Price Versus Financial Stability: A role for money in Taylor rules[☆]

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Abstract

This paper analyzes optimal monetary policy in a standard New-Keynesian model augmented with a financial sector. The banks in the model are subject to shocks which impede their ability and willingness to produce financial assets. We show these financial market supply shocks decrease both the natural rates of output and interest. The implication is that an optimizing central bank with real time data on only inflation, output, interest rate spreads and monetary aggregates will respond positively to the growth rate of monetary aggregates which signal movement in the natural rate from these financial shocks. This simple rule is implementable by central banks as it makes the policy instrument a function of only observables and does not require precise knowledge of the model or the parameters. The key is the use of the Divisia monetary aggregate which provides a *parameter- and estimation- free* approximation to the the true monetary aggregate. We show policy rules reacting to the Divisia monetary aggregate have well-behaved determinacy properties - satisfying a novel Taylor principle for monetary aggregates. Finally, we conclude with a minimax *robust* policy prescription given the uncertainty surrounding parameters driving the financial and other structural shocks.

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JEL Codes: C43, E32, E41, E44, E51, E52, E58, E60

1. Introduction

The New-Keynesian sticky-price (NKSP) framework has become the workhorse model for monetary policy evaluation due to its ability to provide a relevant role for monetary policy while avoiding the well known Lucas critique. The research

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on optimal monetary policy in these models has reached a clear consensus on two fronts. First, price stability is a paramount policy concern while output should only be stabilized at its flexible price level. Second, money is an inferior policy instrument due to persistent money demand shocks and offers no information regarding the natural rates of output or interest. These conclusions, which are robust across many model specifications, have led both academics and policy markers towards "cashless models" of the economy and the monetary transmission mechanism. The result, which is emphasized in Woodford (2003), is that optimal policy can typically be described by an interest rate rule which reacts to the natural rate of interest, inflation and the deviation of output from its flexible price (or natural) level . Cúrdia and Woodford (2009) show this description of optimal policy extends even to a NKSP model with a financial sector for the appropriately defined natural rate of interest.

However, in practice, policy makers know very little about the natural rate of output and interest in real time. This has led researchers to consider optimal simple policy rules which central banks could actually adopt. A rule is considered *simple* (Gali, 2008) if:

1. It makes the policy instrument a function of only observable variables
2. It does not require knowledge of the correct model
3. It does not require knowledge of model specific parameters

The desire to develop simple rules has led researchers to examine interest rate rules which react to inflation and output growth, variables which are readily available at quarterly frequencies, though sometimes with error. The optimality of such simple rules, usually with no output growth response, has been verified in NKSP models under a wide array of model specifications. Simply put, an interest rate rule responding to inflation appears to be the best available simple rule (Schmitt-Grohe and Uribe, 2007).

Relatively less is known about optimal *simple* rules in models with financial sectors. To date though, the literature on optimal simple rules in such models has often found no need to deviate from inflation targeting interest rate rules. For example, in a model with financial frictions, Faia and Monacelli (2007) find that responding to asset prices offers welfare gains when the inflation coefficient is fixed at Taylor's (1993) original values but these gains are eliminated when the inflation coefficient is increased.[3] Similarly, Dellas, Diba, and Loisel (2010) find that reacting to inflation of non-financial products offers the optimal simple rule in a setting where banks are subject to supply shocks. However, none of these models which feature financial sectors have examined the usefulness of monetary aggregates in simple rules.

If money is to serve a useful role in such contexts it seems most likely to do so as an informational variable regarding the natural rate of interest. The

[3]Cúrdia and Woodford (2010) provide similar evidence that such models may call for augmenting the standard Taylor rule (with output and inflation coefficients fixed at Taylor's (1993) original values) with a reaction to the interest rate spread, although fully optimized simple rules are not analyzed.

desire for "simple rules" and the readily available data on monetary aggregates at high frequencies suggests that this is a promising chance to expand the set of useful information to policy makers in real time. However, under typical money demand specifications, money is determined by output, inflation, a single interest rate and money demand shocks.

Two things are worth noting under such a specification. First, money provides no additional information to policy makers regarding the natural rates of output and interest not already contained in output, inflation and policy rate data. Second, such a specification is completely at odds with reality for broader monetary aggregates - inside money - whose equilibrium is determined by both *money demand* which depends on a vector of interest rate spreads and *money supply* by financial institutions and the shocks which originate therein. Hence, under more realistic descriptions, movements in broad monetary aggregates are driven by movements in non-policy rates and financial firms ability and willingness to supply monetary assets.

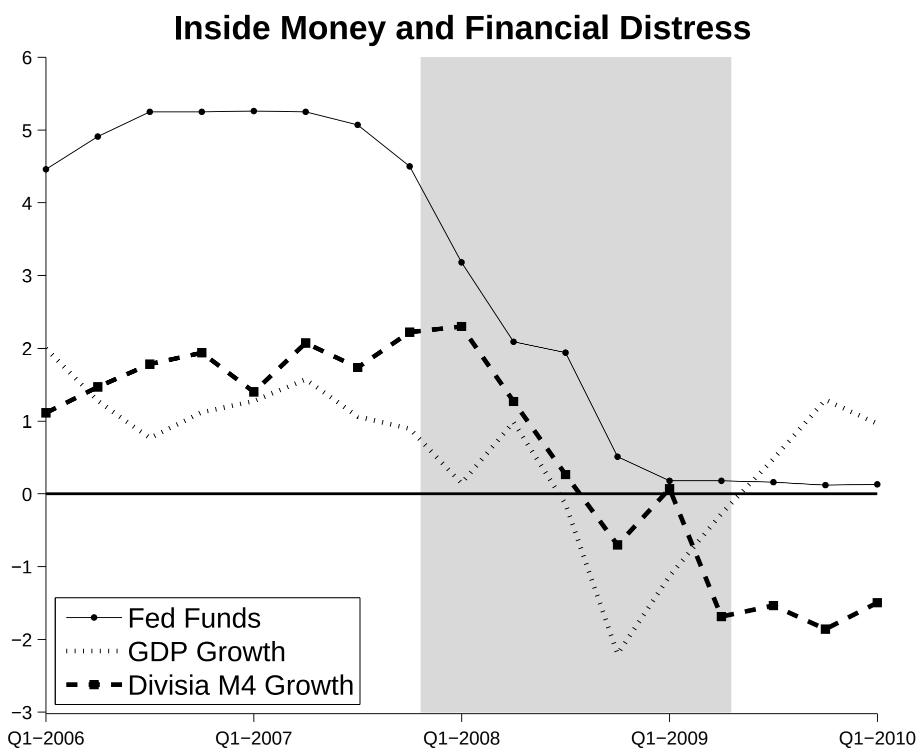

Figure 1: The shaded area marks the NBER recession dates. GDP and Divisia M4 growth rates are quarterly percentages while the Fed Funds rate is in annualized percentage points.

