} \phi_t &\equiv E_t \left[\Delta \log S_{t+1} - (i_t - i_t^*) \right] \\ &= \mathbf{z}_t' \mathbf{A}_1' \mathbf{B}_S \mathbf{A}_2 \mathbf{z}_t \end{aligned}$$

and the foreign exchange risk premium as defined in (5) is given by

$$\begin{aligned} u_t &\equiv E_t\left[\Delta \log S_{t+1} - (i_t - i_t^*)\right] + \frac{1}{2} Var_t(\Delta \log S_{t+1}) \\ &= \mathbf{z}_t' \mathbf{A}_1' \mathbf{B}_S \mathbf{A}_2 \mathbf{z}_t + \frac{1}{2} \mathbf{z}_t' \mathbf{C}_S' \mathbf{C}_S \mathbf{z}_t. \end{aligned}$$

The significant estimates of $\mathbf{B}_S$ and $\mathbf{C}_S$ reported in Tables 1 and 2 hence reject uncovered interest rate parity and constant foreign exchange risk premiums for all the countries we considered.

3.2 The Estimated Monetary Policy Behavior

In addition to the exchange rate equation, it is also interesting to note that the estimated monetary policy reaction function is largely consistent with the conventional view about how policy makers react to changes in different economic variables. More specifically, the identification restrictions imposed on matrix $\mathbf{C}$ in section 2.3 imply that the U.S. and the foreign monetary authorities react contemporaneously to various economic shocks according to (abstracting from all lagged variables)

$$i_t = a_1 \varepsilon_{y,t} + a_2 \varepsilon_{\pi,t} + a_3 \varepsilon_{m,t} + a_4 \varepsilon_{s,t} \tag{15}$$

$$i_t^* = a_1^* \varepsilon_{y,t}^* + a_2^* \varepsilon_{\pi,t}^* + a_3^* \varepsilon_{m,t}^* + a_4^* \varepsilon_{s,t} \tag{16}$$

where $\varepsilon_{y,t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{m,t}$ are exogenous shocks to the U.S. consumption growth, inflation and monetary policy, respectively, while $\varepsilon_{y,t}^*$, $\varepsilon_{\pi,t}^*$ and $\varepsilon_{m,t}^*$ are exogenous shocks to the corresponding foreign variables. Shock $\varepsilon_{s,t}$ is an exogenous innovation to the real exchange rate.[14] The estimates of the contemporaneous policy reaction coefficients a_i and a_i^* $(i = 1, 2, 3, 4)$ are presented in Table 3.

Note that, unlike the usual monetary reaction function, both i_t and i_t^* in the above equations are the ex-post real interest rates, i.e.

$$\begin{aligned} i_t &= \bar{i}_t - \pi_t \\ i_t^* &= \bar{i}_t^* - \pi_t^* \end{aligned}$$

[14]We can identify the policy response to the exchange rate shock because of the restriction that the U.S. (foreign) monetary authority only responds to the U.S. (foreign) output and
inflation shocks. See the zero restrictions imposed on the last two rows of matrix $\mathbf{C}$ in section 2.3.

where $\bar{i}_t$ and π_t are the nominal short-term interest rate and the rate of inflation, respectively, in the U.S., and $\bar{i}_t^*$ and π_t^* are the corresponding foreign variables.

From Table 3, we can see that in all cases except in Japan, the estimates of a_1 and a_1^* are positive and highly significant, indicating that the policy makers raise the interest rates in response to positive innovations in consumption growth. The policy reactions to exogenous shocks to inflation need to be examined with care because $\varepsilon_{\pi,t}$ and $\varepsilon_{\pi,t}^*$ also have immediate impacts on π_t and π_t^*. Therefore, a negative estimate of a_2 or a_2^* does not necessary imply that the central bank cuts its policy interest rate ($\bar{i}_t$ or $\bar{i}_t^*$) in response to a positive inflation shock. In fact, our estimates suggest that in almost all cases, the monetary authorities actually raise the nominal interest rate $\bar{i}_t$ or $\bar{i}_t^*$ in response to exogenous inflation shocks. In the US-Germany case, for instance, as $\varepsilon_{\pi,t}$ or $\varepsilon_{\pi,t}^*$ increases by 1 unit, π_t or π_t^* increases by 5.2 or 6.1 units, respectively. Given the estimates of -3.5 for a_2 and -3.3 for a_2^*, they imply that both the U.S. and German central banks indeed push up the level of their policy instruments ($\bar{i}_t$ and $\bar{i}_t^*$) in response to exogenous inflation shocks.

In addition, the estimates of a_4 and a_4^* might suggest that there are some contemporaneous monetary policy reactions to exogenous shocks to the real exchange rate. In particular, the positive estimates of a_4 imply that the Fed raises the short-term interest rate when the U.S. dollar depreciates in real term against foreign currencies. Monetary authorities in Germany and Britain also respond to the innovations to the real exchange rate in a similar way, as indicated by the negative estimates of a_4^*.[15] It should be noted, however, that such an implication of the monetary policy response to the exchange rate is valid only under the maintained identification restriction that inflation does not respond contemporaneously to the exchange rate shocks. Without this assumption, our model cannot rule out inflation-induced depreciation, and the estimates of a_4 and a_4^* could simply reflect the policy response to inflation shocks. Moreover, we have assumed that the Fed does not respond to the foreign monetary shock $\varepsilon_{m,t}^*$ and the foreign central bank does not respond to the U.S. monetary shock $\varepsilon_{m,t}$, either (see equations (15) and (16) above). It is possible, however, that there are contemporaneous policy interactions across countries. For example, the foreign central bank could responds to a positive U.S. monetary policy shock (a monetary

[15] Note that a positive innovation $\varepsilon_{s,t}$ means a real depreciation of the U.S. currency and a real appreciation of the foreign currency.

tightening) by increasing the foreign interest rate. Under this scenario, the U.S. dollar would appreciate against the foreign currency in response to the positive U.S. monetary policy shock on one hand. On the other hand, the foreign interest rate would also rise as a result of the foreign central bank's reaction to the U.S. monetary shock. Therefore, we would observe as if the foreign interest rate is responding to a depreciation of the foreign currency.

3.3 Variance Decomposition

Now we turn to the main result of this paper. As argued in section 2.1 above, equation (1) implies that we may use the movement of the real exchange rate (-$\Delta \log S_t$) to measure the difference of the marginal utility growth rates ($\log M_t - \log M_t^*$) between the domestic and foreign investors. If those international investors were able to achieve full risk-sharing, $\log M_t$ would be equal to $\log M_t^*$ across every state of nature, and hence, we would obtain $\Delta \log S_t = 0$, i.e. the constant real exchange rate over time. In reality, the lack of complete markets and the existence of non-traded goods as well as transport costs most likely prevents international investors from fully sharing economic risks. When investors face a domestic shock that is difficult to diversify across national borders, the marginal utility growth ($\log M_t$) of the domestic investors would be driven apart from that of foreign investors ($\log M_t^*$), leading to fluctuations in $\Delta \log S_t$. On the other hand, if a particular economic shock is perfectly shared by the domestic and foreign investors, $\log M_t$ and $\log M_t^*$ would move in lockstep in response to the shock, leaving $\Delta \log S_t$ unchanged. As a result, this shock would account for only small portion of the volatility of the real exchange rate. Our structural VAR enables us to examine what macroeconomic risks, each corresponding to a specific fundamental shock ε_t, are shared by international investors and what macroeconomic risks are not.

