What Do We Learn from Blanchard and Quah Decompositions If Aggregate Demand May Not be Long-Run Neutral?

John W. Keating*

University of Kansas
Department of Economics
334 Snow Hall
Lawrence, KS 66045

web page: http://www.people.ku.edu/~jkeating/
e-mail: jkeating@ku.edu
phone: (785)864-2837
fax: (785)864-5270

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Abstract: This paper structurally interprets empirical results obtained with Blanchard and Quah (1989) decompositions of output into permanent and transitory shocks. This is done using assumptions about the qualitative responses of variables to structural shocks that are consistent with many different economic theories. Impulse responses of output to a permanent shock typically behave in a particular way before World War I that is unusual compared to post-World War II responses. Also, permanent shocks typically explain a larger share of output variance in that earlier period. We show these two empirical results provide evidence that a positive (negative) aggregate demand shock had a permanent positive (negative) effect on output in the pre-World War I period. Thus further support is obtained for that hypothesis based on different empirical evidence and less restrictive structural assumptions. Another empirical result is that impulse responses from postwar data are typically qualitatively consistent with the effects of structural shocks from a standard textbook macro model. We show that Blanchard and Quah's statistical model will obtain impulse responses consistent with a textbook model as the parameter measuring aggregate demand's long-run output effect is varied over a specific range. That analysis provides a range of non-neutralities for which Blanchard and Quah-style decompositions will mistakenly appear to be correct.

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I. Introduction

Economists frequently identify VAR models by restricting the qualitative responses of variables to structural shocks. Faust (1998), Canova and De Nicoló (2002) and Uhlig (2005) were early proponents of this methodology.[1] These sign restrictions are based on the idea that economic theory provides qualitative responses that are robust to a variety of alternative structural models, making them potentially more reliable than the restrictions on coefficients that have traditionally been used for identification. Fry and Pagan (2011) provide a critical overview of the approach. Waggoner and Zha (2003), Inoue and Kilian (2011) and Moon et.al. (2011) study issues concerning inference with sign restrictions.

Rather than use sign restrictions for identification purposes, Keating (forthcoming) pioneers an alternative approach. Instead, qualitative responses are used to provide a structural interpretation for the empirical finding that a permanent increase in output is associated with a rise in the price level for many pre-World War I economies.[2] This finding is puzzling because permanent shocks in the Blanchard and Quah (1989) decomposition are supposed to be aggregate supply shocks and a positive aggregate supply shock does not have a positive effect on the price level in the vast majority of economic theories. Thus these pre-World War I impulse responses are inconsistent with most of the structural models one might use to motivate Blanchard and Quah's (1989) decomposition. Keating (forthcoming) reinterprets that statistical model by assuming a positive shock to aggregate supply lowers the price level and in the long run raises output while a positive shock to aggregate demand raises the price level. No assumption is made about the long-run effect of aggregate demand on output. Hence, the assumption originally used to motivate Blanchard and Quah's decomposition, that aggregate demand has no long-run effect on output, is avoided. Neutrality is neither imposed nor ruled-out by these qualitative responses. Based on these structural assumptions, the increase in price level following a permanent output shock implies that a positive (negative) aggregate demand shock had a long-run positive (negative) effect on the level of output in pre-World War I economies. This result is interesting because economic theories exist for which the long-run effect of aggregate demand on output may be positive, negative or zero depending on particular features of the structure.

While a permanent increase in aggregate supply will cause the price level to fall in most structural models, theory can not completely rule out a rise in the price level. For example, a permanent technological improvement that shifts aggregate supply to the right also raises expected future marginal product of capital. That will increase the demand for investment causing aggregate demand to shift right. If this shift is greater than aggregate supply's shift the price level will rise.[3] The reaction of monetary policy may play an important role in determining whether or not aggregate demand responds more than aggregate supply.[4] This possibility calls into question the conclusion that a positive aggregate demand shock had a permanent positive effect on output in per-World War I economies. Is there any other evidence about permanent output shocks in the pre-World War I period that is relevant to the question about long-run non-neutrality and does not require the potentially false assumption about the price level's response to aggregate supply?

This paper finds that the answer to that question is yes. The first empirical finding of interest is that the short-run response of output to a permanent output shock typically exceed its long-run response in prewar (pre-World War I) sample estimates. We label this type of response short-run overshooting. This feature is not observed in a postwar (post-World War II) sample. The second empirical regularity is that the permanent shock tends to explain a larger share of output variance in the prewar period. We show that these two findings provide further support for the type of aggregate demand non-neutrality found previously. And neither result requires assumptions about the price level's response to structural shocks.

The paper also addresses a third empirical finding. The permanent and transitory shock decompositions obtained from postwar economies are usually consistent with a standard textbook macro model.[5] Permanent shocks typically behave like aggregate supply raising output and lowering the price level while transitory shocks typically behave like aggregate demand raising the price level and initially raising output. These findings are often interpreted as evidence that the textbook model does a good job of explaining how postwar economies operate. However, we show that the impulse responses will be consistent with the qualitative responses in the standard textbook model for a range of values of the parameter that when non-zero indicates non-neutrality. This result shows that Blanchard and Quah decompositions are unable to detect a potentially wide range of non-neutralities.

The paper is organized as follows. Section II introduces two observational equivalent representations of the data, Blanchard and Quah's statistical model and the underlying structure. It also shows how each of these representations is mapped into the reduced-form VAR. These mappings are used to relate the statistical model to the structure. Plausible assumptions about how variables qualitatively respond to aggregate demand and aggregate supply shocks are offered in Section III. We discuss the key empirical findings and develop our main results in Section IV. The paper concludes with the implications of these results and potentially important areas for future research.

II. The Blanchard and Quah Decomposition and the Structure

Two potentially different though observationally equivalent representations of the data are developed. One is derived from a statistical model and the other is the actual structure of the economy. The statistical model and the structure are defined in terms of dynamic responses of variables to shocks or moving average representations. The structural moving average representation is the dynamic response of each variable to each structural shock. The statistical moving average representation is the Blanchard and Quah (1989) decomposition - henceforth abbreviated BQ decomposition - of output into temporary and permanent shocks. The BQ decomposition is obtained by imposing a particular set of identification restrictions on the vector autoregression (VAR) model. The empirical findings that we address and the results that we develop are quite different from Keating (forthcoming) although the basic analytical framework is the same.