This point is apparent in the graph above which shows the growth rate of Divisia M4 along with the Federal Funds (policy) rate and output growth during the recent financial crisis. Typical money demand specifications described above would have a hard time explaining why money growth slowed while the Fed was cutting interest rates. One possibility is that weak output growth was suppressing the demand for money, however, this story falls apart in in 2009 when output growth began rising but money growth weakened further. While suggestive, this graph provides some intuition into the behavior of inside money when the economy is subjected to financial market disturbances, thus motivating the question of what (if any) information can policy makers glean from broad monetary aggregates? In particular we ask, is there an exploitable relationship between broad money growth and non-observable variables such as the natural rate of interest when the economy is subject to financial shocks?

1.1. Related Literature

We are not the first to explore the usefulness of money in Taylor rules. Berger and Weber (2012) explore the relationship between a variable they define as the money gap (the difference between the equilibrium quantity of money and estimated money demand) and the natural interest rate in a prototypical NKSP model. They find that with noisy output gap information the optimal money gap response is positive. Our work differs from theirs in at least three regards. First, we search for *simple* rules which as defined above avoid the need to use model specific variables nor estimate any model parameters. This point is especially important since we explore the usefulness of monetary *aggregates* in Taylor rules for which we employ the use of *parameter- and estimation- free* aggregation methods. Second, we examine optimal monetary policy around a largely distorted steady state which requires a second order accurate approximation to the model's equilibrium condition as in Schmitt-Grohe and Uribe (2004). Finally, we perform a complete analysis including numerically examining determinacy regions for Taylor rules which react to Divisia monetary aggregate growth rates and finding a *robustly* optimal policy rule under parametric uncertainty as in Giannoni (2002).

A second related paper is by Andrés, López-Salido, and Nelson (2009) who explore the empirical linkages between dynamic money demand and the natural rate of interest. They find an empirical relationship and suggest the possibility of exploiting this relationship in optimal monetary policy, although they leave this for future work.

McCallum and Nelson (2011) also examine the link between monetary *aggregates* and the natural rate of interest. They argue that the information in monetary aggregates could be used to draw inference on the natural rate of interest provided the demand for the monetary aggregate depends on a vector of interest rates. For example, assuming aggregate demand depends on these real yields, then movements in these rates will affect the real policy rate consistent with stable prices - the natural rate of interest. As they describe, movements in these nominal yields will reflect movements in real yields aside from the policy rate causing the quantity of money to co-move with the natural rate of interest. This is exactly the path we pursue in this work. We spell out this relationship more carefully and provide an implementable *simple* interest rate rule which could be put to use by central banks without knowledge of model specific variables nor structural parameters.

1.2. Outline

The rest of this paper will proceed as follows. Next, we present the New-Keynesian model developed by Belongia and Ireland (2012) to include a role for monetary aggregates and financial firms. Then we define the natural rates of interest and output in the model and show that the financial market disturbances in the model decreases the natural rate of interest which call for countercyclical monetary policy. We then examine the performance of optimal "simple" interest rate rules using a micro-founded welfare metric and a second order approximation to the model's equilibrium conditions. With realistic assumptions regarding the monetary authority's information set we show that reacting to inflation alone, which is typically optimal, fails to respond counter-cyclically to financial market disturbances leading to poor welfare performance.

Instead optimal policy chooses to also react to the growth rate of Divisia monetary aggregates which provide parameter- and estimation- free approximations to the true aggregate. We show that policy rules with a positive Divisia growth response have well-behaved determinacy properties satisfying a novel Taylor principle for monetary aggregates. Moreover, such rules bring about a real policy rate which is highly correlated with the natural rate and maximizes welfare. Interestingly, spread-adjusted Taylor rules are also highly correlated with the natural rate but perform poorly from a welfare standpoint. We offer some insight into this counter-intuitive result by showing that reacting to the interest rate spread fails to provide sufficient liquidity following an adverse financial shock and ultimately induces unwanted volatility in inflation . Finally, we end the paper with a robust policy prescription using a minimax approach, as in Giannoni (2002), given the parameters driving financial and other stochastic shocks remain uncertain.

2. Model

The model used in this analysis was developed by Belongia and Ireland (2012). It's a standard sticky-price New-Keynesian model with the addition of a financial sector comprised of perfectly competitive financial firms. The banks produce interest bearing deposits and loans which requires a varying amount of real resources according to a stochastic process. As is typical in these models, production of the final good requires inputs from intermediate goods producers who have market power and hence can set their price given demand. The presence of quadratic adjustment costs makes prices sticky which in turn makes monetary policy relevant. Finally, the representative household works for, and holds stock in, the intermediate goods firms. The household also demands loans and deposits from the financial firms which depend on the vector of interest rates the household faces.

2.1. The Representative Household

The representative household enters any period $t = 0, 1, 2, ...$ with a portfolio consisting of 3 assets. The household holds maturing bonds B_{t-1}, shares of monopolistically competitive firm $i \in [0,1]$ $s_{t-1}(i)$, and currency totaling M_{t-1}. The timing of transactions requires the household to carefully manage its portfolio of these assets. Central to this is the household's interaction with the representative bank with whom it makes deposits and takes loans. The reason the household deposits and borrows money from the bank at the same time is motivated in part by the description of the typical period t household budget constraints as described by Belongia and Ireland (2012). This budgeting can be described by dividing period t into 2 separate periods: first a securities trading session and then a transactions session.

