Monetary Aggregation

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March 21, 2005

Forthcoming in Steven N. Durlauf and Lawrence Blume (eds.), *The New Palgrave Dictionary of Economics*, 2nd edition, Macmillan

JEL Classifications: E41,G12,C43,C22

1. Introduction

Aggregation theory and index-number theory have been used to generate official governmental data since the 1920s. One exception still exists. The monetary quantity aggregates and interest rate aggregates supplied by many central banks are not based on index-number or aggregation theory, but rather are the simple unweighted sums of the component quantities and quantity-weighted or arithmetic averages of interest rates. The predictable consequence has been induced instability of money demand and supply functions, and a series of "puzzles" in the resulting applied literature. In contrast, the Divisia monetary aggregates, originated by Barnett (1980), are derived directly from economic index-number theory. Financial aggregation and index number theory was first rigorously connected with the literature on microeconomic aggregation and index number theory by Barnett (1980,1987). A collection of many of his contributions to that field is available in Barnett and Serletis (2000).

Data construction and measurement procedures imply the theory that can rationalize the procedure. The assumptions implicit in the data construction procedures must be consistent with the assumptions made in producing the models within which the data are to be used. Unless the theory is internally consistent, the data and its applications are incoherent. Without that coherence between aggregator function structure and the econometric models within which aggregates are embedded, stable structure can appear to be unstable. This phenomenon has been called the "Barnett critique" by Chrystal and MacDonald (1994).

2. Aggregation Theory versus Index Number Theory

The exact aggregates of microeconomic aggregation theory depend on unknown aggregator functions, which typically are utility, production, cost, or distance functions. Such functions must first be econometrically estimated. Hence the resulting exact quantity and price indexes become estimator and specification dependent. This dependency is troublesome to governmental agencies, which therefore view aggregation theory as a research tool rather than a data construction procedure.

Statistical index-number theory, on the other hand, provides indexes which are computable directly from quantity and price data, without estimation of unknown parameters. Such index numbers depend jointly on prices and quantities, but not on unknown parameters. In a sense, index number theory trades joint dependency on prices and quantities for dependence on unknown parameters. Examples of such statistical index numbers are the Laspeyres, Paasche, Divisia, Fisher ideal, and Törnqvist indexes.

The loose link between index number theory and aggregation theory was tightened, when Diewert (1976) defined the class of second-order "superlative" index numbers. Statistical index number theory became part of microeconomic theory, as economic aggregation theory had been for decades, with statistical index numbers judged by their nonparametric tracking ability to the aggregator functions of aggregation theory.

For decades, the link between statistical index number theory and microeconomic aggregation theory was weaker for aggregating over monetary quantities than for aggregating over other goods and asset quantities. Once monetary assets began yielding interest, monetary assets became imperfect substitutes for each other, and the "price" of monetary-asset services was no longer clearly defined. That problem was solved by Barnett (1978,1980), who derived the formula for the user cost of demanded monetary services. Subsequently Barnett (1987) derived the formula for the user cost of supplied monetary services. A regulatory wedge can exist between the demand and supply-side user costs, if nonpayment of interest on required reserves imposes an implicit tax on banks.

Barnett's results on the user cost of the services of monetary assets set the stage for introducing index number theory into monetary economics.

3. The Economic Decision

Consider a decision problem over monetary assets that illustrates the capability of the monetary aggregation theory. The decision problem will be defined so that the relevant literature on economic aggregation over goods is immediately applicable. Initially we shall assume perfect certainty.

Let $\mathbf{m}_t' = (m_{1t}, m_{2t}, \ldots , m_{nt})$ be the vector of real balances of monetary assets during period t, let r_t be the vector of nominal holding-period yields for monetary assets during period t, and let R_t be the one period holding yield on the benchmark asset during period t. The benchmark asset is defined to be a pure investment that provides no services other than its yield, R_t, so that the asset is held solely to accumulate wealth. Thus, R_t is the maximum holding period yield in the economy in period t.