A. The Reduced Form

The reduced form VAR can be written as:

$$\beta(L)\Delta X_t = e_t \, , \tag{1}$$

where $X_t = (Y_t, P_t)'$ is the vector of real output and the price level (both in logarithms), e_t is the vector of VAR residuals, $\beta(L)=I-\beta_1 L-\beta_2 L^2 ...-\beta_\kappa L^\kappa$ is the matrix lag polynomial that represents coefficients on lags of ΔX in the VAR,[6] and each series is first differenced.

Using the change in the logarithm of the price level as the second variable differs from Blanchard and Quah who use the unemployment rate. One advantage of price data is that an aggregate demand and supply structure is usually formulated in terms of the price level, not unemployment. Another advantage is that a theory typically predicts that different kinds of supply shocks will have qualitatively similar effects on price and output and does the same for aggregate demand shocks. Supply shocks typically cause output and price to move in opposite directions while aggregate demand shocks generally cause output and price to move in the same direction, at least for a portion of time after the shock occurs. On the other hand, the unemployment rate may respond in a fundamentally different way to different types of supply shocks. An increase in labor supply raises output and initially raises the unemployment rate. An increase in labor demand, say from a technological improvement, is associated with a rise in output and at some point a reduction in the unemployment rate. If an economy experiences different types of aggregate supply shocks that have fundamentally different effects on the variables in the VAR, then analytical results from Blanchard and Quah (1989) and Faust and Leeper (1997) show that the decomposition based on the unemployment rate would not identify the effects of shocks to aggregate demand and supply.[7]

B. The Structure

The structural moving average representation is:

$$\Delta X_t = \theta(L)\varepsilon_t \tag{2}$$

with $\varepsilon_t = (\varepsilon_t^S, \varepsilon_t^D)'$ the structural shocks to aggregate supply and aggregate demand, respectively, $\theta(L) = \theta_0 + \theta_1 L + \theta_2 L^2 + ... = \sum_{j=0}^{\infty} \theta_j L^j$ the dynamic responses of the variables to structural shocks in which θ_j is a 2×2 matrix of structural parameters, $\theta_j = \begin{bmatrix} \theta_j^{YS} & \theta_j^{YD} \\ \theta_j^{PS} & \theta_j^{PD} \end{bmatrix}$, for all integer j greater than zero. Structural shocks are uncorrelated, as is common in the literature.[8] Each shock is assumed to have variance equal to one, $E\varepsilon_t\varepsilon_t' = I$, which is merely a normalization if shocks are uncorrelated.

The response of the level of each variable to each structural shock is obtained by recursive substitution on equation (2):

$$\frac{\partial X_t}{\partial \varepsilon_{t-k}} = \sum_{j=0}^{k} \theta_j \equiv \Phi_k, \tag{3}$$

with Φ_k defined as the k-th partial sum of $\theta(L)$. Φ_k is written as:

$$\Phi_k = \begin{bmatrix} \Phi_k^{YS} & \Phi_k^{YD} \\ \Phi_k^{PS} & \Phi_k^{PD} \end{bmatrix}. \tag{4}$$

While economists would like to estimate the responses in (3), this paper is concerned with structures in which the BQ decomposition may fail to identify these dynamic effects.

As k goes to infinity in equation (3):

$$\lim_{k\to\infty}\left(\frac{\partial X_t}{\partial \varepsilon_{t-k}}\right) = \sum_{j=0}^{\infty} \theta_j = \theta(1). \tag{5}$$

$\theta(1)$ can be written as:

$$\theta(1) = \begin{bmatrix} \Theta_{YS} & \Theta_{YD} \\ \Theta_{PS} & \Theta_{PD} \end{bmatrix}, \tag{6}$$

and is obtained by setting L equal to 1 in $\theta(L)$. This matrix represents the long-run effect of each shock on each variable. As k gets large, Φ_k will converge to $\theta(1)$. But distinguishing between the effects that occur at finite horizons (Φ_k) and the long run effects ($\theta(1)$) will be useful in our analysis. $\Theta_{YD} = 0$ has been the usual assumption. But later we develop alternative structural assumptions that allow this parameter to differ from zero. When not equal to zero, this parameter indicates a type of non-neutrality.

C. Blanchard and Quah's Decomposition

The moving average representation for the BQ decomposition is:

$$\Delta X_t = C(L)\mu_t \tag{7}$$

with $C(L) = C_0 + C_1 L + C_2 L^2 + ... = \sum_{j=0}^{\infty} C_j L^j$ the responses of variables to shocks, each C_j a 2×2 matrix and $\mu_t = (\mu_t^P, \mu_t^T)'$ represents the permanent and transitory shocks. These shocks are assumed to be uncorrelated, as is customary, and the variance of each shock is normalized to one: $E\mu_t\mu_t' = I$.

Responses of output and the price level to permanent and transitory shocks are derived by recursive substitution on equation (7):

$$\frac{\partial X_t}{\partial \mu_{t-k}} = \sum_{j=0}^{k} C_j \ . \tag{8}$$

Letting k get large yields the long-run response of each variable to these shocks:

$$\lim_{k\to\infty}\left(\frac{\partial X_t}{\partial \mu_{t-k}}\right) = \sum_{j=0}^{\infty} C_j = C(1) \ . \tag{9}$$

C(1) represents the sum of C(L) coefficients, and it can be written as:

$$C(1) = \begin{bmatrix} C_{YP} & 0 \\ C_{PP} & C_{PT} \end{bmatrix}. \tag{10}$$

C(1) is lower triangular because the second shock has no permanent effect on output.

D. The Relationship between the Structure and the Statistical Model

An easy way to derive the relationship between the structure and the statistical model is to map each of the moving average representations into the reduced form. Blanchard and Quah's statistical model is transformed into the VAR by multiplying equation (7) by $C_0C(L)^{-1}$. Assuming the structure is invertible,[9] it is mapped into the VAR by multiplying equation (2) by $\theta_0\theta(L)^{-1}$. These mappings indicate how the VAR's residuals:

$$e_t = C_0\mu_t = \theta_0\varepsilon_t \tag{11}$$

and coefficients:

$$\beta(L) = C_0C(L)^{-1} = \theta_0\theta(L)^{-1} \tag{12}$$

are related to both the statistical model and the structure. These equations will allow us to examine the structural information provided by the statistical model even when the hypothesis of long-run neutrality for aggregate demand is false.