2.1.1. Securities Trading Session

In the first part of period t the household purchases new securities which consists of bonds B_t which pay one nominal unit of currency in period t for price $1/r_t$, where r_t is the gross nominal rate of interest, and shares of monopolistically competitive firm i, $s_t(i)$ for a price of $Q_t(i)$ per share. In this first portion of period t the household also acquires the liquidity needed for the transactions period by securing loans from the representative bank totaling L_t. The household ends this securities trading session by allocating its loans and the currency remaining after trading securities between deposits and currency. Since deposits pay interest they dominate currency in return, however the household will hold currency in equilibrium due to the increased liquidity currency offers. The timing of these transactions is summarized in the securities trading session budget constraint in (1) below.

$$D_t + N_t = M_{t-1} + B_{t-1} - \int_0^1 Q_t(i)(s_t(i) - s_{t-1}(i))di - B_t/r_t + L_t \qquad (1)$$

2.1.2. Transactions Session

In the second portion of period t the household's deposits mature yielding $r_t^D D_t$ units of currency which are then added to the currency the household set aside at the end of the securities trading session - N_t. The household adds to this currency by supplying h_t total hours of labor to intermediate goods producing firm for a nominal wage rate $P_t W_t$. At the same time each intermediate goods producing firm $i \in [0,1]$ makes a dividend payment of $F_t(i)$ for each share owned by the household. The household must also pay back to the bank all loans with interest totaling $r_t^L L_t$. The household then optimally allocates the remaining currency between consumption goods $P_t C_t$ and currency to be carried into next period M_t. These activities are summarized in the transactions session budget constraint in (2) below.

$$M_t = N_t - P_t C_t + W_t h_t + \int_0^1 F_t(i)s_t(i)di + r_t^D D_t - r_t^L L_t. \qquad (2)$$

2.1.3. Household Preferences

The true monetary aggregate which enters the household's reduced from utility function is given by the CES aggregator

$$M_t^A = \left[\nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{3}$$

where ν calibrates the relative expenditure shares on currency and deposits and ω calibrates the elasticity of substitution between the two monetary assets. In general, we need only assume that the monetary aggregate is block-wise weakly separable within the household's utility function. The approach taken by Belongia and Ireland (2012) it to specify a shopping time friction of the form

$$h_t^s = \frac{1}{\chi}\left(\frac{\upsilon_t P_t C_t}{M_t^A}\right)^{\chi} \tag{4}$$

where υ_t is a shock to the demand for monetary services following a first order autoregressive process (in logs)

$$ln(\upsilon_t) = (1-\rho_\upsilon)ln(\upsilon) + \rho_\upsilon ln(\upsilon_{t-1}) + \varepsilon_t^\upsilon where\ \varepsilon_t^\upsilon \sim i.i.d.\ (0, \sigma_\upsilon^2). \tag{5}$$

We take this as our baseline calibration however the non additive-separability of the monetary aggregate implies a real balance affect which has been challenged empirically (See for example Ireland (2004a)). To show that our results do not hinge on this feature of the household's preferences we also consider the possibility that the monetary aggregate enters the household's utility function in an additively-separable form so that the term

$$\eta_m \upsilon_t ln\left(\frac{M_t^A}{P_t}\right)$$

is added to the household's utility function over consumption and leisure. In either case we can define the household's preferences recursively by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[ln(C_t) - \eta(h_t + h_t^s)\right]$$

or

$$U = E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[ln(C_t) - \eta h_t + \eta_m \upsilon_t ln\left(\frac{M_t^A}{P_t}\right)\right]$$

where a_t is a preference shock which follows a first order auto-regressive process (in logs)

$$ln(a_t) = \rho_a ln(a_{t-1}) + \varepsilon_t^a where\ \varepsilon_t^a \sim i.i.d.\ (0, \sigma_a^2). \tag{6}$$

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. The household's optimization problem and the resulting first order necessary conditions are given in the appendix using Bellman's equation.

2.2. The Representative Financial Firm

The representative bank creates demand deposits and originates loans for the representative household in a purely competitive market. Specifically, in period $t = 0, 1, 2, \ldots$, the representative bank creates interest bearing deposits in the amount D_t and originates loans in the amount L_t. The pure-competition assumption implies the representative bank takes as given the gross nominal interest rate it pays on deposits r_t^D and the gross nominal interest rate it charges on loans r_t^L. The bank not only pays interest on deposits but also bears a time-varying real cost $c_t(\frac{D_t}{P_t})$ in order to create and service deposits defined by

$$c_t(\frac{D_t}{P_t}) = x_t \frac{D_t}{P_t}.$$

The x_t term is what makes the cost of producing deposits time varying, and in this case stochastic, as this deposit cost function evolves according to the first order auto-regressive process (in logs)

$$ln(x_t) = (1 - \rho_x) ln(x) + \rho_x ln(x_{t-1}) + \varepsilon_t^x where\ \varepsilon_t^x \sim i.i.d.\ N(0, \sigma_x^2). \tag{7}$$

The representative bank is also subject to the balance sheet constraint defined by the identity,

$$L_t = (1 - \tau_t) D_t \tag{8}$$

where τ_t represents reserves held by the bank. In this model, the banks demand for reserves varies stochastically according the first order auto-regressive process (in logs)

$$ln(\tau_t) = (1 - \rho_\tau) ln(\tau) + \rho_\tau ln(\tau_{t-1}) + \varepsilon_t^\tau where\ \varepsilon_t^\tau \sim i.i.d.\ N(0, \sigma_\tau^2). \tag{9}$$

Taking x_t and τ_t as given, the profit maximization problem facing the representative bank is defined by

$$\max_{D_t, L_t} \Pi_t^B = (r_t^L - 1) L_t - (r_t^D - 1) D_t - P_t x_t \frac{D_t}{P_t}\ subject\, to\, (8)$$

Substituting (8) into the objective function is a simple way to express the bank's problem and leads to the first order necessary condition for profit maximization

$$r_t^D = 1 + (r_t^L - 1)(1 - \tau_t) - x_t. \tag{10}$$

or

$$\mathcal{S}_t = 1 + (r_t^L - r_t^D) = 1 + r_t^L (1 - \tau_t) + x_t \tag{11}$$

Equation (11) shows that the (gross) spread between deposits and loans varies endogenously according to the market determined loan rate and varies exogenously according to the banks demand for reserves and the marginal cost of producing deposits. The exogenous process for reserves demand simulates times of financial distress when banks choose to decrease lending activity and hoard deposits. The deposit cost shock acts to simulate negative banking productivity shocks effectively raising the bank's marginal cost of producing deposits. Just as in Cúrdia and Woodford (2009), the existence of a time-varying loan-deposit spread has implications for the natural rate of interest[4] and ultimately optimal monetary policy.

2.3. The Representative Final Goods Producing Firm

The representative final goods producing firm maximizes period t profits for $t = 0, 1, 2, ...$ using a constant returns CES technology defined by

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{12}$$

where $Y_t(i)$ is an input from intermediate goods producing firm $i's$ output. As is standard, we assume the final goods market is purely competitive leaving the representative final goods producing firm with no market power. Hence, behaving purely as a price taker in both the output market P_t and the input markets $P_t(i)\,\forall\, i \in [0,1]$ the representative final goods producing firm solves

$$\max_{Y_t, \{Y_t(i)\}_{i\in[0,1]}} \Pi_t^F = P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \tag{13}$$

subject to (12). This constrained maximization problem can easily be transformed into an unconstrained problem by substituting the constraint into the objective function to eliminate the choice variable Y_t. The resulting first order necessary condition defines the factor demand for each input $Y_t(i)$ as

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \tag{14}$$

$\forall\, i \in [0,1]$.