For the above purpose, we calculate a variance decomposition for $\Delta \log S_t$, which is treated as a proxy for $\log M_t^* - \log M_t$. For conventional linear VAR models, the variance decomposition can be obtained as a transformation of the model parameters. For nonlinear models, however, no such a simple relation exists. Therefore, the variance decomposition is computed based on Monte Carlo simulations. Specifically, random shocks (ε_{t+j}, $j = 1, \cdots, 4$) are drawn and the forecasting errors for $\Delta \log S_t$ are calculated from the estimated exchange rate equation. This process is repeated 500 times. The sample variances of the forecast errors due to each component of ε_{t+j} (namely

$\varepsilon_{Y,t+j}$, $\varepsilon_{\Pi,t+j}$, $\varepsilon_{M,t+j}$ and $\varepsilon_{S,t+j}$) are then computed. Since the variances are state-dependent due to the nonlinearity of the exchange rate movement, we first compute the variance decomposition conditional on each observation of $\mathbf{z}_t$ in our sample period (1973 to 1996). We then take the average of the variances across different states. Table 4 reports those variances as percentages of the overall volatility of the forecast errors of $\Delta \log S_t$, or $\log M_t^* - \log M_t$, at each time horizon.[16]

The most striking finding from the above exercise is that exogenous financial shocks, the shocks to the exchange rate and stock market returns that are orthogonal to other macroeconomic shocks, jointly account for only a tiny fraction of the volatility of the US/foreign marginal utility growth differential ($\log M_t - \log M_t^*$). In the US/Germany case, the financial shocks jointly account for about 0.6% of the volatility of $\log M_t^* - \log M_t$ four quarters ahead. Similar results are found for other countries as well, namely 3.4% in the US/UK case and 0.3% in the US/Japan case. Almost all the variances of the marginal utility growth differentials are due to exogenous shocks to consumption, inflation and monetary policies.

To show the identified financial shocks are indeed important sources of asset market volatilities, we also compute the variance-decomposition for the domestic and foreign stock returns, using the US/Germany, US/UK and US/Japan data. The results are reported in Tables 5, 6 and 7. Across all countries, we find that the financial shocks in fact account for the largest share (25% and up) of the variance of stock market returns, a result in sharp contrast with the finding reported in Table 4 that the financial shocks appear to have little impact on the US/foreign marginal utility growth differentials. In the US/Germany case, the financial shocks jointly account for 28% and 36% of the volatility of the U.S. and German stock market returns, respectively (Table 5). In the other two cases, the percentages are 65% and 61%, respectively, for the U.S. and British stock market returns (Table 6), and 31% and 25%, respectively, for the U.S. and Japan stock market returns (Table 7).

In summary, the results imply that while investors seem to be facing a significant amount of financial risks, these risks are diversified or shared very well across countries. The underlying financial shocks have little impact on the domestic/foreign marginal utility growth differentials. At the same time, however, these international investors do not appear to fully share other

[16] The variance decompositions are reported up to 4 quarters in Table 4. A longer horizon beyond 4 quarters yields no change in the main result.

macroeconomic risks such as exogenous shocks to consumption, inflation and monetary policies, as each of these shocks contributes significantly to the volatility of the marginal utility growth differential across countries.

The above results are consistent with our knowledge about the incompleteness of financial markets. Full risk sharing through the existing financial markets requires that asset returns span the space of the underlying economic shocks, a proposition that is strongly rejected by empirical evidence (Davis et al., 2000). For example, while labor earnings account for a major portion of national income, they are not securitized because of the non-marketable nature of human capital. Labor income in turn is shown to have near-zero correlation with aggregate equity returns [see Fama and Schwert (1977) and Botazzi et al. (1996)]. On the other hand, aggregate supply and demand shocks as well as monetary policy shocks are probably most responsible for the uncertainties in labor income.

This lack of perfect risk sharing may also reflect the impact of non-traded consumption goods and transport cost, which can be affected significantly by shocks to aggregate consumption, inflation and monetary policies. Suppose that a typical investor's utility function is characterized by non-separability between traded and non-traded consumption goods. The macroeconomic shocks would then drive the marginal utility growth of domestic investors away from that of foreign investors. As a result, there would be large fluctuations in the movement of the real exchange rate.

Brandt et al. (2001) obtain a starkly different result from those in the previous studies of international risk sharing based on aggregate consumption. It shows that there appear to be a lot of risks if one looks at asset market data. The implied volatility of the marginal utility growth is very high. However, most of the risks seem to be shared very well by international investors, as measured by the implied correlation of the marginal utility growth rates across countries. On the other hand, the weak correlation of aggregate consumption growth rates among different countries as documented and analyzed in many previous studies seems to indicate very poor international risk sharing. The current paper does not intend to reconcile these two stylized facts. Our decomposition of risk, however, may provide one potential explanation of the apparently contradicting observations.

In particular, the results from our exercise suggest that if we only look at the aggregate consumption data,[17] we would overlook the fact that international investors share most of the financial market risks and conclude that risk sharing is poor across countries. On the other hand, if the exogenous financial market shocks are the most important source of risk that international investors face ,[18] our results would lead to the same conclusion as in Brandt et al. (2001) that international investors share most of the risks they face.

3.4 Some Caveats

One caveat of our empirical exercise is the reliance on the movement of the real exchange rate ($\Delta \log S_t$) as a proxy of the marginal utility growth differential ($\log M_t - \log M_t^*$) across countries (the key equation in section 2.1). This implicitly makes strong assumptions about how investors behave in terms of their international portfolio decisions (such as rational expectation). Even if one resorts to a weaker assumption of no-arbitrage in international asset markets, equation (1) will not hold for arbitrary pairs of domestic and foreign stochastic discount factors (M_t and M_t^*) if markets are incomplete.[19] In such a case, $\log M_t$ and $\log M_t^*$ in equation (1) should be interpreted as the linear projection of the marginal utility growth onto the space of asset returns as pointed out by Brandt et al. (2001). Therefore, the movements of the real exchange rate may not be exactly linked to the fluctuations in the marginal utility growth differential. The movement of investors' marginal utility growth orthogonal to asset returns is not captured by our empirical model. For example, risk sharing can also be achieved through international government transfers and aids, rather than through financial markets. Our empirical exercise therefore only addresses the issue of what macroeconomic risks are or are not shared by international investors through the *existing* asset markets.

In fact, some of the results reported in Table 4 might just be an outcome of such approximation errors, as the results apparently suggest that nominal shocks (inflation and monetary policy shocks) account for most of the

[17] Note that consumption growth is not the same as marginal utility growth unless the power utility function is assumed.

[18] Note that our approach does not measure the relative magnitude of different macroeconomic risks since $\log M_t$ and $\log M_t^*$ are not separately observable, only their difference is.

[19] Absence of arbitrage, however, is sufficient for the existence of the stochastic discount factors. See, for example, Harrison and Kreps (1979).

variations in the marginal utility growth differential across countries. While this could reflect some impact of nominal rigidities, it is more likely due to the fact that the movement of the real exchange rate is only linked to the "asset-return" part of the marginal utility growth.

Moreover, the presence of non-traded consumption goods and transport cost could further weaken the link between the real exchange rate and the marginal utility growth. When economic shocks change the relative prices of traded and non-traded goods, the stochastic discount factors and the real exchange rate could move not as a result of a lack of risk sharing. Our empirical model cannot distinguish this type of exchange rate movements from those due to undiversified idiosyncratic risks. For example, Sercu, Uppal and Van Hulle (1995) show that even if financial markets are perfectly integrated, complete and frictionless (hence complete financial risk sharing), the real exchange rate could still fluctuate in a no-trade zone because of the presence of transport costs.