Using equation (11) along with the identity covariance matrix assumption for shocks, the residuals' covariance matrix:

$$\Sigma_e = C_0C_0' = \theta_0\theta_0' \ , \tag{13}$$

relates the statistical model's short-run coefficients to the short-run structural parameters. Letting L=1 in equation (12):

$$\beta(1) = C_0C(1)^{-1} = \theta_0\theta(1)^{-1} \ , \tag{14}$$

yields a relationship between the statistical decomposition, the structure and $\beta(1)$ which is a matrix characterized by the sum of VAR coefficients. $C_0 = \beta(1)C(1)$ from the first identity in (14). Insert that equation into the first identity of (13), and simplify:

$$C(1)C(1)' = \beta(1)^{-1}\Sigma_e\beta(1)'^{-1}. \tag{15}$$

C(1) parameters can be estimated by the appropriate Cholesky decomposition. Given the C(1) coefficients, the dynamic responses of variables to shocks are derived directly from the VAR.[10]

The following steps are necessary to finally arrive at the relationship between the BQ decomposition and the structure. Obtain $C_0 = \theta_0\theta(1)^{-1}C(1)$ from the last identify in (14). Inserting that equation into the second equality from (13), and simplifying yields:

$$C(1)C(1)' = \theta(1)\theta(1)'\ . \tag{16}$$

It is well known that if the standard assumption from textbook theory is correct, the BQ decomposition will identify structural effects of aggregate demand and aggregate supply shocks.[11] However, we are interested in allowing for the possibility that Θ_{YD} is not equal to zero. A relationship between the BQ model's impulse responses and the structural responses is obtained by combining the last identities in equations (12) and (14):

$$C(L) = \theta(L)\theta(1)^{-1}C(1). \tag{17}$$

An equivalent way of writing this equation is:

$$C_j = \theta_j\theta(1)^{-1}C(1) \qquad \text{for integer } j \geq 0. \tag{18}$$

We need to obtain the relationship between the coefficients in C(1) and θ(1). Inserting (6) and (10) into equation (16) and solving for each coefficient yields:[12]

$$C_{YP} = \left(\Theta_{YS}^2 + \Theta_{YD}^2\right)^{1/2},\ C_{PP} = \frac{\Theta_{YS}\Theta_{PS} + \Theta_{YD}\Theta_{PD}}{\left(\Theta_{YS}^2 + \Theta_{YD}^2\right)^{1/2}} \text{ and } C_{PT} = \frac{\Theta_{YS}\Theta_{PD} - \Theta_{YD}\Theta_{PS}}{\left(\Theta_{YS}^2 + \Theta_{YD}^2\right)^{1/2}}.$$

With these solutions it is easy to determine that:

$$\theta(1)^{-1}C(1)=\frac{\begin{bmatrix}\Theta_{YS} & -\Theta_{YD}\\ \Theta_{YD} & \Theta_{YS}\end{bmatrix}}{\left(\Theta_{YS}^2+\Theta_{YD}^2\right)^{1/2}}\ . \tag{19}$$

By inserting (18) into (8) and using the definition of structural responses from equation (3), we obtain:

$$\frac{\partial X_t}{\partial \mu_{t-k}}=\sum_{j=0}^{k} C_j\ =\sum_{j=0}^{k}\theta_j\theta(1)^{-1}C(1)=\Phi_k\theta(1)^{-1}C(1)\ . \tag{20}$$

And finally, using (19) and (4) in this equation yields:

$$\frac{\partial X_t}{\partial \mu_{t-k}}=\frac{\begin{bmatrix}\Phi_k^{YS} & \Phi_k^{YD}\\ \Phi_k^{PS} & \Phi_k^{PD}\end{bmatrix}\begin{bmatrix}\Theta_{YS} & -\Theta_{YD}\\ \Theta_{YD} & \Theta_{YS}\end{bmatrix}}{\left(\Theta_{YS}^2+\Theta_{YD}^2\right)^{1/2}}\ . \tag{21}$$

Equation (21) shows precisely how the statistical model’s impulse response functions are related to the structure. If $\Theta_{YD} \neq 0$, the statistical model’s coefficients are nonlinear functions of the structural parameters, and therefore the model is unable to obtain consistent estimates of the structure, in general. But, in spite of this misspecification, the BQ decomposition may still be used to infer information about the structure of an economy.

III. Structural Assumptions

Blanchard and Quah, and many others, have taken the position that aggregate demand is long-run neutral and used that assumption to interpret their statistical model. Even when that is not a valid structural assumption, the BQ decomposition still provides a statistical model of permanent and transitory shocks to output. But, if aggregate demand may have a long-run effect on output, then other structural assumptions are needed to interpret their statistical model.

Fortunately alternative assumptions are available. Economic theory frequently places bounds on the qualitative effects structural shocks have on variables. For example, most theories predict output rises and the price level falls after a positive shock to aggregate supply:

A1: $$\frac{\partial Y_t}{\partial \varepsilon^S_{t-k}} = \Phi^{YS}_k > 0 \quad \text{for integer } k \geq 0;$$

A2: $$\frac{\partial P_t}{\partial \varepsilon^S_{t-k}} = \Phi^{PS}_k < 0 \quad \text{for integer } k \geq 0.$$

These assumptions do not rule out the possibility that a supply shock also causes the aggregate demand curve to shift. If both curves shift in the same direction, assumption A2 requires the aggregate demand curve to shift by less than aggregate supply. As we discussed earlier, assumption A2 is somewhat controversial. Fortunately, for some of our purposes assumption A2 can be avoided. In particular, we will show that the effects structural shocks have on the price level play no role in determining output's response to a permanent output shock.

If the long-run aggregate supply curve is vertical, then a shift in the aggregate demand curve will not affect output in the long run unless it also has a permanent effect on aggregate supply. A number of different theories predict that aggregate demand may be non-neutral in the long run. Table 1 summarizes some of these theories and provides a small sampling of important relevant references.[13] Some theories predict an increase in aggregate demand will cause output to be higher in the long run while others make the opposite prediction. Given that long-run non-neutrality is a possibility, we make the weaker assumption that a positive aggregate demand shock raises output for a finite number of periods:

A3: $$\frac{\partial Y_t}{\partial \varepsilon^D_{t-k}} = \Phi^{YD}_k > 0 \quad \text{for } k=0,1...,K \quad \text{with} \quad 0 < K < \infty.$$

This assumption is uncontroversial as K is an arbitrary positive integer and A3 allows aggregate demand's long-run output effect to take on any value.

Assumption A3 does not rule out a possible decline in output at various points in time following a positive aggregate demand shock. Clearly output must eventually fall if the long-run response is negative. In any case, for some theories output will exhibit cycles as it transitions to a steady state. Cycling makes it possible for the level of output to fall below its initial level, even if the long-run effect is non-negative, and Assumption A3 allows for that.