2.4. The Representative Intermediate Goods Producing Firm

Unlike the final goods market the intermediate goods market is not purely competitive. Instead each intermediate goods producing firm $i \in [0,1]$ produces a differentiated product leading to some degree of market power. To permit aggregation and allow for the consideration of a representative intermediate goods producing firm i, we assume all such firms have the same constant returns

[4] See for example within their paper the natural rate of interest under financial frictions denoted $r_t^{n,FF}$.

to scale technology which implies linearity in the single input labor $h_t(i)$,

$$Y_t(i) = Z_t h_t(i). \tag{15}$$

In each period $t = 0, 1, 2, ...$ the representative intermediate goods producing firm rents $h_t(i)$ units of labor from the representative household for a nominal market determined wage rate, $P_t W_t$. The Z_t term in (15) is an aggregate technology shock that follows a random walk with drift (in logs)

$$ln(Z_t) = ln(Z) + ln(Z_{t-1}) + \varepsilon_t^z where\ \varepsilon_t^z \sim i.i.d.\ (0, \sigma_z^2). \tag{16}$$

The market power of each intermediate goods producing firm i leads to the ability for each firm to set the price $P_t(i)$ of its output $Y_t(i)$ each period t. The price setting ability of each firm is constrained in two ways. First, each intermediate goods producing firm faces a demand for its product from the representative final goods producing firm defined in (14). Second, each intermediate goods producing firm faces a convex cost of price adjustment proportional one nominal unit of the final good defined by Rotemberg (1982) to take the form

$$\Phi(P_t(i), P_{t-1}(i), P_t, Y_t) = \frac{\phi}{2}\left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1\right]^2 Y_t P_t. \tag{17}$$

Every intermediate goods producing firm $i \in [0, 1]$ maximizes its period t real price per share denoted by $\frac{Q_t(i)}{P_t}$. Though the firm maximizes period t share price, the costly price adjustment constraint makes the intermediate goods producing firm's problem dynamic (and recursive) as shown in the appendix. Mathematically summarizing, each intermediate goods producing firm solves to following dynamic problem,

$$\max_{\{h_t(i), P_t(i)\}_{t=0}^{\infty}} \frac{Q_t(i)}{P_t}$$

subject to the constraints (14), (15) and (17). In a symmetric equilibrium the log-linearized first order condition of the above problem takes the form of a New-Keynesian Phillips Curve (NKPC) relating current inflation to the average real marginal cost and expected future inflation. The resulting NKPC can be calibrated to match the NKPC derived from Calvo style price adjustment based on the frequency of price changes[5].

2.5. The Central Bank

We close the model by specifying the general class of monetary policy rules we consider by

[5]However, the two pricing assumptions will in general result in different NKPCs up to a second order approximation. This difference will generally lead to different welfare-loss functions when approximated around a distorted steady-state as shown in Lombardo and Vestin (2008).

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y (\tilde{Y}_t - \tilde{Y}_{t-1}) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{\mathcal{S}}_t \tag{18}$$

in log-deviations from steady state[6] with ρ_r in $[0, 1]$ and ϕ_π, ϕ_y, ϕ_m and ϕ_s in $[0, \infty)$ and $\mathcal{S} = r_t^L - r_t^D$. The policy rule is restricted to be both linear in logs and react only to observable model non-specific variables. The second restriction is key for the policy rule to be implementable as stressed in Schmitt-Grohe and Uribe (2007) and Faia and Monacelli (2007). For this reason we include the growth rate of output instead of deviations of output from its natural level.[7] Orphanides (2003) stresses the latter is not available to policy makers in real time without significant measurement error. For robustness, we also examine the case when the efficient level of output is available to policy makers in real time (See section 4.2.3). The implementability restriction also requires that we provide a measure of the monetary aggregate that doesn't require knowledge of its functional form nor the values of its parameters. We discuss this issue below.

2.5.1. The Monetary Aggregation Problem

The general problem of tracking an unknown aggregator function without estimation is not new. The solution lies in statistical index number theory as advanced by Diewert (1976) and specifically applied to monetary aggregation by Barnett (1978, 1980). The focus of this field is to provide parameter- and estimation-free aggregates. One such index number performs this task with a known level of accuracy. The Divisia monetary aggregate provides a second-order accurate approximation to the growth rate of M_t^A.[8] We now define the Divisia monetary aggregate in this model.

Definition 1. The growth rate of the Divisia monetary aggregate is defined by

$$ln(\mu_t^{Divisia}) = \left(\frac{s_t^N + s_{t-1}^N}{2}\right) ln\left(\frac{N_t}{N_{t-1}}\right) + \left(\frac{s_t^D + s_{t-1}^D}{2}\right) ln\left(\frac{D_t}{D_{t-1}}\right) \tag{19}$$

where s_t^N and s_t^D are the expenditure shares of currency and interest bearing deposits respectively defined by

$$s_t^N = \frac{u_t^N N_t}{u_t^N N_t + u_t^D D_t} = \frac{(r_t^L - 1)N_t}{(r_t^L - 1)N_t + (r_t^L - r_t^D)D_t} \tag{20}$$

and

$$s_t^D = \frac{u_t^D D_t}{u_t^N N_t + u_t^D D_t} = \frac{(r_t^L - r_t^D)D_t}{(r_t^L - 1)N_t + (r_t^L - r_t^D)D_t}. \tag{21}$$

Since the definition of the Divisia monetary aggregate only requires knowledge of current and one period lagged monetary component quantities and in-

[6] To be clear, for any variable X_t, $\tilde{X}_t = ln(X_t) - ln(\bar{X})$.

[7] This natural level of output is defined in section 3 below.

[8] The ability to track any function which is homogeneous of degree one (as all sensible aggregator functions are) to second order accuracy places the Divisia aggregate in Diewert's (1976) class of *superlative index numbers*.

terest rates our policy rule specified in (18) is a simple rule which could actually be implemented by central banks facing limited real time information. For example in the U.S., the St. Louis Fed's MSI series provides Divisia monetary aggregates for M1 and M2 at a monthly frequency[9].

We embed the Divisia approximation to the true aggregate, as opposed to alternative simple-sum approximations, in the policy rule due to the superiority of the Divisia monetary aggregate in tracking the true aggregate - as shown in this model by Belongia and Ireland (2012). [10] For thoroughness we examine the performance of the more common simple-sum aggregate

$$ln(\mu^{Simple-Sum}) = ln\left(\frac{N_t + D_t}{N_{t-1} + D_{t-1}}\right) \tag{22}$$

in place of the Divisia aggregate in the above policy rule. The rule results in indeterminacy in most of the parameter space. In the appendix we show the determinacy region and show through linearization the size of the error of the simple-sum aggregate in tracking the true aggregate in this model.