Another problem of the model is the use of aggregate consumption and aggregate stock market returns to identify the fundamental economic risks faced by investors. Recent research suggests that the uninsurable idiosyncratic consumption risk of individual investors can play a very important role in reconciling consumption-based asset pricing models with stock returns (e.g. Duffie and Constantinides, 1996).[20] Using aggregate consumption, one ignores those risks and hence could overestimate the amount of risk sharing across countries. On the other hand, most investors only hold stocks of a few individual firms, rather than the market portfolio. Using aggregate stock market returns, one ignores the possible reduction of systematic risk of individual firms (Chari and Henry, 2002) and hence could underestimate the amount of risk sharing among international investors.

In the case of limited participation in asset markets, the use of aggregate data could introduce yet another potential bias in our model, which treats the U.S. and other countries symmetrically in determining the market prices of risk (in particular, the third assumption used to identify $\mathbf{\Gamma}$ and $\mathbf{\Gamma}^*$ in Section 2.3). This is because the U.S. has probably a larger fraction of consumers who actually trade in financial markets than any other countries. Therefore, the aggregate consumption in the U.S. has less bias than the

[20] In fact the variance decomposition of stock returns indicates that there is a large portion of the variation in stock returns that is not "explained" by either aggregate consumption, inflation or monetary policy shocks in our empirical model. This may suggest the need to use disaggregated data to fully understand asset market phenomena.

foreign one as a proxy of the consumption of the country's active investors, and hence could affect the market prices of risk differently than the foreign consumption does. We address this issue in the next section.

An attempt to fix these potential problems would require a carefully specified structural model. It is unlikely, however, that such an attempt would completely change the basic result about the *relative* degrees of international risk sharing in response to different kinds of economic shocks, as reported in Table 4. That is, compared to such macroeconomic risks as those associated with consumption, inflation and monetary policy shocks, the risk associated with exogenous financial market shocks seems to be better diversified among international investors.

As in all VAR-based empirical studies, one crucial element of our model is the identification assumption. In the next section, we check the sensitivity of the result to different identification assumptions.

3.5 Alternative Identification Restrictions

Our VAR model postulates 9 fundamental macroeconomic shocks, which we rewrite here for convenience

$$\varepsilon_t = (\varepsilon'_{Y,t}, \varepsilon'_{\Pi,t}, \varepsilon'_{M,t}, \varepsilon'_{S,t})'$$

where $\varepsilon_{Y,t} = (\varepsilon_{y,t}, \varepsilon^*_{y,t})'$ and $\varepsilon_{\Pi,t} = (\varepsilon_{\pi,t}, \varepsilon^*_{\pi,t})'$ are exogenous shocks to the home and the foreign country's consumption growth and inflation, respectively, $\varepsilon_{M,t} = (\varepsilon_{m,t}, \varepsilon^*_{m,t})'$ stands for exogenous shocks to the monetary policies in the two countries and $\varepsilon_{S,t} = (\varepsilon_{s,t}, \varepsilon_{r,t}, \varepsilon^*_{r,t})'$ represents exogenous financial shocks, including shocks to the foreign exchange rate and the domestic and foreign stock market returns. In this paper, we are not interested in identifying individual shocks included in ε_t separately. Our focus, instead, is on the dynamic effects of the financial shocks $\varepsilon_{S,t}$ as a group.

Our baseline model given in section 2.3 allows unrestricted contemporaneous interaction between $\varepsilon_{M,t}$ and $\varepsilon_{S,t}$ while imposing restrictions on how $\varepsilon_{M,t}$ interacts with $\varepsilon_{Y,t}$ and $\varepsilon_{\Pi,t}$. One natural alternative to the baseline identification scheme is to disallow any contemporaneous interaction between $\varepsilon_{M,t}$ and $\varepsilon_{S,t}$ while relaxing all the restrictions on how monetary policies respond to consumption and inflation shocks ($\varepsilon_{Y,t}$ and $\varepsilon_{\Pi,t}$). This is a plausible assumption because it is unusual for central banks to systematically feedback on the foreign exchange rate or stock market returns.

One maintained assumption of the above two identification schemes is that neither consumption growth nor inflation responds contemporaneously to the financial shocks. It is possible, however, that movements in the foreign exchange rate may have an immediate impact on inflation, particularly when quarterly data are used. Therefore, the second alternative identification scheme we consider is the one that allows contemporaneous feedback of inflation (but not monetary policies) on the exchange rate shocks, while imposing restrictions on policy response to $\varepsilon_{Y,t}$ and $\varepsilon_{\Pi,t}$ simlar to the baseline model. That is, the U.S. monetary policy only feedbacks on the U.S. inflation shocks, but not on the foreign inflation shocks etc.

In our baseline model, the contemporaneous interaction between the monetary policy shocks $\varepsilon_{M,t}$ and the financial shocks $\varepsilon_{S,t}$ is through the feedback of monetary policies on the foreign exchange rate. An alternative is through the feedback of monetary policies on the stock returns, rather than the exchange rate, though this seems less plausible than the baseline model. This is our third identification scheme.

We estimate our model under the above three alternative identification schemes. The results are reported, respectively, in Tables 8, 9 and 10. In almost all cases, we find the result qualitatively same as the one in our baseline model. That is, the macroeconomic factors (consumption, inflation and monetary policy shocks) jointly account for almost the entire variance of the marginal utility growth differential across countries, and little is left for the financial shocks. These results confirm our earlier conclusion about relative risk sharing is not sensitive to different identification restrictions on the structural shocks.

As noted in the last section, our model also makes assumptions to identify the parameters in the market prices of risk in addition to those necessary for identifying the structural shocks. In particular, we made an assumption that the impact of the U.S. consumption on the market price of foreign consumption risk is equal in size to the impact of the foreign consumption on the market price of the U.S. consumption risk, and so on. Under an alternative identification scheme, we relax this restriction and allow those impacts to be different across countries. In other words, the off-diagonal elements (C_{12}, C_{34}, C_{56} and C_{89}) of matrix $\mathbf{C}_S$ are no longer restricted to be zero to reflect this kind of asymmetry.[21] We then estimate the model

[21] Since the model is just identified, we need, however, to assume that the impact of the U.S. consumption on the market price of the *U.S.* consumption risk is equal in size to the impact of the foreign consumption on the market price of the *foreign* consumption risk,

under this new assumption about $\mathbf{\Gamma}$ and $\mathbf{\Gamma}^*$ in combination with the four different identification schemes for the structural shocks (i.e. the baseline model plus the above three different sets of restrictions on ε_t). The results for the baseline case are reported in Table 11 and 12. Table 11 includes the estimates of the parameters in matrix $\mathbf{C_S}$ (see Appendix B for the definition of $\mathbf{C_S}$) and Table 12 reports the results of variance decomposition for the movement of the real exchange rate.[22]

As we can see from Table 11, many of the estimates of the off-diagonal elements (C_{12}, C_{34}, C_{56} and C_{89}) of $\mathbf{C}_S$ are statistically significant and economically important. Moreover, the results also seem to vary a lot across the three country pairs. Similar results are obtained for all other cases. This suggests not only that the impacts of the U.S. and the foreign macro variables on the risk premiums are likely different, but also that the degree of asymmetry varies across countries. Nevertheless, the main result regarding the relative risk sharing remains the same under this alternative identification scheme for $\mathbf{C}_S$, as shown in Table 12. In fact, for the three country pairs, the exogenous financial shocks still account for the least of the volatility of the marginal utility growth differential in all the cases we considered.

4 Concluding Remarks

International risk sharing has been an important research topic in international macroeconomics and finance. While most of the previous studies have focused on the degree of international risk sharing, the current paper identifies various macroeconomic risks faced by investors and asks whether or not some risks are better diversified than others internationally.