Finally, we assume the price level rises following a positive aggregate demand shock:[14]

$$\text{A4:} \qquad \frac{\partial P_t}{\partial \varepsilon^{D}_{t-k}} = \Phi^{PD}_{k} > 0 \quad \text{for integer } k \geq 0 .$$

If the long-run aggregate supply curve shifts left due to a positive aggregate demand shock ($\Theta_{YD} < 0$), then A4 will hold. Movement of each curve would put upward pressure on prices. In contrast, if long-run aggregate supply shifts right ($\Theta_{YD} > 0$), then the shift in aggregate demand must be larger for A4 to hold.

While assumptions A1 through A4 do not hold in all cases, they are consistent with a wide array of economic structures. And these assumptions leave Θ_{YD} unconstrained, which is an advantage because many economic theories reject long-run neutrality.

IV. Results

The first empirical finding we address is that prewar output responses to permanent output shocks tend to have fundamentally different shapes compared with postwar responses. Keating and Nye (1998) found in prewar data for 7 of the 10 countries in their study that the immediate effect on output of a permanent shock is larger than the long-run effect (Column 2 of Table 2).[15] In contrast, short-run overshooting is almost never observed in postwar subsamples for those 10 countries. Francis and Ramey (2006) and Bordo, Lane and Redish (2009) have also observed short-run overshooting responses for output using different VAR specifications.[16] What can explain this unusual pattern in prewar impulse responses?

Proposition 1: If aggregate supply shocks do not cause short-run overshooting of output, permanent output shocks will exhibit short-run overshooting, if and only if $\Theta_{YD} > 0$.

To prove this proposition, begin by examining the response of output to a permanent shock which comes directly from equation (21):

$$\frac{\partial Y_t}{\partial \mu^P_{t-k}} = \frac{\Phi^{YS}_k \Theta_{YS} + \Phi^{YD}_k \Theta_{YD}}{\left(\Theta^2_{YS} + \Theta^2_{YD}\right)^{1/2}} \quad . \tag{22}$$

We only need assumptions A1 and A3 to analyze this response of output to a permanent shock as the price level responses to structural shocks are irrelevant. If Θ_{YD}=0, then clearly output's response to a permanent shock is identical to its response to aggregate supply. There does not appear to be any theory in which the short-run effect of a supply shock on output exceeds the long-run effect. Output tends to gradually rise in response to a permanent beneficial supply shock. Cyclical dynamics may occur as output approaches its steady state in some structural models, but these dynamics do not resemble short-run overshooting. Therefore, Θ_{YD}=0 can not explain the short-run overshooting observed in prewar data.

Next consider the case of Θ_{YD}<0. Combining this assumption with A1 and A3 in (22) yields:

$$\frac{\partial Y_t}{\partial \mu^P_{t-k}} < \Phi^{YS}_k \quad \text{for } 0 \le k \le K \ .$$

This inequality obtains because when Θ_{YD} is negative the coefficient on Φ^{YS}_k in (22) is less than one and the other term in that equation is negative. This means the response of output to a permanent shock is smaller than it's response to an aggregate supply shock up to the K-th response. In the long run, the following inequality holds for any non-zero Θ_{YD}:

$$\lim_{k\to\infty}\left(\frac{\partial Y_t}{\partial \mu^P_{t-k}}\right)=C_{YP}=(\Theta^2_{YS}+\Theta^2_{YD})^{1/2}>\Theta_{YS}\,.$$

Hence, the permanent shock's long-run output effect exceeds that of aggregate supply. Combining that with our earlier result we see that when $\Theta_{YD}<0$, the permanent shock has a smaller short-run output effect than aggregate supply and a larger long-run effect on output. Thus, given that aggregate supply is incapable of causing short run overshooting, it will not occur when $\Theta_{YD}<0$.

The final case to consider is $\Theta_{YD}>0$. Equation (22) shows that, in general, the response of output to it's permanent shock is a linear combination of output responses to aggregate supply and aggregate demand shocks. The coefficient on Φ^{YD}_k in equation (22) is positive if and only if Θ_{YD} is positive. Macroeconomic theories often predict an aggregate demand shock will have its peak effect on output within a year or so, an effect that is qualitatively similar to the short-run overshooting observed in models with annual prewar data. If Θ_{YD} is positive and sufficiently large, the response of output to a permanent shock could inherit short-run over-shooting behavior from the dynamic response of output to an aggregate demand shock. Therefore, the short-run overshooting observed in prewar economies is evidence in support of the hypothesis that a permanent increase in aggregate demand had a permanent positive effect on output.

Another interesting finding is that the fraction of output variance explained by permanent shocks is typically larger in the pre-1914 period than in the post-World War II period. This finding is obtained for all 3 countries by Keating and Nye (1999) who follow Blanchard and Quah and use the unemployment rate in the statistical model (Column 4 of Table 2).[17] It is also obtained for 7 of 10 countries by Keating and Nye (1998) who use inflation in place of the unemployment rate (Column 3 of Table 2). This difference in variance across time periods can only occur at finite horizons because as the forecast horizon goes to infinity the permanent shocks will, by construction, explain 100% of the variance of output. Could different long-run output effects of aggregate demand explain these differences in variance decomposition between prewar and postwar periods?

Proposition 2. If aggregate demand shocks are long-run neutral in Economy A but may have a long-run effect on output in Economy B, and in all other respects the two economies are identical, then the fraction of output variance associated with permanent shocks is larger for Economy B, if and only if $\Theta_{YD} > 0$ in Economy B.

Proof of Proposition 2: See the Appendix.

The proof only requires assumptions A1 and A3 because the output variance explained by permanent shocks is a function of the effects of structural shocks on output.[18]

The vast majority of research with permanent and transitory shock decompositions has used postwar data from developed economies. These data typically yield permanent shocks that can be given an aggregate supply interpretation and transitory shocks that can be given an aggregate demand interpretation. That evidence suggests the textbook model may provide a good description of how developed postwar economies operate. For the moment, lets assume that this textbook structural model holds for postwar economies. Let Economy A be the postwar for a country and let Economy B be the prewar period for that same country. Proposition 2 shows that $\Theta_{YD} > 0$ in prewar economies can explain why permanent shocks account for more output variance in the prewar period.[19] A similar result should hold even if aggregate demand is not exactly neutral but close to it in the postwar.[20] These variance decompositions provide additional evidence that is consistent with this particular type of non-neutrality in prewar economies.