Let y_t be the real value of total budgeted expenditure on monetary services during period t. Under simplifying assumptions for data within one country, the conversion between nominal and real expenditure on the monetary services of one or more assets is accomplished using the true cost of living index on consumer goods. But for multicountry data or on data aggregated across heterogeneous regions, the correct deflator can be found in Barnett (2003,2005). The optimal portfolio allocation decision is:

$$\text{maximize } u(\mathbf{m}_t) \qquad (1)$$

$$\text{subject to } \boldsymbol{\pi}_t' \mathbf{m}_t = y_t,$$

where $\boldsymbol{\pi}_t' = (\pi_{1t}, \ldots, \pi_{nt})$ is the vector of monetary-asset real user costs, with

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t}. \qquad (2)$$

This function u is the decision maker's utility function, assumed to be monotonically increasing and strictly concave. The user cost formula (2), derived by Barnett (1978, 1980), measures the foregone interest or opportunity cost of holding a unit of monetary asset i, when the higher yielding benchmark asset could have been held.

To be an admissible quantity aggregator function, the function u must be weakly separable within the consumer's complete utility function over all goods and services. Producing a reliable test for weak separability is the subject of much intensive research by an international group of econometricians. See, e.g., Jones, Dutkowsky, and Elger (2005), Fleissig and Whitney (2003), and De Peretti (2005). Two approaches exist. One approach uses stochastic extensions of nonparametric revealed preference tests, while the other uses parametric econometric models.

Let $\mathbf{m}_t^*$ be derived by solving decision (1). Under the assumption of linearly homogeneous utility, the exact monetary aggregate of economic theory is the utility level associated with holding the portfolio, and hence is the optimized value of the decision's objective function:

$$M_t = u(\mathbf{m}_t^*). \qquad (3)$$

4. The Divisia Index

Although equation (3) is exactly correct, it depends upon the unknown function, u. Nevertheless, statistical index-number theory enables us to track M_t exactly, without estimating the unknown function, u. In continuous time, the exact monetary aggregate, $M_t = u(\mathbf{m}_t^*)$, can be tracked exactly by the Divisia index, which solves the differential equation

$$\frac{d \log M_t}{dt} = \sum_{\text{i}} s_{it} \frac{d \log m_{it}^*}{dt} \tag{4}$$

for M_t, where

$$s_{it} = \frac{\pi_{it} m_{it}^*}{y_t}$$

is the i'th asset's share in expenditure on the total portfolio's service flow. In equation (4), it is understood that the result is in continuous time, so the time subscripts are a shorthand for functions of time. We use t to be the time period in discrete time, but the instant of time in continuous time. The dual user cost price aggregate $\Pi_t = \Pi(\boldsymbol{\pi}_t)$, can be tracked exactly by the Divisia price index, which solves the differential equation

$$\frac{d \log \Pi_t}{dt} = \sum_{\text{i}} s_{it} \frac{d \log \pi_{it}}{dt} . \tag{5}$$

The user cost dual satisfies Fisher's factor reversal in continuous time:

$$\Pi_t M_t = \boldsymbol{\pi}_t' \mathbf{m}_t. \tag{6}$$

As a formula for aggregating over quantities of perishable consumer goods, that index was first proposed by Francois Divisia (1925) with market prices of those goods inserted in place of the user costs in equation (4). In continuous time, the Divisia index, under conventional neoclassical assumptions, is exact. In discrete time, the Törnqvist approximation is:

$$\log M_t - \log M_{t-1} = \sum_{\text{i}} \overline{s}_{it} (\log m_{it}^* - \log m_{i,t-1}^*) , \tag{7}$$

where

$$\overline{s}_{it} = \frac{1}{2}(s_{it} + s_{i,t-1}) .$$

In discrete time, we often call equation (7) simply the Divisia quantity index. After the quantity index is computed from (7), the user cost aggregate most commonly is computed directly from equation (6).

Diewert (1976) defines a ”superlative index number” to be one that is exactly correct for a quadratic approximation to the aggregator function. The discretization (7) to the Divisia index is in the superlative class, since it is exact for the quadratic translog specification to an aggregator function. With weekly or monthly monetary data, Barnett (1980) has shown that the Divisia index growth rates, (7), are accurate to within three decimal places. In addition, the difference between the Fisher ideal index and the discrete Divisia index growth rates are third order and comparably small. That third-order differential error typically is smaller than the round-off error in the component data.