2.6. Market Clearing

It is now possible to define the equilibrium conditions which close the model. Market clearing in the labor market requires that labor supply equal labor demand, or

$$h_t = \int_0^1 h_t(i)di. \tag{23}$$

Equilibrium in the final goods market requires that the accounting identity

$$Y_t = C_t + x_t\frac{D_t}{P_t} + \frac{\phi}{2}\left[\frac{\Pi_t}{\Pi} - 1\right]^2 Y_t \tag{24}$$

holds as well. Equilibrium in the money market, equity market and bond market requires that at all times

$$M_t = M_{t-1} \tag{25}$$

$$s_t(i) = s_{t-1}(i) = 1 \tag{26}$$

$$B_t = B_{t-1} = 0 \tag{27}$$

respectively. Finally, imposing the symmetry among the intermediate goods producing firms requires that in equilibrium

$$Y_t(i) = Y_t,\ P_t(i) = P_t,\ F_t(i) = F_t,\ and\ Q_t(i) = Q_t. \tag{28}$$

[9]Private organizations such as the Center for Financial Stability have recently begun providing broader Divisia monetary aggregates at monthly frequencies as well.

[10]For more general research examining the Divisia monetary aggregate's properties relative to alternative simple-sum measures see the following works. At paper length Barnett and Chauvet (2011b); Belongia (1996) and at book length Barnett and Singleton (1987); Belongia and Binner (2000); Barnett and Serletis (2000); Barnett and Chauvet (2011a); Barnett (2012).

2.7. Welfare Relevant Natural Rates of Output and Interest

Central to the analysis of optimal monetary policy in NKSP models are the concepts of the natural rate of output and interest. These measures respectively represent the level of output and the real interest rate in an identical economy as the one described without the presence of sticky prices. Woodford (2003) has shown that these concepts play the key role in optimal monetary policy, hence we provide the relevant definitions in this model to show that financial market supply shocks affect these variables and hence impact optimal monetary policy rules - similar to the model of Cúrdia and Woodford (2009). For cohesiveness, we use Woodford's (2003, pg. 302) definition of the natural rate of interest in a monetary economy.

Definition 2. The natural rate of output is the equilibrium level of output at each point in time that would prevail under flexible prices, given a monetary policy that maintains a constant interest rate spread $u_t^A = r_t^L - r_t^A$ between non-monetary (bonds or loans) and monetary riskless short-term assets (currency and deposits).

The aggregate interest rate on monetary riskless short-term assets r_t^A in Definition 2 is the nominal return for holding one unit of the monetary aggregate in period t. This interest rate can be derived from first principles[11] providing a coherent way to think about interest rates in an economy with multiple monetary assets each with different rates of return. Therefore the aggregate user-cost $u_t^A = r_t^L - r_t^A$ provides a natural analogue to the the interest rate spread between bonds and a single monetary asset, the environment considered in Woodford (2003, Ch. 4). Applying this definition to the log-linearized equilibrium conditions results in a natural rate of output that depends only on the model's stochastic disturbances. As shown in the appendix, the resulting expression for the natural rate of output (in log deviations from steady state[12]) is given by

$$\tilde{Y}_t^n = \tilde{Z}_t - \Psi_\upsilon^y \tilde{\upsilon}_t - \Psi_x^y \tilde{x}_t - \Psi_\tau^y \tilde{\tau}_t \tag{29}$$

where Ψ_x^y and Ψ_τ^y are positive in all reasonable calibrations [13]. As is standard from the real business cycle literature, positive technology shocks increase the productive capacity of the economy under flexible prices. Novel here is the appearance of the financial shocks in this expression. Adverse shocks to the financial intermediary's cost of producing monetary assets and willingness to supply loans will behave as negative technology shocks and lower the natural rate of output.

However, there is a striking difference between financial and goods market supply shocks from a policy standpoint. This difference lies in how these shocks affect the natural rate of interest. The expression for the natural rate - given the

[11] We carefully define this in the appendix. See Eq. A.18.

[12] To be clear, for any variable X_t, $\tilde{X}_t = ln(X_t) - ln(\bar{X})$.

[13] The sign of Ψ_υ^y changes depending on the specification of preferences. For additively separable utility it is always positive however for non additively-separable utility it is negative.

above definition of the natural rate of output - is shown below (in log deviations from steady state [14])

$$\tilde{r}_t^n = (1-\rho_a)\tilde{a}_t - (1-\rho_z)\tilde{Z}_t - \Psi_x^r(1-\rho_x)\tilde{x}_t - \Psi_\tau^r(1-\rho_\tau)\tilde{\tau}_t \tag{30}$$

where $\Psi_x^r > 0$ and $\Psi_\tau^r > 0$ and independent of how money enters the utility function. The implication for monetary policy is that responding to adverse financial supply shocks calls for *countercyclical* policy, meanwhile responding to adverse technology shocks calls for *procyclical* policy. The challenge facing the monetary authority is how to form a policy rule which is optimal in this environment given the lack of information they have on the natural rate. We show the monetary aggregate provides valuable information regarding movements in this key variable.

3. Calibration and Solution Strategy

At this point it is useful to proceed by assigning numerical values to the model's parameters. Following the calibration strategy pioneered by Kydland and Prescott (1982) we assign values to parameters in a fashion that allows us to match key features of U.S. data. Since the model used here was developed by Belongia and Ireland (2012) we take many of the values used in their study. Moreover, the model is similar in many regards to the ones estimated by Ireland (2004a,b) providing reliable estimates of the parameters defining the stochastic processes. Table D.10 in the appendix summarizes the choice of parameter values. We end the section with a brief discussion of the solution procedure.

3.1. Calibration

The model is calibrated to the U.S. economy so that one period represents one quarter. We set $\beta = .99$ and we set $\bar{Z} = 1.005$ which is consistent with annual real GDP growth of 2%. These facts imply an annual real interest rate of 6% which is in line with the RBC calibration literature (Kydland and Prescott, 1982). We set $\pi = 1.005$ which matches the official inflation target of the Fed of 2% per year. The disutility of work parameter $\eta = 2.5$ so that one-third of the household's time is spent working. When money is non-separable setting $\chi = 2$ makes the shopping time specification quadratic. As for the CES monetary aggregator M_t^A, the calibrations of $\omega = 1.5$ specifies more substitutability between currency and deposits than the Cobb-Douglas specification and setting $\nu = .225$ pins down the ratio of steady state ratio $\frac{N}{N+D} = .1$, the U.S. average ratio of currency to simple-sum M2 since 1959. Similarly, $\bar{\upsilon} = .4$ matches the steady state ratio $\frac{N+D}{PC} = 3.3$, the average of simple-sum M2 to nominal consumption expenditures over the same period.