Using an asset-pricing approach, we find that international investors share most of the risk of exogenous financial market shocks. However, other macroeconomic risks such as those associated with exogenous shocks to consumption growth, inflation and monetary policies are not fully shared across countries. This asset-market-based approach to international risk sharing allows us to avoid the possibility of making too stringent assumption of the

which makes the diagonal elements (C_{11}, C_{33}, C_{55} and C_{99})of $\mathbf{C}_S$ zero. See Appendix B for details.

[22]The results for other cases are not reported here due to space limitation, but are available upon request. The main result is, however, little changed in all cases.

utility function as well as the difficult task of distinguishing and measuring traded and non-traded consumption goods and transport costs. The empirical exercise helps us understand the apparently contradicting pictures of international risk sharing painted by asset market returns and by aggregate consumption growth across countries.

Appendix A: Background for 2.1

Absence of arbitrage in asset markets implies that there exists a positive stochastic discount factor M_{t+1} such that for any domestic asset (Harrison and Kreps, 1979),

$$1 = E_t(M_{t+1}R_{t+1}) \tag{A1}$$

where R_{t+1} is the real gross rate of return on the domestic asset between time t and $t+1$, and expectation is taken with respect to the investors' information set at time t. In various versions of the consumption-based asset pricing model, M_{t+1} is equal to $u'(c_{t+1})/u'(c_t)$ where $u'(c_t)$ is the marginal utility of consumption at time t. Hence, $\log M_{t+1}$ is simply the growth rate of marginal utility of domestic investors.

Let S_t be the real exchange rate (in units of domestic goods/foreign goods) between the two countries. Then for any foreign asset that can be purchased by domestic investors, $(A1)$ implies

$$1 = E_t\left[M_{t+1}\left(\frac{S_{t+1}}{S_t}\right)R^*_{t+1}\right] \tag{A2}$$

where R^*_{t+1} is the real gross rate of return in terms of foreign goods. But for foreign investors, absence of arbitrage implies that there must also exist a foreign stochastic discount factor satisfying

$$1 = E_t(M^*_{t+1}R^*_{t+1}). \tag{A3}$$

where $\log M^*_{t+1}$ may be similarly interpreted as the growth rate of marginal utility of foreign investors.

If markets are complete, M_{t+1} and M^*_{t+1} are unique. Therefore, $(A2)$ and $(A3)$ imply that

$$\frac{S_{t+1}}{S_t} = \frac{M^*_{t+1}}{M_{t+1}} \tag{A4}$$

or, in terms of logarithms,

$$\log S_{t+1} - \log S_t = -(\log M_{t+1} - \log M^*_{t+1}). \tag{A5}$$

If markets are incomplete, there would be multiple stochastic discount factors that satisfy (A1) and (A3), and equation (A5) would not hold for arbitrary pairs of domestic and foreign discount factors. However, as pointed out by Brandt et al. (2001), the projection of the discount factor (or the marginal utility growth) on the space of asset returns is unique. Therefore (A5) still holds for the so-called minimum variance discount factors.

We assume that M_{t+1} and M^*_{t+1} follow the log-normal distribution according to

$$\log M_{t+1} = \mu_t - \lambda_t^{'}\varepsilon_{t+1} \tag{A6}$$

$$\log M^*_{t+1} = \mu^*_t - \lambda_t^{*'}\varepsilon_{t+1} \tag{A7}$$

where ε_t is distributed as $\mathcal{N}(\mathbf{0}, \mathbf{I})$. Now consider a one-period risk-free pure discount bond with continuously compounded real interest rates i_t and i^*_t in the home and foreign country, respectively. $(A1)$ and $(A3)$ implies that

$$i_t = -\log(E_t M_{t+1}) \tag{A8}$$

$$i^*_t = -\log(E_t M^*_{t+1}) \tag{A9}$$

Using $(A6)$ - $(A9)$, we can express μ_t and μ^*_t as

$$\mu_t = -(i_t + \frac{1}{2}\lambda_t^{'}\lambda_t) \tag{A10}$$

$$\mu^*_t = -(i^*_t + \frac{1}{2}\lambda_t^{*'}\lambda^*_t). \tag{A11}$$

Note that if M_{t+1} and M^*_{t+1} are not distributed as log-normal, $(A10)$ and $(A11)$ still hold as the second order approximations to $(A8)$ and $(A9)$ respectively, as shown in Backus et al (2001). Equations (A10) and (A11) together with $(A5)$ imply

$$\triangle \log S_{t+1} - (i_t - i^*_t) = \frac{1}{2}(\lambda_t^{'}\lambda_t - \lambda_t^{*'}\lambda^*_t) + (\lambda_t - \lambda^*_t)^{'}\varepsilon_{t+1}. \tag{A12}$$

Note the UIP deviation ϕ_t can be expressed as a quadratic function of the home and foreign country's market prices of risk

$$\phi_t = \frac{1}{2}(\lambda_t^{'}\lambda_t - \lambda_t^{*'}\lambda^*_t). \tag{A13}$$

Decompose ϕ_t as $\phi_t = u_t + v_t$, where $u_t = (\lambda_t - \lambda^*_t)'\lambda_t$ and $v_t = -\frac{1}{2}(\lambda_t - \lambda^*_t)'(\lambda_t - \lambda^*_t)$. Note that, using equation $(A6)$ and $(A12)$, u_t can be expressed as

$$u_t = Cov_t[\Delta \log S_{t+1} - (i_t - i^*_t), \ -\log M_{t+1}], \tag{A14}$$

The second term v_t is simply the Jensen's inequality term when taking logarithm of the foreign exchange return, or

$$v_t = -\frac{1}{2}Var_t[\triangle \ln S_{t+1} - (i_t - i^*_t)] \tag{A15}$$

This term does not have any economic significance and disappears in a continuous time setting.[23]

Next, we assume that the domestic and foreign real gross stock returns R_t and R_t^* also have log-normal distribution

$$\log R_{t+1} = E_t(r_{t+1}) + \sigma' \varepsilon_{t+1} \tag{A16}$$
$$\log R^*_{t+1} = E_t(r^*_{t+1}) + \sigma^{*\prime} \varepsilon_{t+1} \tag{A17}$$

where $r_{t+1} = \log R_{t+1}$ and $r^*_{t+1} = \log R^*_{t+1}$. Using $(A1)$ and $(A3)$ again, we have

$$r_{t+1} - i_t = -\frac{1}{2}\sigma'\sigma + \sigma'\lambda_t + \sigma'\varepsilon_{t+1} \tag{A18}$$
$$r^*_{t+1} - i^*_t = -\frac{1}{2}\sigma^{*'}\sigma^* + \sigma^{*'}\lambda^*_t + \sigma^{*'}\varepsilon_{t+1}. \tag{A19}$$

Since quarterly data are used in our exercise, it is assumed for simplicity that the stock returns have constant volatilities (i.e. σ and σ^* are independent of time). Note that the risk premiums are still time-varying under this specification because of the presence of the market prices of risk λ_t and λ^*_t.