5. Prior Applications

Divisia monetary aggregates were first constructed for the United States by Barnett (1980), when he was on the staff of the Special Studies Section of the Board of Governors of the Federal Reserve System, and now are maintained by the Federal Reserve Bank of Saint Louis in its data base, called FRED. See Anderson, Jones, and Nesmith (1997), who produced the Divisia data for FRED. A Divisia monetary-aggregates data base also has been produced for the UK by the Bank of England. An overview of Divisia data maintained by many central banks throughout the world can be found in Belongia and Binner (2000,2005) and in Barnett, Fisher, and Serletis (1992), along with a survey of empirical results with that data. The most extensive collection of relevant applied and theoretical research in that area is in Barnett and Serletis (2000) and Barnett and Binner (2004).

6. The State of the Art

6.1. Multilateral Aggregation

The European Central Bank is implementing a multilateral extension of the Divisia monetary aggregates for monetary quantity and interest rate aggregation within the Euro area. This aggregation is multilateral in the recursive sense that it permits aggregation of monetary service flows first within countries, followed by aggregation over countries. The resulting aggregation will be in a strictly nested, internally consistent manner. The multilateral extension of the theory was produced by Barnett (2003,2005). This extension was produced under three increasingly strong sets of assumptions, (1) with the weakest being produced from heterogeneous agents theory, (2) followed by the somewhat stronger assumption of existence of a multilateral representative agent, and (3) finally with the strongest being the assumption of existence of a unilateral representative agent. The intent is to move from the weakest towards the strongest assumptions, as progress is made within the European Monetary Union towards its harmonization and economic convergence goals. Since Barnett's three assumption structures are nested, construction of the data under the most general heterogeneous countries approach would continue to be valid, as the stronger assumptions become more reasonable and are attained within the Euro area.

6.2. Extension to Risk

Extension of index number theory to the case of risk was introduced by Barnett, Liu, and Jensen (2000), who derived the extended theory from Euler equations, rather than from the perfect-certainty first-order conditions used in the earlier index number theory literature. Since that extension is based upon the consumption capital-asset-pricing model (CCAPM), the extension is subject to the "equity premium puzzle" problem of smaller than necessary adjustment for risk. We believe that the undercorrection produced by CCAPM results from its assumption of intertemporal blockwise strong separability of goods and services within preferences. Barnett and Wu (2005) have extended Barnett, Liu, and Jensen's result to the case of risk aversion with intertemporally nonseparable tastes.

The extension to risk is likely to be especially important to countries in which its residents hold significant deposits in foreign denominated assets, since exchange rate risk can cause rates of return on monetary assets to be subject to non-negligible risk. With the recent trend towards financial integration in many parts of the world, exchange rate risk is likely to grow in importance in monetary aggregation. In many countries, the largest holder of foreign denominated deposits is the central bank itself. Within the United States, the extension to risk is highly relevant to the so called "missing M2" episode of the early 1990s, when substitutability among small time deposits, stock funds, and bond funds produced "puzzles."

7. Duality

User cost aggregates are duals to monetary quantity aggregates. Either implies the other uniquely. In addition, user-cost aggregates imply the corresponding interest rate aggregates uniquely. The interest-rate aggregate r_t implied by the user-cost aggregate Π_t is the solution for r_t to the equation:

$$\frac{R_t - r_t}{1 + R_t} = \Pi_t.$$

Accordingly any monetary policy that operates through the opportunity cost of money (i.e., interest rates) has a dual policy operating through the monetary quantity aggregate, and vice versa. Aggregation theory implies no preference for either of the two dual policy procedures, or for any other approach to policy, so long as the policy does not violate principles of aggregation theory.

8. Conclusion

Aggregation theory is about measurement, and has little, if anything, to say about the choice of policy instrument, such as the funds rate or the base. But accurate measurement, through proper application of aggregation theory, has much to say about the transmission of policy, modeling of structure, and the measurement of intermediate targets (if any) and final targets.

Policies that violate aggregation theoretic principles include the following oversimplified approaches: (1) inflation targeting that targets one arbitrary consumer-good price as a final target, while ignoring all other consumer goods prices, rather than targeting the true cost-of-living index over all consumer goods prices, (2) interest rate targeting that analogously targets one arbitrary interest rate as an intermediate target, while ignoring all other interest rates, rather than targeting the aggregation-theoretic interest-rate or user-cost aggregate over a weakly-separable collection of monetary assets, (3) monetary quantity targeting that targets a simple-sum monetary aggregate as an intermediate target, rather than the aggregator function over a weakly-separable collection of monetary assets, and (4) policy simulations using money-demand or money-supply functions containing simple-sum monetary aggregates or quantity-weighted interest-rate aggregates. The measurement defects in the above four cases are unrelated to the choice of the funds rate or monetary base as an instrument of policy. Unlike intermediate targets, final targets, and variables in models, the chosen instruments of policy tend to be highly-controllable, disaggregated variables, presenting few serious measurement problems.