On the production side of the economy we set the elasticity of substitution between intermediate goods $\theta = 6$ yielding a steady state mark-up of 20% over marginal cost for the monopolistic firm following Ireland's estimates (2000; 2004a; 2004b). The setting for the cost of price adjustment $\phi = 50$ implies

[14] To be clear, for any variable X_t, $\tilde{X}_t = ln(X_t) - ln(\bar{X})$.

a NKPC that coincides with Calvo style pricing dynamics when the average duration of a price is slightly more than one year (Ireland, 2004b). Regarding the production of financial assets and services setting $\tau = .02$ sets the steady level of reserves to 2%, the average ratio of St. Louis adjusted reserves to the simple-sum deposit components of M2 since 1959 (Ireland, 2011). From the goods market clearing condition, setting $x = .01$ implies in steady-state 3% of output is devoted to banking activities, slightly less than the 3.6% derived from "Federal Reserve banks, credit intermediation, and related activities" on average over the last decade according to the Bureau of Economic Analysis. The fact that our model implies a lower average is accurate considering the limited scope of the activities carried out by the financial sector in this model (Belongia and Ireland, 2006, 2012).[15]

Many of the stochastic processes driving the model have been estimated in Ireland (2004a). Specifically, we use Ireland's estimates of the persistence and standard error of the money demand, preference and technology[16] shocks. However, the parameters driving the financial shocks have not yet been estimated in the literature. Dellas, Diba, and Loisel (2010) calibrate a model with reserves demand and bank cost shocks interpreting the calibrated values from a frequentists perspective. Following their approach to calibrating the financial shocks standard errors we set σ_τ and σ_x so that an increase in banks demand for reserves and an increase in the benchmark-M2 aggregate interest rate spread of the magnitude witnessed during the recent financial crisis occurs once every 80 years on average in our model, assuming normality.[17] The time period of 80 years acknowledges the time span between the Great Depression and the recent financial crisis. This specification of σ_τ is conditional on a value of ρ_τ. When calibrating ρ_τ and ρ_x we once again follow Belongia and Ireland (2012) and set $\rho_\tau = .5$ and $\rho_x = .5$. However since these are only calibrated values and not estimates in section 5.2 we perform a *robust* policy calculation allowing ρ_τ and ρ_x to vary between $[0, .99]$. Moreover, in this section we relax the normality assumption used here to calibrate σ_τ and σ_x to a student's t-distribution

[15] As previously mentioned, the deposit-cost shock also dictates the spread between the benchmark interest rate r_t^L and the deposit rate r_t^D. To confirm the logic of the calibration note the annualized spread between the benchmark rate and the aggregate rate $r^L - r^A$ in the model's steady-state is about 4.6%, which is very close to the average spread between the benchmark rate and the Divisia M2 aggregate interest rate in U.S. data running from January 1967 to September 2009 equal to 4.2% using data from the Center for Financial Stability. Hence, the ability of the model to emulate the data along this added dimension reconfirms the calibration put forth by Belongia and Ireland (2012).

[16] The persistence parameter for the technology shock is set to 1 throughout the paper.

[17] More specifically reserves measured using the ratio of St. Louis adjusted reserves to the deposit components of M2 spiked from slightly over .02 in September 2008 to over .21 in June of 2011. Hence we set σ_τ so that $P(\tau > ln(.21) - ln(.02)) = 1/320$ where $\tau \sim N(0, \sigma_\tau^2 \sum_{i=0}^{11} \rho_\tau^{2i})$. Similarly, the spread between the benchmark rate and the Divisia M2 aggregate rate stood at just 2.8% in July 2008 and in just one quarter the spread spiked to 4.47%. In order to simulate such liquidity shocks we set σ_x so that $P(x > ln(4.47) - ln(2.8)) = 1/320$ where $x \sim N(0, \sigma_x^2)$. The value obtained for σ_x is very close the one obtained by Belongia and Ireland (2006) when they calibrate an RBC version of the model to match their estimated VAR standard errors.

with significantly "fatter-tails" as called for by much of the finance literature (Mandelbrot, 1963; Fama, 1965).

3.2. Solution Strategy

Solving the full non-linear model is not possible hence we resort to approximation techniques. Two main difficulties are encountered. First, the model as presented does not exhibit a deterministic steady-state due to the unit root in the technology process and the price level. To deal with this we detrend most nominal variables by the price level and the technology shock. Details of this detrending are handled in the appendix. Second, a first order approximation is adequate for questions of local determinacy however for proper welfare rankings we must use (at least) a second order approximation[18] (Schmitt-Grohe and Uribe, 2004). Therefore we analyze determinacy using a linear approximation to the model around the steady state implied by the competitive equilibrium and we evaluate welfare using a second-order approximation around the Ramsey planner's steady state with all distortions in place as in Schmitt-Grohe and Uribe (2007).[19]We discuss this choice and provide more details regarding the welfare evaluation in the section below.

4. Welfare Evaluation of Monetary Policy Rules

We now present our main results of the paper. Most importantly, optimal monetary policy features a positive response to the growth rate of the Divisia monetary aggregate. We show that policy rules which respond to the growth rate of money result in real policy rates which are highly correlated with the natural rate of interest. However, this condition appears to be only necessary, not sufficient for good policy. Specifically, responding to the interest rate spread increases the correlation between the real and natural rates of interest but fails to deliver good policy. Instead, such rules increase the variance of inflation and the resources allocated to financial intermediation - leading to detrimental welfare effects.

4.1. Welfare Evaluation Methodology

The focus of the current literature on optimal simple monetary policy rules has been to use a micro-founded measure of welfare without imposing any upper bound on the size of distortions generated in the competitive equilibrium. Following this line of work we use a second order approximation to the household's utility function as our metric for ranking alternative policy rules. Moreover, the appropriate expectation of welfare is the *conditional* as this measure takes into account welfare gains and losses accrued while transitioning from the deterministic steady state to the stochastic steady state implied by the given policy rule (Kim, Kim, Schaumburg, and Sims, 2005). This does however give meaning to

[18]An exception to this fact is when the steady state distortions are small. This case is the focus of Woodford (2003) and Cúrdia and Woodford (2010).