As mentioned previously, researchers frequently find empirical evidence that is consistent with the textbook theory when they use postwar data samples. The qualitative features of impulse response functions are the primary means by which that conclusion is reached. But interpreting this postwar evidence as supporting the textbook theory could be a serious mistake.

Proposition 3: Impulse responses to permanent and transitory shocks qualitatively resemble effects of the standard textbook model's aggregate supply and aggregate demand shocks, respectively, over a potentially wide range of positive and negative values for Θ_{YD}.

Proof: Use the four responses in equation (21) to calculate the bounds on Θ_{YD} such that the permanent shock causes output to rise and the price level to fall and the temporary shock initially raises output and causes the price level to rise. The following discussion presents and interprets these bounds.

The temporary shock causes output and the price level to rise, respectively, when

$$\frac{\Phi_k^{YD}\Theta_{YS}}{\Phi_k^{YS}} > \Theta_{YD} > \frac{\Phi_k^{PD}\Theta_{YS}}{\Phi_k^{PS}}. \tag{23}$$

Given assumptions A1 through A4, if Θ_{YD} is greater than the negative value on the right for all non-negative k and less than the positive number on the left for all non-negative k less than or equal to K (from assumption A3), then a temporary shock that initially raises output will cause the price level to rise. This pattern of responses resembles the effects of aggregate demand shocks in the textbook macro model. But the textbook macro theory is not necessarily correct because the long-run effect of aggregate demand on output could fall anywhere within this range of positive and negative values.

If assumptions A1 through A4 are valid and the temporary shock initially moves output and price in opposite directions, then Θ_{YD} must fall outside the range of values in (23), and therefore it could not be equal to zero. Keating and Nye (1998) found that a temporary shock causes opposite movements in price and output for half of the prewar economies in their study.[21] Hence, these unusual prewar findings provide further evidence against long-run neutrality of aggregate demand in the prewar economies. Unfortunately, we can not infer whether aggregate demand's long-run effect on output is positive or negative based on that evidence alone.

From (21), the permanent positive output shock will have a negative effect on price when:

$$\frac{-\Phi_k^{PS}\Theta_{YS}}{\Phi_k^{PD}} > \Theta_{YD}\,. \tag{24}$$

Given A1, A2 and A4, (24) sets a positive upper bound on Θ_{YD} under which the permanent shock to output will cause a drop in the price level. Keating (forthcoming) addresses pre-World War I economies for which this inequality is violated (Column 5 of Table 2), and concludes Θ_{YD} is greater than zero for those economies. In postwar estimates, the permanent shock to output almost always drives the price level downward. But these negative price responses do not rule out the non-zero values of Θ_{YD} given by (24) for the postwar economies.

Finally, we determine the conditions under which the permanent shock always has a positive effect on output. As discussed earlier, this response is positive as $k \to \infty$ in (21) for all values of Θ_{YD}. Now consider finite k for which either positive or negative values of Φ_k^{YD} are possible. First, consider a positive response. That occurs for $k \in [0,K]$ based on A3, and it may also occur for some k>K. From (21), we see that:

$$\Theta_{YD} > \frac{-\Phi_k^{YS}\Theta_{YS}}{\Phi_k^{YD}} \quad \text{when} \quad \Phi_k^{YD} > 0\,. \tag{25}$$

Given A1 and the assumption that Φ_k^{YD} is positive, Θ_{YD} must be greater than the negative number in (25) to guarantee that the response of output to a permanent shock is positive.

Now consider the possibility that Φ_k^{YD} is negative. This may only occur for values of k>K. In this case:

$$\Theta_{YD} < \frac{-\Phi_k^{YS}\Theta_{YS}}{\Phi_k^{YD}} \quad \text{when} \quad \Phi_k^{YD} < 0\,. \tag{26}$$

This inequality is fundamentally different from (25) because division by a negative-valued Φ_k^{YD} reverses the inequality and makes the right-hand side a positive number. This new condition establishes a positive upper bound on Θ_{YD}. Thus (25) and (26) provide a range of positive and negative values for Θ_{YD} over which a permanent shock always has a positive effect on output, an empirical finding that is almost always obtained.

Many studies assume long-run neutrality of aggregate demand to justify a statistical model of permanent and transitory output shocks. That assumption seems to have a significant amount of credibility in the economics profession based on models used in most introductory and intermediate macroeconomics textbooks. So it was natural for economists to interpret postwar findings from a host of different studies as support for the textbook model. But, the responses to temporary and permanent shocks will appear to behave like the textbook model's aggregate demand and supply shocks, respectively, over the potentially wide range of values for Θ_{YD} established by equations (23), (24), (25) and (26). If Assumption A2 is unacceptable, then only (25), (26) and the first inequality of (23) would be relevant.[22] In either case, the qualitative features of the statistical model's impulse response functions do not serve as a reliable means of determining that the long-run neutrality assumption is valid. If Θ_{YD} is not close to zero, the impulse responses and variance decompositions from the statistical decomposition of output will almost certainly differ substantially from the effects that structural shocks have on variables.

V. Conclusions

A deeper understanding of permanent and transitory shock decompositions of output is provided by this paper. We show that even though the BQ decomposition has serious limitations when the crucial identifying restriction is not a valid structural assumption, Blanchard and Quah's (1989) statistical model may still be used to infer important facts about economic structure.[23]

This study yields two primary conclusions. The first is that additional empirical evidence supports the hypothesis that an increase (decrease) in aggregate demand had a permanent positive (negative) effect on output in pre-World War I economies. The short-run overshooting responses of output to a permanent output shock are explained by that hypothesis. The tendency for permanent shocks to explain a greater amount of variance in the prewar sample period also supports that hypothesis. In fact, the difference in variance decomposition between the two periods could be explained solely by aggregate demand having this particular type of long-run non-neutrality in prewar economies.[24] These findings add further support to the primary conclusion in Keating (forthcoming) and do so using a set of less restrictive structural assumptions.

This paper develops two conditions in which a positive (negative) shock to aggregate demand will have a permanent positive (negative) effect on output. Keating (forthcoming) developed a different condition. Table 2 summarizes the pre-World War I evidence that is pertinent to these 3 conditions. The countries in the table have data thought to be of relatively high-quality that covers a relatively long span of time during the prewar period. Evidence consistent with this particular structural hypothesis about aggregate demand is found for every one of these 10 countries. For half of them - the US, the UK, Sweden, Italy and France - all available evidence supports that hypothesis. In all but one case the evidence for Germany and Denmark is supportive. Norway, Japan and Canada each have one piece of evidence that supports $\Theta_{YD} > 0$.This accumulation of different types of evidence provides a strong case for that particular type of long-run non-neutrality in the prewar period.