Similar interpretations can be made for the excess stock returns in (A18) and (A19). The risk premiums on the domestic and foreign stocks can be expressed as, after adjusting for the Jensen's inequality term,

$$\begin{aligned} E_t(r_{t+1} - i_t) + \frac{1}{2}\sigma'\sigma &= \mathrm{Cov}_t(r_{t+1} - i_t, -\log M_{t+1}) \\ &= \sum_{i=1}^{N} \lambda_{i,t} \cdot \mathrm{Cov}_t[r_{t+1} - i_t, \varepsilon_{i,t+1}] \end{aligned} \tag{A20}$$

$$\begin{aligned} E_t(r^*_{t+1} - i^*_t) + \frac{1}{2}\sigma^{*'}\sigma^* &= \mathrm{Cov}_t(r^*_{t+1} - i^*_t, -\log M^*_{t+1}) \\ &= \sum_{i=1}^{N} \lambda^*_{i,t} \cdot \mathrm{Cov}_t[r^*_{t+1} - i^*_t, \varepsilon_{i,t+1}]. \end{aligned} \tag{A21}$$

[23]It is, however, interesting to note that both the conditional volatility of the exchange rate and the risk premium are determined by the home and foreign country's market price of risk. Since in the finance literature the market price of risk is routinely treated as time-varying, it is not surprising that movements of the exchange rate are characterized by stochastic volatilities and time-varying risk premiums.

Appendix B: Identification of $\boldsymbol{\Gamma}$ and $\boldsymbol{\Gamma}^*$

We rewrite (12) and (13) here for convenience

$$r_t = -\frac{1}{2}\sigma'\sigma + \beta'\mathbf{z}_{t-1} + i_{t-1} + \sigma'\varepsilon_t \tag{B1}$$

$$r_t^* = -\frac{1}{2}\sigma^{*'}\sigma^* + \beta^{*'}\mathbf{z}_{t-1} + i_{t-1}^* + \sigma^{*'}\varepsilon_t \tag{B2}$$

The first identifying restriction is based on the assumption that home and foreign investors price risk factors in a symmetrical fashion as described below. For example, let us consider the first two elements of ε_t: the shocks to the home and foreign country's consumption growth ($\varepsilon_{y,t}$ and $\varepsilon_{y,t}^*$). To investors in the home country, the foreign exchange risk associated with the shock to home consumption is $\mathrm{Cov}_{t-1}[\triangle \log S_t,\ \varepsilon_{y,t}]$,[24] while to investors in the foreign country the foreign exchange risk associated with foreign consumption is $\mathrm{Cov}_{t-1}[-\triangle \log S_t,\ \varepsilon_{y,t}^*]$. We assume that if the market price of the risk (or the expected excess rate of return per unit of the covariance) in the home country is given by

$$\lambda_{1,t} = \Gamma_{11}y_t + \Gamma_{12}y_t^* + \Gamma_{13}\pi_t + \Gamma_{14}\pi_t^* + \Gamma_{15}i_t + \Gamma_{16}i_t^* + \Gamma_{17}\triangle \log S_t + \Gamma_{18}r_t + \Gamma_{19}r_t^*$$

then the foreign counterpart is given by

$$\lambda_{2,t}^* = \Gamma_{12}y_t + \Gamma_{11}y_t^* + \Gamma_{14}\pi_t + \Gamma_{13}\pi_t^* + \Gamma_{16}i_t + \Gamma_{15}i_t^* - \Gamma_{17}\triangle \log S_t + \Gamma_{19}r_t + \Gamma_{18}r_t^*$$

where Γ_{ij} refers to the element on the ith row and jth column of matrix $\boldsymbol{\Gamma}$. And similar parameterizations apply to $\lambda_{i,t}$ and $\lambda_{i,t}^*$ for $i = 2, \cdots, 6$ and $i = 8, 9$. For $i = 7$ we need a little different condition. Note that if the foreign exchange risk due to the exogenous shock $\varepsilon_{S,t}$ is expressed as $\mathrm{Cov}_{t-1}[\triangle \log S_t,\ \varepsilon_{S,t}]$ for home investors, its foreign analogy would be $\mathrm{Cov}_{t-1}[-\triangle \log S_t,\ -\varepsilon_{S,t}]$. Hence, the symmetric assumption implies that if $\varepsilon_{S,t}$ has a market price of risk in the home country given by

$$\lambda_{7,t} = \Gamma_{71}y_t + \Gamma_{72}y_t^* + \Gamma_{73}\pi_t + \Gamma_{74}\pi_t^* + \Gamma_{75}i_t + \Gamma_{76}i_t^* + \Gamma_{77}\triangle \log S_t + \Gamma_{78}r_t + \Gamma_{79}r_t^*$$

then in the foreign country its market price of risk must be

$$\lambda_{7,t}^* = -\Gamma_{72}y_t - \Gamma_{71}y_t^* - \Gamma_{74}\pi_t - \Gamma_{73}\pi_t^* - \Gamma_{76}i_t - \Gamma_{75}i_t^* + \Gamma_{77}\triangle \log S_t - \Gamma_{79}r_t - \Gamma_{78}r_t^*$$

In summary, the symmetric treatment of the market prices of risk across countries implies that $\boldsymbol{\Gamma}^* = \mathbf{A}\boldsymbol{\Gamma}\mathbf{A}$ where $\mathbf{A} = \mathrm{diag}\{\ \mathbf{J},\ \ \mathbf{J},\ \ \mathbf{J},\ \ -1,\ \ \mathbf{J}\ \}$

[24] We have ignored the term $(i_{t-1} - i_{t-1}^*)$ here since it does not affect the conditional covariance.

with $\mathbf{J} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Pre and post multiplication of matrix $\mathbf{A}$ has an effect on matrix $\mathbf{\Gamma}$ in the following manner: First it changes the position of the first and second rows, the third and fourth rows, the fifth and sixth rows, and the eighth and ninth rows of matrix $\mathbf{\Gamma}$ and then changes the sign of the seventh row. Second, it changes the position of the first and second columns, the third and fourth columns, the fifth and sixth columns, and eighth and ninth columns of matrix $\mathbf{\Gamma}$ and then changes the sign of the seventh column.

With this restriction, equation (11) can be expressed as

$$\triangle \log S_t = \mathbf{z}_{t-1}^{'}\mathbf{A}_1^{'}\mathbf{B}_S\mathbf{A}_2\mathbf{z}_{t-1} + \mathbf{b}^{'}\mathbf{z}_{t-1} + (\mathbf{C}_S\mathbf{z}_{t-1})^{'}\varepsilon_t \tag{B3}$$

where $\mathbf{B}_S$ and $\mathbf{C}_S$ are respectively 5×4 and 9×9 matrices whose elements are to be estimated, $\mathbf{b} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{pmatrix}^{'}$, and the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are given by

$$\mathbf{A}_1 = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0-1 & 1 \end{pmatrix}$$

and

$$\mathbf{A}_2 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

To see this, note that $\mathbf{z}'(\mathbf{\Gamma}'\mathbf{\Gamma} - \mathbf{\Gamma}^{*\prime}\mathbf{\Gamma}^{*})\mathbf{z} = \mathbf{z}'(\mathbf{\Gamma}'\mathbf{\Gamma} - \mathbf{A}\mathbf{\Gamma}'\mathbf{\Gamma}\mathbf{A})\mathbf{z} = \mathbf{z}'\mathbf{\Gamma}'\mathbf{\Gamma}\mathbf{z} - \widetilde{\mathbf{z}}'\mathbf{\Gamma}'\mathbf{\Gamma}\widetilde{\mathbf{z}} = (\mathbf{z} - \widetilde{\mathbf{z}})'\mathbf{\Gamma}'\mathbf{\Gamma}(\mathbf{z} + \widetilde{\mathbf{z}})$, where $\widetilde{\mathbf{z}} = \mathbf{A}\mathbf{z}$. Now

$$\mathbf{z} - \widetilde{\mathbf{z}} = \begin{bmatrix} z_1 - z_2 \\ z_2 - z_1 \\ z_3 - z_4 \\ z_4 - z_3 \\ z_5 - z_6 \\ z_6 - z_5 \\ 2z_7 \\ z_8 - z_9 \\ z_9 - z_8 \end{bmatrix} \quad \text{and} \quad \mathbf{z} + \widetilde{\mathbf{z}} = \begin{bmatrix} z_1 + z_2 \\ z_2 + z_1 \\ z_3 + z_4 \\ z_4 + z_3 \\ z_5 + z_6 \\ z_6 + z_5 \\ 0 \\ z_8 + z_9 \\ z_9 + z_8 \end{bmatrix}$$