[19]It's worth noting, the determinacy results also hold around the Ramsey planner's steady state and the welfare results are qualitatively identical if we approximate the model around the competitive equilibrium's steady state.

the initial state at which policy is evaluated. For this reason we evaluate all policies around the Ramsey planner's steady state. Hence, we generally have the following welfare metric, regardless of how preferences over money are specified.

$$\mathcal{W}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t, h_t, \frac{M_t^A}{P_t}, a_t, \upsilon_t) \right] \tag{31}$$

We take a second order approximation to $u(C_t, h_t, \frac{M_t^A}{P_t}, a_t, \upsilon_t)$ around the Ramsey steady state allowing for us to express welfare as the weighted sum of conditional means and covariances of the arguments in the utility function. We then evaluate these conditional moments using decision rules found from the second order approximation to the equilibrium conditions.[20] We report our results as consumption equivalent shares. Specifically if $\mathcal{W}_0^*$ is the welfare obtained under an optimal policy then Ω is the share of the generic consumption stream the household would need to equate welfare under the sub-optimal policy to the optimal welfare

$$\mathcal{W}_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t(1+\Omega), h_t, \frac{M_t^A}{P_t}, a_t, \upsilon_t) \right]. \tag{32}$$

Given this definition, Ω is simply the welfare cost in consumption terms of sub-optimal policies.

4.2. Optimal Simple Rules

We now turn our attention to optimal simple rules of the form

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y(\tilde{Y}_t - \tilde{Y}_{t-1}) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{\mathcal{S}}_t$$

in log-deviations from steady state with ρ_r in $[0,1]$ and ϕ_π, ϕ_y, ϕ_m and ϕ_s in $[0,\infty)$ and $\mathcal{S}_t = 1 + (r_t^L - r_t^D)$. The most significant departure from policy rules specified in the literature on optimal simple rules is the inclusion of the (growth rate of the) Divisia monetary aggregate and the spread between loans and deposits. We show below that optimal policy calls for a response to the Divisia monetary aggregate but not the interest rate spread. In particular, we highlight the increased correlation of the real interest rate with the natural rate of interest when policy reacts to the Divisia aggregate. Moreover, reacting to Divisia achieves this high correlation at a low inflation volatility compared to rules which react to the interest rate spread. We also examine optimal policy under additively separable preferences over money and under the assumption

[20] To prevent the order of the forecast, and the forecast itself, from exploding we implement the pruning method proposed in Kim, Kim, Schaumburg, and Sims (2005). More specifically, we use a first order approximation to the equilibrium conditions to evaluate the conditional variances and then use these conditional variances in the second order approximation to evaluate the means. See Kim, Kim, Schaumburg, and Sims (2005) and specifically Section 7 therein.

policy makers have knowledge to the efficient output gap and find our results to be robust.

4.2.1. Baseline Results

We first present our baseline results assuming policy makers only have knowledge of the growth rate of output, current and past interest rates, inflation and the growth rate of the monetary aggregate. The results of the central bank's optimization are presented in Table 1.

Table 1: Welfare Results for Optimal Simple Rules

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y \left(\tilde{Y}_t - \tilde{Y}_{t-1}\right) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{\mathcal{S}}_t$$

Policy Rule	*Optimal Policy Rule Coefficients*					*Welfare Cost*
	ρ_r^*	ϕ_π^*	ϕ_y^*	ϕ_m^*	ϕ_s^*	Ω^1
Optimal	0	3.234	0	2.366	0	0
Optimal $\phi_\pi = 0$	0	–	0	4.217	0	.0304
Optimal $\phi_m = 0$	0	3.6780	0.8982	–	0	.1551
Optimal $\phi_m = \phi_y = 0$	0	4.070	–	–	0	.1736

[1] Ω is the % of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see Eq. (32)). Welfare is calculated as conditional to the initial deterministic Ramsey steady state.

Several points are worth noting. *(i.)* First, responding to the Divisia monetary aggregate is essential to achieve optimal welfare. Notice the welfare cost of not responding to the Divisia aggregate is nearly 5 times as large as the welfare cost of not responding to inflation. One reason for the importance of responding to the Divisia monetary aggregate is due to its ability to provide an indicator in movement of the natural rate. Section 2.7 shows the natural rate of interest falls in response to financial shocks calling for expansionary policy. Only focusing on inflation, or even output growth, fails to provide sufficient expansion. To verify this, Table 2 presents the correlation between the natural rate (See Eq. (30)) and real interest rate under optimal rules with and witohut money.

Table 2: Correlation of Real and Natural Interest Rates

Policy Rule	*Corr*$(\tilde{r}_t^n, \tilde{r}_t - E_t\tilde{\pi}_{t+1})$
Optimal	.8281
Optimal $\phi_\pi = 0$	.8355
Optimal $\phi_m = 0$	.0377

(ii.) Second, unlike in Schmitt-Grohe and Uribe (2007) and Faia and Monacelli (2007) - without considering monetary aggregates ($\phi_m = 0$) - responding to output growth is welfare enhancing. The reason for the difference lies in the stochastic rank of our economy versus theirs. Their economies are driven by aggregate demand and technology shocks, which affect the natural and efficent levels of output symmetrically. In our economy on the other hand, the presence

of financial shocks - which affect the natural level of output but no the efficient level - induces policy makers to deviate from strict inflation targeting. Our third point *(iii.)* responding to the interest rate (loan-deposit) spread decreases welfare. We expand on this counter-intuitive point below where we consider non-optimized rules, including a spread-adjusted Taylor rule as advocated by Taylor (2008).

4.2.2. Results with Additively Separable Utility

One may wonder if we arrive at the above results due to the specification of non-additively-separable preferences over money and consumption. As Ireland (2004a) highlights this assumption results in money showing up directly in the IS equation and the NKPC. The resulting real-balance effect may lead to a bias to stabilize the monetary aggregate. In this section we directly address this concern. The results below verify that money should enter the policy rule due to the information it conveys to policy makers regarding developments in financial markets, not because of the specification of preferences over money.

Table 3: Optimal Simple Rules with Additively Separable Preferences

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y \left(\tilde{Y}_t - \tilde{Y}_{t-1} \right) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{S}_t$$

	Optimal Policy Rule Coefficients					*Welfare Cost*
Policy Rule	ρ_r^*	ϕ_π^*	ϕ_y^*	ϕ_m^*	ϕ_s^*	Ω^1
Optimal	0	3.350	0	2.803	0	0
Optimal $\phi_\pi = 0$	0	–	0	5.012	0	.0290
Optimal $\phi_m = 0$	0	5.7601	0.881	–	0	.2258
Optimal $\phi_m = \phi_y = 0$	0	4.044	–	–	0	.2398

[1] Ω is the % of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see Eq. (32)). Welfare is calculated as conditional to the initial deterministic Ramsey steady state.