The second main conclusion is that the apparent successes of BQ decompositions with postwar data do not necessarily mean non-neutralities are unimportant in that period. We have shown the BQ decomposition may appear consistent with the standard textbook model's predictions even when the long-run effect of aggregate demand on output is different from zero. It is possible that the source(s) of non-neutrality in prewar economies might still be relevant but have smaller effects in the postwar period. Improvements in policy or structural changes in modern economies may have attenuated a prewar factor.

Or perhaps the postwar economies have experienced fundamentally different types of non-neutrality.

This paper's findings call for empirical research to formally test for the specific structural mechanisms that allow aggregate demand to be non-neutral. Economic theory offers many ways that this might occur, and Table 1 summarizes important theories from that literature. Tests on prewar data are particularly important since the evidence against neutrality is strongest in that sample period. But empirical tests should also be performed on postwar data as this paper has shown the BQ decomposition is unable to detect certain types of non-neutrality when present in an economy.

Notes

1. Jim Stock presented a general overview of structural VAR methods at the 2008 NBER Summer Institute: http://www.nber.org/WNE/slides7-14-08/Lecture7.pdf which included a good introduction to the sign restrictions approach.

2. The last column in Table 2 indicates which countries obtain the puzzling result that a permanent shock to output is associated with an increase in the price level in the pre-World War I period. Section 4 in Keating and Nye (1998) argues that problems with the quality and consistency of pre-1914 data are unable to explain their findings.

3. Marginal tax rates may affect aggregate supply and aggregate demand. A shock to labor supply may affect them both if, for example, it also affects the demand for consumption.

4. In a standard New-Keynesian DSGE model Galí, López-Salido and Vallés (2003) show how the price level may rise or fall following a permanent increase in productivity depending on the policy rule.

5. We define "textbook model" to mean the class of simple economic theories found in most undergraduate textbooks that yield long-run neutrality for aggregate demand.

6. Deterministic elements were omitted without loss of generality.

7. Keating and Nye (1999) find evidence that suggests both types of aggregate supply shock have been at work in the G7 countries using Blanchard and Quah's original formulation based on the unemployment rate. One solution to this problem is to explicitly allow for multiple types of supply shocks in a model along with more variables. Shapiro and Watson (1988), King Plosser, Stock and Watson (1991), Gamber and Joutz (1993b) and Amed, Ickes, Wang and Yoo (1993) are examples.

8. Cover, Enders and Hueng (2006) present a method for identifying the model when structural shocks are correlated.

9. Most users of structural VAR models have either explicitly assumed invertibility or ignored invertibility issues altogether. However, Chari, Kehoe and McGrattan (2008) and others show that invertibility is not necessarily a feature of a structural model and examined some of the problems that may result. In response to this criticism, Christiano, Eichenbaum and Vigfusson (2007) present a more sanguine view of the structural VAR's usefulness. Fernández-Villaverde, Rubio-Ramírez, Sargent and Watson (2007) develop a general condition that can be used to check for invertibility when a system of structural equations is put into state space form, the number of structural shocks is equal to the number of variables in the structure and the VAR contains all of these variables.

10. To obtain dynamic responses of each variable to each shock, insert C(1) into the first equality from (14), solve for C_0, insert the solution for C_0 into the first equality in (12), and solve for C(L).

11. More formally, if aggregate demand shocks are long-run neutral, then $\Theta_{YD} = 0$. This condition makes $\theta(1)$ lower triangular. If $\theta(1)$ is lower triangular, equation (16) yields $C(1) = \theta(1)$ because the lower triangular factor of a symmetric matrix is unique according to Hamilton (1994, p.91). Hence, Blanchard and Quah's statistical model and the structure are identical in this case.

12. The positive solution for C_{YP} is chosen since economists almost always report responses of variables to a permanent positive output shock. In practice, the solution for C_{PT} can also be chosen so that the immediate effect on output of a temporary shock is positive.

13. And recently, Bai, Ríos-Rull and Storesletten (2012) have developed an model of search frictions. They find that shocks to consumption or investment demand can have a productive role. They show that demand shocks explain more output variance than either the technology shocks or the shocks to the marginal rate of substitution in an otherwise fairly standard RBC model. Their search friction prevents a perfect matching between producers and consumers. Greater demand leads to more intensive search which increases the number of successful matches and thus causes more output to be produced. One thing that remains to be determined is whether or not demand shocks in their model will raise or lower the price level given that they now also serve as productivity shocks. If demand shocks cause the price level to fall, then that would provide another rationale for investigating the long-run effect of aggregate demand on prewar economies using the results we develop in this paper which require no assumptions about price level responses to structural shocks.

14. We could have also allowed $\Phi_k^{PD} = 0$ for small k so that prices could be sticky for a year (or more) in response to aggregate demand. If there is no initial effect on price and A2 holds, then a permanent increase in output unambiguously lowers the price level for small k. There is some evidence that prices are sticky in the postwar, and in nearly all postwar estimates the price level falls with a permanent increase in output. Furthermore, in 8 of 10 prewar countries price immediately rises with a permanent increase in output (Column 5 Table 2). Thus, these prewar findings are not consistent with $\Phi_k^{PD} = 0$ if A2 is valid. These different price responses are consistent with the view of some economists that price adjustment was relatively fast in the prewar and relatively slow in the postwar. See Calomiris and Hubbard (1989) for a discussion and references.

15. Impulse responses for the US were reported in the body of Keating and Nye (1998). Responses for all countries were in an appendix to the working paper. The appendix was removed from the published version. These responses are available on request.

16. Francis and Ramey found overshooting even when the estimate their model over a broader sample period that includes the prewar along with subsequent years.

17. The results in Gamber and Joutz (1993a) suggest using data of higher frequency in Blanchard and Quah's decomposition may raise the variance of output explained by permanent shocks. Keating and Nye (1999) used quarterly data in postwar estimates and annual data in prewar models. If that implication of Gamber and Joutz's generally held, then Keating and Nye's estimates would be biased toward finding greater output variance explained by permanent shocks in the postwar period. Thus that paper’s finding that permanent shocks explain more output variance in the prewar period can not be explained by the use of postwar data of higher frequency.