The objective of the Divisia monetary aggregates is measurement of the economy's monetary service flow and its dual opportunity cost (user cost) and implied interest rate aggregate, not advocacy of any particular policy use of the correctly measured variables. But all uses of data are adversely affected by improper measurement, and a long series of "puzzles" in monetary economics have been shown to have been produced by improper measurement. See, for example, chapter 24 of Barnett and Serletis (2000).

REFERENCES

Anderson, Richard G., Barry E. Jones, and Travis Nesmith (1997), "Building New Monetary Services Indexes: Concepts, Data and Methods," Federal Reserve Bank of St. Louis *Review*, January/February, pp. 53-82.

Barnett, W. A. (1978), "The User Cost of Money," *Economics Letters*, vol 1, pp. 145-149.

Barnett, W. A. (1980), "Economic Monetary Aggregates: An Application of Aggregation and Index Number Theory," *Journal of Econometrics*, vol 14, pp. 11-48.

Barnett, W. A. (1987), "The Microeconomic Theory of Monetary Aggregation," in: W. A. Barnett and K. J. Singleton (eds.), *New Approaches in Monetary Economics*, Cambidge University Press, Cambridge, pp. 115 – 168.

Barnett, William A. (2003), "Aggregation-Theoretic Monetary Aggregation over the Euro Area, when Countries are Heterogeneous," European Central Bank Working Paper No. 260.

Barnett, William A. (2005), "Multilateral Aggregation-Theoretic Monetary Aggregation over Heterogeneous Contries," *Journal of Econometrics*, forthcoming.

Barnett, William A. and Jane Binner (2004), *Functional Structure and Approximation in Econometrics*, North Holland, Amsterdam.

Barnett, William A. and Apostolos Serletis (eds.) (2000), *The Theory of Monetary Aggregation*, North Holland, Amsterdam.

Barnett, William A. and Shu Wu (2005), "On User Costs of Risky Monetary Assets," *Annals of Finance*, vol. 1, no. 1, January, pp. 35-50.

Barnett, W. A., Douglas Fisher, and Apostolos Serletis (1992), "Consumer Theory and the Demand for Money," *Journal of Economic Literature* 30, pp. 2086-2119.

Barnett, W. A., Y. Liu and M. Jensen (2000), "CAPM Risk Adjustment for Exact Aggregation over Financial Assets", *Macroeconomic Dynamics* 1, pp. 485-512.

Belongia, Michael T. and Jane M. Binner (2000), *Divisia Monetary Aggregates: Theory and Practice*, Palgrave, London.

Belongia, Michael T. and Jane Binner (2005), *Money, Measurement, and Computation*, Palgrave, London.

Chrystal, A. K. and R. MacDonald (1994), "Empirical Evidence on the Recent Behaviour and Usefulness of Simple-Sum and Weighted Measures of the Money Stock," Federal Reserve Bank of St. Louis *Review*, vol. 76, pp. 73-109.

De Peretti, P. (2005), “Testing the Significance of the Departures from Utility Maximization,” *Macroeconomic Dynamics*, June, vol 9, no 3, forthcoming.

Diewert, W. Erwin (1976), “Exact and Superlative Index Numbers,” *Journal of Econometrics*, vol. 4, pp. 115-145.

Divisia, Francois (1925), “L’Indice Monétaire et la Théorie de la Monnaie,” *Revue d’Economie Politique* 39, pp. 980-1008.

Fleissig, A. and G. Whitney (2003), “A New PC-Based Test for Varian’s Weak Separability Conditions,” *Journal of Business and Economic Statistics*, 21, pp. 133-144.

Jones, B., D. Dutkowsky, and T. Elger (2005), “Sweep Programs and Optimal Monetary Aggregation,” *Journal of Banking and Finance* 29, pp. 483-508.