Note also that

$$\begin{bmatrix} z_1 - z_2 \\ z_3 - z_4 \\ z_5 - z_6 \\ z_7 \\ z_8 - z_9 \end{bmatrix} = \mathbf{A}_1\mathbf{z} \quad \text{and} \quad \begin{bmatrix} z_1 + z_2 \\ z_3 + z_4 \\ z_5 + z_6 \\ z_8 + z_9 \end{bmatrix} = \mathbf{A}_2\mathbf{z}.$$

Therefore, if there is no restriction on $\mathbf{\Gamma}$, we can express the original quadratic form as $\mathbf{z}'(\mathbf{\Gamma}'\mathbf{\Gamma} - \mathbf{\Gamma}^{*\prime}\mathbf{\Gamma}^*)\mathbf{z} = \mathbf{z}'\mathbf{A}_1'\mathbf{B}_S\mathbf{A}_2\mathbf{z}$ as claimed.

The second set of restrictions is based on another type of symmetric assumption to simplify the expression of matrix $\mathbf{C}_S$. We assume, for example, the contribution of y^* to the market price of home consumption risk is assumed to be equal in size to the contribution of y to the market price of foreign consumption risk. This type of symmetric assumption implies restrictions on matrix $\mathbf{\Gamma}$ in the form of $\Gamma_{1+2i,1+2j} = \Gamma_{2+2i,2+2j}$ and $\Gamma_{1+2k,2+2l} = \Gamma_{2+2k,1+2l}$ for $i,j,k,l = 0,1,2$ and $i \neq j$. It makes all off-diagonal elements of $\mathbf{C}_S$ except the seventh row and the seventh column equal to zero. The resulting matrix $\mathbf{C}_S$ becomes

$$\mathbf{C}_S = \begin{pmatrix} C_{11} & 0 & 0 & 0 & 0 & 0 & C_{17} & 0 & 0 \\ 0 & -C_{11} & 0 & 0 & 0 & 0 & C_{17} & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 & C_{37} & 0 & 0 \\ 0 & 0 & 0 & -C_{33} & 0 & 0 & C_{37} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 & C_{57} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -C_{55} & C_{57} & 0 & 0 \\ C_{17} & C_{17} & C_{37} & C_{37} & C_{57} & C_{57} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{99} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_{99} \end{pmatrix}$$

However, it is possible that the impact of the U.S. consumption (y_t) on the market price of the foreign consumption risk is different from the impact of the foreign consumption (y_t^*) on the market price of the U.S. consumption risk. In such a case, we need to assume that the impact of y_t on the market price of the *U.S.* consumption risk is the same as the impact of y_t^* on the *foreign* consumption risk in order to make the model just identified. Under this alternative identification scheme, the matrix $\mathbf{C}_S$ becomes

$$\mathbf{C}_S = \begin{pmatrix} 0 & C_{12} & 0 & 0 & 0 & 0 & C_{17} & 0 & 0 \\ -C_{12} & 0 & 0 & 0 & 0 & 0 & C_{17} & 0 & 0 \\ 0 & 0 & 0 & C_{34} & 0 & 0 & C_{37} & 0 & 0 \\ 0 & 0 & -C_{34} & 0 & 0 & 0 & C_{37} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{56} & C_{57} & 0 & 0 \\ 0 & 0 & 0 & 0 & -C_{56} & 0 & C_{57} & 0 & 0 \\ C_{17} & C_{17} & C_{37} & C_{37} & C_{57} & C_{57} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{89} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_{89} & 0 \end{pmatrix}$$

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Table 1: Estimates of the matrix $\mathbf{C}_S$

	US/Ger Ex-rate	US/UK Ex-rate	US/Jap Ex-rate
C_{11}	0.1786 (0.3813)	0.2844 (0.3400)	-2.4416 (0.7682)
C_{33}	0.3442 (0.4275)	0.6377 (0.3011)	-1.1371 (0.5393)
C_{55}	-3.2899 (0.6164)	-2.4110 (0.2759)	5.1335 (1.0445)
C_{17}	0.3838 (0.2200)	0.3945 (0.1497)	1.0454 (0.3814)
C_{37}	1.5146 (0.1869)	1.1181 (0.1075)	2.1055 (0.2340)
C_{57}	0.9513 (0.1620)	0.4839 (0.0763)	-1.8144 (0.2882)
C_{99}	0.0716 (0.0441)	1.4175 (1.9024)	-0.1615 (0.0593)

Note: This table reports the estimates of the elements of the 9×9 matrix $\mathbf{C}_S$, whose definition can be found in Appendix B. The figures in parentheses are the robust standard errors. Under the symmetry assumption, $\mathbf{C}_S$ has 7 unknown parameters. C_{ij} represents the element on the ith row and jth column of the matrix. The exchange rate equation is given by $\triangle \ln S_t = \mathbf{z}_{t-1}^{'} \mathbf{A_1}^{'} \mathbf{B_S} \mathbf{A_2} \mathbf{z}_{t-1} + b' \mathbf{z}_{t-1} + (\mathbf{C}_s \mathbf{z}_{t-1})' \varepsilon_t$. Hence $\mathbf{C}_S$ determines the conditional variance of $\triangle \ln S_t$, which can be obtained as: $Var_{t-1}(\Delta S_t) = \mathbf{z}_{t-1}' \mathbf{C}_S' \mathbf{C}_S \mathbf{z}_{t-1}$.

Table 2: Estimates of the matrix $\mathbf{B}_S$

	US/Ger Ex-rate	US/UK Ex-rate	US/Jap Ex-rate
B_{11}	0.3331 (0.2949)	0.6418 (0.3022)	0.0216 (0.1768)
B_{12}	0.5860 (0.5621)	1.2597 (0.4950)	-0.1329 (0.3112)
B_{13}	0.3300 (0.5856)	0.6363 (0.6706)	-0.5993 (0.5357)
B_{14}	-0.0169 (0.0634)	0.0299 (0.0531)	0.0764 (0.0453)
B_{21}	-0.0757 (0.0546)	0.0376 (0.0435)	-0.1412 (0.0587)
B_{22}	-0.4080 (0.1795)	0.0512 (0.1565)	-0.0360 (0.1543)
B_{23}	-0.6946 (0.2880)	-0.4888 (0.2047)	-0.6465 (0.2134)
B_{24}	-0.5229 (0.3567)	-0.2903 (0.3144)	-0.7658 (0.2248)
B_{31}	-0.0464 (0.0314)	0.0297 (0.0222)	0.0285 (0.0256)
B_{32}	-0.0343 (0.0288)	-0.0371 (0.0230)	0.0008 (0.0196)
B_{33}	-0.0932 (0.3650)	0.1599 (0.4494)	0.1997 (0.3585)
B_{34}	-1.1361 (0.7890)	-1.6291 (0.5772)	-0.0295 (0.5373)
B_{41}	-0.7173 (0.6940)	-1.7538 (0.6665)	0.0765 (0.4993)
B_{42}	0.0497 (0.0890)	-0.0319 (0.0667)	-0.0585 (0.0674)
B_{43}	-0.0613 (0.0594)	0.0786 (0.0422)	0.0454 (0.0575)
B_{44}	0.0920 (0.0490)	0.0718 (0.0464)	-0.0149 (0.0231)
B_{51}	0.1484 (0.1043)	-0.0831 (0.0376)	0.0486 (0.0290)
B_{52}	0.0449 (0.0867)	-0.0247 (0.0600)	0.0657 (0.0326)
B_{53}	-0.0001 (0.0055)	0.0047 (0.0026)	0.0031 (0.0048)
B_{54}	-0.0021 (0.0038)	-0.0021 (0.0032)	0.0065 (0.0038)