18. The Appendix shows that certain negative values of Θ_{YD} may, in principle, explain the variance relationship in Proposition 2. It then shows that these negative values can be ruled out if a permanent shock always has a positive effect on output. The empirical results of Keating and Nye (1998,1999) provide overwhelming support for this assumption.

19. Some economists believe attempts at fine tuning an economy may actually be destabilizing. Lags in the response of the economy to policy as emphasized by Friedman (1961), for example, may be the cause of this instability. This suggests more activist postwar policies may have raised the amount of output variance explained by aggregate demand shocks relative to the prewar period. While this story is able to explain the difference between prewar and postwar variances, it is unable to explain why a permanent shock to output causes short-run overshooting of output or why this shock raises the price level in many prewar economies.

20. We have not tried to prove that as Θ_{YD} approaches zero in the postwar economy something akin to Proposition 2 will hold, but this seems obvious. A proof of that might be similar in some ways to Blanchard and Quah's proof that as Θ_{YD} approaches zero the permanent and transitory shock decomposition approaches the structural decomposition of aggregate supply and demand shocks. Thus the gain from proving this result is modest while the expected cost seems much higher. The proof would be longer and probably more tedious than the lengthy proof of Proposition 2 in this paper's appendix. And, it may also require auxiliary assumptions that are difficult to justify.

21. They obtain this finding for the UK, Norway, Italy, France and Canada in the prewar but for none of the ten developed countries in the postwar. However, Morling (2002) obtains a similar empirical result for many developing economies in the postwar period. This evidence suggests that aggregate demand is more likely to have a permanent effect on output in less developed or in developing economies than in a country that is highly developed by modern standards.

22. And with fewer inequalities involved, a possibly larger range of values for Θ_{YD} allows the statistical model to obtain impulse response that are qualitatively consistent with the textbook model.

23. The analysis in this paper has only been applied to Blanchard and Quah-style decompositions. It would be interesting to see if qualitative structural assumptions can be used to structurally interpret a decomposition of permanent and transitory shocks based on unobserved components. As Morley, Nelson and Zivot (2003) show these two decompositions can in some ways perform very differently.

24. This explanation for variance decompositions is appealing in light of Occam's Razor which asserts that a simpler theory is preferable unless an alternative theory with greater explanatory power exists. Our aggregate demand hypothesis is simple because it depends only on the sign of a single long-run structural parameter. And this hypothesis can explain, in addition to the different variance decompositions, the other empirical regularities we have discussed.

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Table 1: Potential Long-Run Effects of Aggregate Demand on Output

Non-Superneutrality	A permanent increase in the growth rate of money raises or lowers output in the long run depending on certain features in the model. Mundell (1963), Tobin (1965), Orphanides and Solow (1990)
Long-Run Fiscal Policy Effects	An increase in government spending crowds-out or crowds-in investment in the long-run. This ultimately influences long-run aggregate supply by its effect on the stock of capital. Baxter and King (1993)
Hysteresis	The natural unemployment rate depends on past levels of the unemployment rate. This allows aggregate demand to affect the natural rate, and as a result full employment output is affected. Blanchard and Summers (1986), Ball (1999)
Coordination Failure	Coordination problems yield multiple equilibria, allowing aggregate demand to possibly affect the equilibrium level of output. Cooper and John (1988), Cooper (1999)
Deflation and Destabilizing Price Flexibility	Deflation raises real interest rates and that reduces interest sensitive spending. More flexible price adjustment exacerbates this effect. Fisher (1933), DeLong and Summers (1986), Tobin (1993)

Table 2: Summary of Empirical Findings with Permanent Shocks to Output

(1) Country	(2) Short-run Overshooting in the Prewar Output Response to a Permanent Shock?	(3) Prewar Output Variance Explained by Permanent Shocks Exceeds Postwar Variance (Inflation Rate is the Second Variable)?	(4) Prewar Output Variance Explained by Permanent Shocks Exceeds Postwar Variance (Unemployment Rate is the Second Variable)?	(5) Positive Prewar Price Response to a Permanent Increase in Output?
United States	Y	Y	Y-initially	Y*
United Kingdom	Y	Y	Y	Y*
Sweden	Y	Y-initially	n/a	Y
Norway	N	N-virtually the same	n/a	Y*
Japan	Y	N-virtually the same	n/a	N*
Italy	Y	Y-initially	n/a	Y-initially
Germany	Y	Y	Y	N
France	Y	Y	n/a	Y*-initially
Denmark	N	Y	n/a	Y*
Canada	N	N-virtually the same	n/a	Y*

Notes:
A: Y=yes; N=no; Occasionally additional information is provided.
B: Comparison of prewar and postwar variances in the third column was done only with samples in which VAR coefficients are stable according to an Andrews and Ploberger (1994) test.
C: n/a in the fourth column indicates when a country's prewar unemployment rate series was either unavailable or of inadequate length.
D: * in the fifth column indicates a price level impulse response to the permanent shock that is statistically significant at the 5% level.

Sources:
Columns 2,3 and 5: Keating and Nye (1998).
Column 4: Keating and Nye (1999). This paper also finds short-run overshooting for the US, the UK and Germany supporting the results in Column 2 for each of these countries.

Appendix: Proof of Proposition 2.

Structural assumptions:

I. In Economy A the textbook aggregate demand and supply model describes the structure: $\Theta_{YD} = 0$ and the BQ decomposition identifies structural effects.

II. In Economy B aggregate demand shocks have permanent effects on output: $\Theta_{YD} \neq 0$ and the permanent-transitory shock decomposition is unable to identify effects of aggregate demand and supply.

III. Otherwise both economies are identical, and so Φ_j^{ik} in Economy A is the same as Φ_j^{ik} in Economy B for finite $j \leq J$ (where J must be at least as large as K from Assumption A3).