Note: This table reports the estimates of the elements of the 5×4 matrix $\mathbf{B_S}$. B_{ij} represent the element on the ith row and jth column of the matrix. The figures in parentheses are the robust standard errors. The exchange rate equation is given by $\triangle \ln S_t = \mathbf{z}_{t-1}^{'}\mathbf{A_1}^{'}\mathbf{B_S}\mathbf{A_2}\mathbf{z}_{t-1} + (i_t - i_t^*) + (\mathbf{C}_s\mathbf{z}_{t-1})'\varepsilon_t$. Hence $\mathbf{B}_S$ determines the ex-ante UIP deviation, which can be obtained as $E_t[\Delta \log S_{t+1} - (i_t - i_t^*)] = \mathbf{z}_t'\mathbf{A}_1'\mathbf{B_S}\mathbf{A_2}\mathbf{z_t}$, where the matrixes $\mathbf{A_1}$ and $\mathbf{A_2}$ are given in Appendix B. Moreover, the foreign exchange risk premium as defined in (5) can be obtained as $E_t(\Delta \log S_{t+1} - (i_t - i_t^*)) + \frac{1}{2}Var_t(\Delta \log S_{t+1}) = \mathbf{z}_t'\mathbf{A}_1'\mathbf{B}_S\mathbf{A}_2\mathbf{z}_t + \frac{1}{2}\mathbf{z}_t'\mathbf{C}_S'\mathbf{C}_S\mathbf{z}_t$

Table 3: Estimates of monetary policy reaction coefficients

	US and Germany		US and UK		US and Japan	
	i_t	i_t^*	i_t	i_t^*	i_t	i_t^*
$\varepsilon_{y,t}$	3.5985		3.2418		2.2922	
	(0.8334)		(0.9486)		(0.5824)	
$\varepsilon_{\pi,t}$	-3.5481		-3.2276		-6.7962	
	(0.7383)		(0.6460)		(0.9396)	
$\varepsilon_{m,t}$	3.0904		2.5172		3.3539	
	(0.3946)		(0.2835)		(0.4183)	
$\varepsilon^*_{y,t}$		1.4934		5.0047		-1.4364
		(0.9392)		(1.8782)		(1.1912)
$\varepsilon^*_{\pi,t}$		-3.2714		-12.0673		-10.0580
		(0.9740)		(1.5322)		(1.3390)
$\varepsilon^*_{m,t}$		3.5071		3.6488		2.1443
		(0.5634)		(0.4700)		(0.3099)
$\varepsilon_{s,t}$	0.8022	-1.0837	0.6998	-0.4823	1.6478	1.4875
	(0.4501)	(0.4495)	(0.1610)	(0.2685)	(0.3633)	(0.2109)

Note: The reported figures are the estimates of the instantaneous reactions of the monetary policy instrument to various exogenous shocks in each country. The figures in parentheses are the robust standard errors. $\varepsilon_{y,t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{m,t}$ are exogenous shocks to the U.S. consumption growth, inflation and monetary policy respectively, $\varepsilon^*_{y,t}$, $\varepsilon^*_{\pi,t}$ and $\varepsilon^*_{m,t}$ are exogenous shocks to the corresponding foreign variables, and $\varepsilon_{s,t}$ is an exogenous shock to the real exchange rate. Abstracting from all lagged variables, the monetary policy reaction function is $i_t = a_1\varepsilon_{y,t} + a_2\varepsilon_{\pi,t} + a_3\varepsilon_{m,t} + a_4\varepsilon_{s,t}$ for the U.S. For the foreign country, the monetary policy reaction function is given by $i_t^* = a_1^*\varepsilon^*_{y,t} + a_2^*\varepsilon^*_{\pi,t} + a_3^*\varepsilon_{m,t} + a_4^*\varepsilon_{s,t}$. Both i_t and i_t^* are the ex-post real short-term interest rates (in one-tenth of a percentage point), i.e. $i_t = \bar{i}_t - \pi_t$ and $i_t^* = \bar{i}_t^* - \pi_t^*$ where $\bar{i}_t$ and $\bar{i}_t^*$ are the U.S. and foreign nominal interest rates and π_t and π_t^* are the rates of inflation.

Table 4: Variance decomposition of $\Delta \log S_t$

	US vs Germany			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0315	0.4608	0.3079	0.1998
2 quarter	0.0586	0.6132	0.2853	0.0429
3 quarter	0.0502	0.6659	0.2732	0.0107
4 quarter	0.0512	0.6665	0.2764	0.0060
	US vs UK			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0591	0.4380	0.2519	0.2511
2 quarter	0.1053	0.5983	0.2080	0.0884
3 quarter	0.1168	0.6534	0.1826	0.0472
4 quarter	0.1146	0.6847	0.1667	0.0340
	US vs Japan			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.1144	0.3683	0.3644	0.1530
2 quarter	0.1292	0.4800	0.3713	0.0195
3 quarter	0.1297	0.4992	0.3674	0.0037
4 quarter	0.1411	0.4794	0.3764	0.0031

Note: This table reports the results of variance decomposition for the US/foreign marginal utility growth differential $\log M_t - \log M_t^*$, which is approximated by the movement of the real exchange rate $-\Delta \log S_t$. The VAR identification scheme assumes that consumption growth and inflation do not respond to financial shocks, and that domestic (foreign) monetary policy feedbacks only on the domestic (foreign) inflation and consumption growth, but they both feedback on the exchange rate. The same identification assumptions also apply to Table 5 - 7.

Table 5: Variance decomposition of the US and German stock returns

	The U.S. stock market returns			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0382	0.1305	0.0697	0.7617
2 quarter	0.1312	0.1813	0.1490	0.5385
3 quarter	0.2014	0.2210	0.2098	0.3678
4 quarter	0.2375	0.2427	0.2397	0.2802
	German stock market returns			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0336	0.2281	0.0962	0.6421
2 quarter	0.0877	0.2298	0.1348	0.5477
3 quarter	0.1259	0.2361	0.1639	0.4742
4 quarter	0.1872	0.2427	0.2064	0.3638

Table 6: Variance decomposition of the US and UK stock returns

	The U.S. stock market returns			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0445	0.0799	0.0300	0.8456
2 quarter	0.0955	0.1200	0.0831	0.7014
3 quarter	0.1028	0.1298	0.0920	0.6755
4 quarter	0.1120	0.1354	0.1019	0.6508
	U.K. stock market returns			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0124	0.1973	0.0810	0.7094
2 quarter	0.0453	0.2024	0.1033	0.6490
3 quarter	0.0539	0.2046	0.1081	0.6334
4 quarter	0.0662	0.2086	0.1192	0.6060

Table 7: Variance decomposition of the US and Japan stock returns

	The U.S. stock market returns			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0872	0.0006	0.0594	0.8527
2 quarter	0.1751	0.1346	0.1611	0.5291
3 quarter	0.2009	0.1732	0.1902	0.4358
4 quarter	0.2331	0.2246	0.2311	0.3112
	Japan stock market returns			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0629	0.0861	0.0320	0.8190
2 quarter	0.1824	0.1903	0.1718	0.4555
3 quarter	0.2352	0.2374	0.2331	0.2943
4 quarter	0.2489	0.2491	0.2486	0.2534