Calculate the k-step forecast error for each economy. For Economy A, the decomposition of permanent and transitory shocks identifies the structure, and so equation (2) is used to derive the k-step forecast error:

$$X_t - E_{t-k}X_t = \sum_{j=0}^{k-1} \Phi_j \varepsilon_{t-j} = \sum_{j=0}^{k-1} \begin{bmatrix} \Phi_j^{YS} & \Phi_j^{YD} \\ \Phi_j^{PS} & \Phi_j^{PD} \end{bmatrix} \begin{bmatrix} \varepsilon_{t-j}^{S} \\ \varepsilon_{t-j}^{D} \end{bmatrix}. \tag{i}$$

For economy B, the permanent and transitory shock decomposition fails to identify the structure because aggregate demand has a permanent effect on output. In this case, equation (21) is used to determine the k-step forecast error:

$$X_t - E_{t-k}X_t = \sum_{j=0}^{k-1} \Phi_j \theta(1) C(1)^{-1} \mu_t = \frac{\sum_{j=0}^{k-1} \begin{bmatrix} \Phi_j^{YS} & \Phi_j^{YD} \\ \Phi_j^{PS} & \Phi_j^{PD} \end{bmatrix} \begin{bmatrix} \Theta_{YS} & -\Theta_{YD} \\ \Theta_{YD} & \Theta_{YS} \end{bmatrix} \begin{bmatrix} \mu_{t-j}^{P} \\ \mu_{t-j}^{T} \end{bmatrix}}{\left(\Theta_{YS}^2 + \Theta_{YD}^2\right)^{1/2}}. \tag{ii}$$

Since these two economies are different in the long run, the Φ_j parameters can't be the same for all possible j. Given Assumption III, however, the Φ_j parameters are the same for j=0,1,2,...,k-1.

The fraction of k-step forecast error variance for output associated with permanent shocks in Economy A is obtained from equation (i):

$$\frac{\sum_{j=0}^{k-1} (\Phi_j^{YS})^2}{\sum_{j=0}^{k-1} (\Phi_j^{YS})^2 + (\Phi_j^{YD})^2}, \tag{iii}$$

and for Economy B it is obtained from equation (ii):

$$\frac{\sum_{j=0}^{k-1} \left(\Phi_j^{YS}\Theta_{YS}\right)^2 + \left(\Phi_j^{YD}\Theta_{YD}\right)^2 + 2\Phi_j^{YS}\Theta_{YS}\Phi_j^{YD}\Theta_{YD}}{\sum_{j=0}^{k-1} \left(\left(\Phi_j^{YS}\right)^2 + \left(\Phi_j^{YD}\right)^2\right)\left(\Theta_{YS}^2 + \Theta_{YD}^2\right)}. \tag{iv}$$

What conditions on Θ_{YD} guarantee that permanent shocks explain a larger fraction of output variance in Economy B? Multiplying by positive denominators, collecting terms and simplifying, one can show that equation (iv) is greater than equation (iii) when:

$$2\Theta_{YS}\Theta_{YD}\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD} + \Theta_{YD}^2\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right] \;>\; 0$$

It is convenient to divide by Θ_{YS} squared, and factor the expression as follows:

$$\left(\frac{\Theta_{YD}}{\Theta_{YS}}\right)\left\{2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD} + \left(\frac{\Theta_{YD}}{\Theta_{YS}}\right)\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]\right\} \;>\; 0. \qquad \text{(v)}$$

From assumptions A1 and A3 we know that $\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}$ is positive. However, $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]$ may be zero, negative or positive depending on the relative importance of supply and demand shocks for output during the first k periods. Consider each of these cases:

1. If $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right] = 0$, $\frac{\Theta_{YD}}{\Theta_{YS}} \;>\; 0$ satisfies (v);

2. If $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right] < 0$, $0 < \frac{\Theta_{YD}}{\Theta_{YS}} \;<\; \frac{-2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}}{\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]} \;>\; 0$ satisfies (v);

3. If $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right] > 0$, (v) is satisfied by either:

$$\frac{\Theta_{YD}}{\Theta_{YS}} > 0$$

$$\text{OR} \qquad \frac{\Theta_{YD}}{\Theta_{YS}} \;<\; \frac{-2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}}{\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]} \;<\; 0.$$

In 3 of these 4 conditions, $\frac{\Theta_{YD}}{\Theta_{YS}}$ is positive, and therefore Θ_{YD} is positive. However, the fourth condition is satisfied by negative values for Θ_{YD}. If we can show that these negative Θ_{YD} values are irrelevant, that will complete a proof that Θ_{YD} is positive.

We will show that these negative values can be ruled out by assuming that permanent output shocks always have a positive effect on output. This is true for all model estimates in Keating and Nye

(1998), with the exception of Japan using the 1972 to 1994 sample period. In that exceptional case, the largest root in the VAR is complex and explosive and so the output response will end up negative at some point with probability one. This unusual response suggests some type of specification error is involved and so we discount that finding (But even here the response of Japan's output to a permanent shock is positive initially and for 6 years after, so the following result holds in that sample for $k \leq 6$).

Assuming the output response to a permanent shock is always positive places a lower bound on Θ_{YD}, a negative number that we will show is not as negative as the values of Θ_{YD} in the last condition for case 3. Hence, the negative values of Θ_{YD} that satisfy (v) are so negative that a permanent shock would cause output to fall at some point in the impulse response, a condition that is ruled out by the evidence.

The response of output to a permanent shock is given by equation (22), and if the responses for the first k periods are positive this equation implies:

$$\frac{\Theta_{YD}}{\Theta_{YS}} > \frac{-\Phi_j^{YS}}{\Phi_j^{YD}} \quad \text{for } j=0,1,\ldots,k.$$

Let $\rho_j = \dfrac{\Phi_j^{YS}}{\Phi_j^{YD}}$. Each ρ_j is positive because of assumptions A1 and A3. Define ρ_* as the minimum over all ρ_j for j = 0, 1, 2,..., k-1. Based on the previous inequality we can see that if $\dfrac{\Theta_{YD}}{\Theta_{YS}}$ were smaller than $-\rho_*$, at least one of output's responses to a permanent shock would be negative. If output does not fall in response to a permanent shock, $-\rho_*$ sets a lower bound for $\dfrac{\Theta_{YD}}{\Theta_{YS}}$. Therefore, the negative values in case 3 above are ruled out if:

$$\frac{-2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}}{\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2-\left(\Phi_j^{YS}\right)^2\right]} < -\rho_* . \tag{vi}$$

To show that (vi) must hold, use the definition of ρ_j to eliminate Φ_j^{YS} and, given that $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2-\left(\Phi_j^{YS}\right)^2\right]$ is positive in this case, we can manipulate (vi) into the following inequality:

$$\sum_{j=0}^{k-1}(2\rho_j - \rho_* + \rho_*\rho_j^2)\left(\Phi_j^{YD}\right)^2 \quad > \quad 0.$$

This inequality holds because $\rho_j > 0$ and $\rho_j \geq \rho_*$ for all j. Thus (vi) holds and this rules out all the cases of $\Theta_{YD} < 0$ that satisfy (v). This completes a proof of Proposition 2.