Table 8: Variance decomposition of $\Delta \log S_t$
(Identification scheme: recursive)

	US vs German			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0218	0.4157	0.3161	0.2464
2 quarter	0.0371	0.5224	0.4111	0.0293
3 quarter	0.0302	0.5507	0.4139	0.0051
4 quarter	0.0345	0.5320	0.4294	0.0040
	US vs UK			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.1065	0.3747	0.1844	0.3344
2 quarter	0.1610	0.5065	0.2487	0.0838
3 quarter	0.1674	0.5485	0.2539	0.0302
4 quarter	0.1818	0.5460	0.2497	0.0225
	US vs Japan			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0851	0.3214	0.3464	0.2471
2 quarter	0.1030	0.4248	0.4434	0.0288
3 quarter	0.0978	0.4375	0.4605	0.0042
4 quarter	0.1008	0.4211	0.4761	0.0020

Note: This table reports the results of variance decomposition for the US/foreign marginal utility growth differential $\log M_t - \log M_t^*$, approximated by the movement of the real exchange rate $-\Delta \log S_t$. The identification restrictions assume that there is no feedback of monetary policies, consumption growth or inflation on exogenous financial shocks. No restrictions on how monetary policies respond to inflation and consumption shocks are imposed.

Table 9: Variance decomposition of $\Delta \log S_t$
(Identification scheme: inflation feedbacks on the exchange rate)

	US vs German			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0318	0.6246	0.3156	0.0280
2 quarter	0.1357	0.4644	0.2670	0.1330
3 quarter	0.1402	0.4643	0.2566	0.1389
4 quarter	0.1426	0.4611	0.2551	0.1411
	US vs UK			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0432	0.5158	0.2014	0.2396
2 quarter	0.0445	0.7502	0.1185	0.0868
3 quarter	0.0341	0.8478	0.0738	0.0443
4 quarter	0.0331	0.8723	0.0601	0.0344
	US vs Japan			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.4350	0.0485	0.4272	0.0892
2 quarter	0.7419	0.0992	0.1202	0.0388
3 quarter	0.8268	0.0929	0.0694	0.0109
4 quarter	0.8319	0.1037	0.0574	0.0070

Note: This table reports the results of variance decomposition for the US/foreign marginal utility growth differential $\log M_t - \log M_t^*$, approximated by the movement of the real exchange rate $-\Delta \log S_t$. The identification restrictions assume that there is a contemporaneous feedback of inflation on exogenous exchange rate shocks, while domestic (foreign) monetary policy feedbacks only on the domestic (foreign) inflation and consumption growth.

Table 10: Variance decomposition of $\Delta \log S_t$
(Identification scheme: monetary policy feedbacks on stock returns)

	US vs German			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0351	0.2945	0.4609	0.2095
2 quarter	0.0596	0.3616	0.5349	0.0439
3 quarter	0.0487	0.3892	0.5537	0.0083
4 quarter	0.0441	0.3941	0.5578	0.0039
	US vs UK			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0068	0.2701	0.3738	0.3493
2 quarter	0.0336	0.3758	0.4857	0.1048
3 quarter	0.0250	0.3921	0.5466	0.0363
4 quarter	0.0267	0.4036	0.5442	0.0255
	US vs Japan			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.1761	0.0147	0.6294	0.1798
2 quarter	0.2294	0.0254	0.6900	0.0552
3 quarter	0.2339	0.0087	0.7476	0.0098
4 quarter	0.2475	0.0082	0.7376	0.0068

Note: This table reports the results of variance decomposition for the US/foreign marginal utility growth differential $\log M_t - \log M_t^*$, approximated by the movement of the real exchange rate $-\Delta \log S_t$. The identification restrictions assume that there is contemporaneous response of monetary policies on exogenous shocks to stock market returns, and that domestic (foreign) monetary policy feedbacks only on the domestic (foreign) inflation and consumption growth.

Table 11: Alternative estimates of the matrix $\mathbf{C}_S$

	US/Ger Ex-rate	US/UK Ex-rate	US/Jap Ex-rate
C_{12}	-1.6396 (0.4389)	0.0212 (0.4446)	0.0799 (2.0948)
C_{34}	-0.5108 (0.4435)	0.5101 (0.3081)	-2.0522 (2.6659)
C_{56}	4.8518 (1.0103)	-2.0989 (0.3687)	0.5273 (2.0245)
C_{17}	-0.3184 (0.3503)	0.4759 (0.1672)	3.5347 (1.2964)
C_{37}	0.8920 (0.2047)	1.1725 (0.1041)	4.0031 (0.9731)
C_{57}	1.7892 (0.2046)	0.4911 (0.0897)	1.7634 (0.5258)
C_{89}	0.3610 (0.0825)	0.0256 (0.0457)	-0.4550 (0.1827)

Note: This table reports the estimates of the elements of the 9×9 matrix $\mathbf{C}_S$, whose definition can be found in Appendix B. The identification assumption is that the impact of y_t on the market price of the U.S. consumption risk is the same as the impact of y_t^* on the market price of the foreign consumption risk. In the meantime, we allow that the impact of y_t on the market price of the *foreign* consumption risk to be different from the impact of y_t^* on the market price of textitthe U.S. consumption risk. The figures in parentheses are the robust standard errors. Under the symmetry assumption, $\mathbf{C}_S$ has 7 unknown parameters. C_{ij} represents the element on the ith row and jth column of the matrix. The exchange rate equation is given by $\triangle \ln S_t = \mathbf{z}'_{t-1}\mathbf{A_1}'\mathbf{B_S}\mathbf{A_2}\mathbf{z}_{t-1} + b'\mathbf{z}_{t-1} + (\mathbf{C}_s\mathbf{z}_{t-1})'\varepsilon_t$. Hence $\mathbf{C}_S$ determines the conditional variance of $\triangle \ln S_t$, which can be obtained as: $Var_{t-1}(\Delta S_t) = \mathbf{z}'_{t-1}\mathbf{C}'_S\mathbf{C}_S\mathbf{z}_{t-1}$.

Table 12: Variance decomposition of $\Delta \log S_t$
(the baseline case with the alternative identification scheme for C_S)

	US vs German			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0566	0.1415	0.6376	0.1642
2 quarter	0.0386	0.1875	0.7381	0.0358
3 quarter	0.0286	0.2003	0.7644	0.0067
4 quarter	0.0280	0.2157	0.7522	0.0040
	US vs UK			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.0740	0.4241	0.2103	0.2917
2 quarter	0.1358	0.5940	0.1746	0.0955
3 quarter	0.1404	0.6554	0.1584	0.0458
4 quarter	0.1439	0.6704	0.1535	0.0321
	US vs Japan			
	Consumption shock	inflation shock	monetary shock	financial shock
1 quarter	0.2835	0.3673	0.0709	0.2783
2 quarter	0.3917	0.4984	0.1007	0.0092
3 quarter	0.3988	0.4955	0.1046	0.0012
4 quarter	0.3847	0.5042	0.1099	0.0012

Note: This table reports the results of variance decomposition for the US/foreign marginal utility growth differential $\log M_t - \log M_t^*$, approximated by the movement of the real exchange rate $-\Delta \log S_t$. The identification restrictions for the structural shocks ε_t are the same as those in the baseline case. However, we impose the alternative restriction on the matrix $\mathbf{C}_S$ as discussed in Section 3.5. The results for all other cases are available upon request.