Interestingly when money enters the policy rule, it ends up in the IS and NKPC as pointed out by McCallum and Nelson (2011). Hence, Table 3 suggests money *should* enter these equations from a normative standpoint, regardless of the empirical motivation for money entering these equations (See e.g. (Ireland, 2004a)). Specifically, under additive separability the optimal coefficients are qualitatively similar to the optimal coefficients under our baseline specification. Qualitatively however, optimal policy continues to respond only to inflation and Divisia. What's more, under these preferences the welfare cost of not responding to Divisia continues to be nearly 6 times the cost of not responding to inflation.

4.2.3. Results when the Efficient Output Gap is Observable

To this point we have followed the mainstream literature studying optimal *simple* monetary policy rules and assumed that no measure of the output gap is observable. Although this is the most conservative assumption regarding the real time information policy makers, one may argue that assumption is too restrictive. For example, central banks can use standard filtering techniques to generate measures of the trend and cyclical components of output.[21]The trend component presumably represents technological changes to which monetary policy should not attempt to affect. [22] Indeed, there is *some* evidence policy makers have this information. Gali, Lopez-Salido, and Valles (2003) argue the Volcker-Greenspan Fed's response to technology shocks was consistent with optimal policy. In this section we examine the robustness of our results when we relax this constraint on the policy maker's information set. In particular, we ask, is it still necessary for optimal policy to respond to the monetary aggregate when the efficient output gap[23] is observable?

Table 4: Optimal Simple Rules when the *Efficient Output Gap*[1] is observable.

$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_g \tilde{G}^e_t + \phi_m \tilde{\mu}^{Divisia}_t - \phi_s \tilde{\mathcal{S}}_t$					
	Optimal Policy Rule Coefficients				
	ρ^*_r	ϕ^*_π	ϕ^*_g	ϕ^*_m	ϕ^*_s
Optimal	0	3.234	0	2.366	0

[1] The Efficient Output Gap in levels is given by $G^e_t = \frac{Y_t}{Y^e_t} = \eta \frac{Y_t}{Z_t}$ as shwon in the appendix (See Eq. (A.32)).

The results in this section show two things. First, when the efficient output gap is observable optimal policy still responds to the Divisia monetary aggregate. Second, and more to the point, knowledge of the efficient output gap has no impact on the optimal policy coefficients - the only variables in the policy rule with a non-zero coefficient are inflation and the growth rate of the Divisia aggregate. This result stresses the importance of financial shocks shaping the optimal simple policy rule. In particular, policy makers must now deal with inefficient resource costs from financial intermediation in addition to inefficient resource cost stemming from price adjustment and inefficient rents due to monopolist's market power.

4.3. Non-Optimized Rules

It is insightful at this point to compare the performance of policy rules which have historically provided a description of actual Fed policy with our optimal rules. In particular, we can infer the welfare gains of switching from current policy to our prescribed optimal rule. We examine the performance of the original Taylor rule (Taylor, 1993) and the forward-looking rule estimated in Clarida, Galí, and Gertler (2000) which provides a good description of Fed

[21]Woodford (2003 p. 615-616) acknowledges that central bank forecast's of the output gap, "...are usually measures of real GDP relative to some fairly smooth trend." He goes on to say that the measurement appropriate in an optimal policy rule " ... is the difference between real GDP and a target level that should vary in response to real disturbances of many sorts..., and it is not obvious these real factors should all be expected to evolve as a smooth trend."

[22]This is the case in our model as well which features a stochastic trend.

[23]The level of the efficient output gap is given by $G^e_t = \frac{Y_t}{Y^e_t} = \eta \frac{Y_t}{Z_t}$, as shown in the appendix (See Eq. (A.32)).

policy during the Volcker-Greenspan era. Not surprisingly, the above rules lack an aggressive enough response to the financial market shocks and hence lack the necessary correlation with the natural rate of interest. Addressing this issue, Taylor (2008) recently suggested allowing the intercept of the Taylor rule to vary with the interest rate spread (SATR). In Table 5 we present the welfare performance of the ad-hoc policy rules.

Table 5: Non-Optimized Rules

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi E_t \tilde{\pi}_{t+i} + \phi_g E_t \tilde{G}^e_{t+i} - \phi_s \tilde{\mathcal{S}}_t$$

	Policy Rule Coefficients					*Welfare Cost*
PolicyRule[1]	i	ρ_r	ϕ_π	ϕ_g	ϕ_s	Ω^2
Taylor Rule	0	0	1.5	.125	0	.2084
CGG[3]	1	0.79	.4515	.1953	0	.2316
SATR $\phi_s = 2$	0	0	1.5	.125	2	.3834

[1] Here we generalize our previous policy rule allowing for forward (or backward) looking behavior. We use the level of the efficient output gap in this rule due to indeterminacy problems when $\phi_s > 0$ associated with the growth rate rule specified in Eq. (18).
[2] Ω is the % of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see Eq. (32)). Welfare is calculated as conditional to the initial deterministic Ramsey steady state.
[3] CGG refers to Clarida, Gali and Gertler's (2000) estimated forward-looking Taylor rule.

Table 6 shows that policy rules which lack a response to financial variables lack correlation with the natural rate and perform poorly. What is perhaps more shocking is the poor performance of the SATR - despite bringing about a real policy rate which is highly correlated with the natural rate of interest. This reiterates our previous point; good policy rules in this economy are correlated with the natural rate of interest, but correlation with the natural rate of interest is not sufficient to guarantee good policy.

Table 6: Correlation of Real and Natural Interest Rates

Policy Rule	$Corr(\tilde{r}^n_t, \tilde{r}_t - E_t \tilde{\pi}_{t+1})$
Taylor	.1644
CGG	.1785
SATR $\phi_s = 2$	.7269

4.3.1. Creating liquidity without inflation: Why responding to Money Growth (Not Interest Rate Spreads) Is Optimal

This poor welfare performance of spread adjusted Taylor rules (SATR) at first seems counter-intuitive. Table 2 suggests countercyclical policy is key to optimally responding to financial shocks, however the point is bit more subtle. Indeed, responding directly the the interest rate spread by lowering the policy rate fails to deliver good policy. The reason for this failure can be seen by examining the impulse responses following a negative banking productivity shock as shown in Figure 2 below. The solid lines show the dynamics under the optimal policy which reacts to inflation and the Divisia aggregate while the dotted and diamond lines are the dynamics under various SATR's. Notice the dynamics for inflation are driven primarily by the Fisher equation and the monetary policy rule. That is, inflation dynamics look very similar to the variables in the respective policy rules.

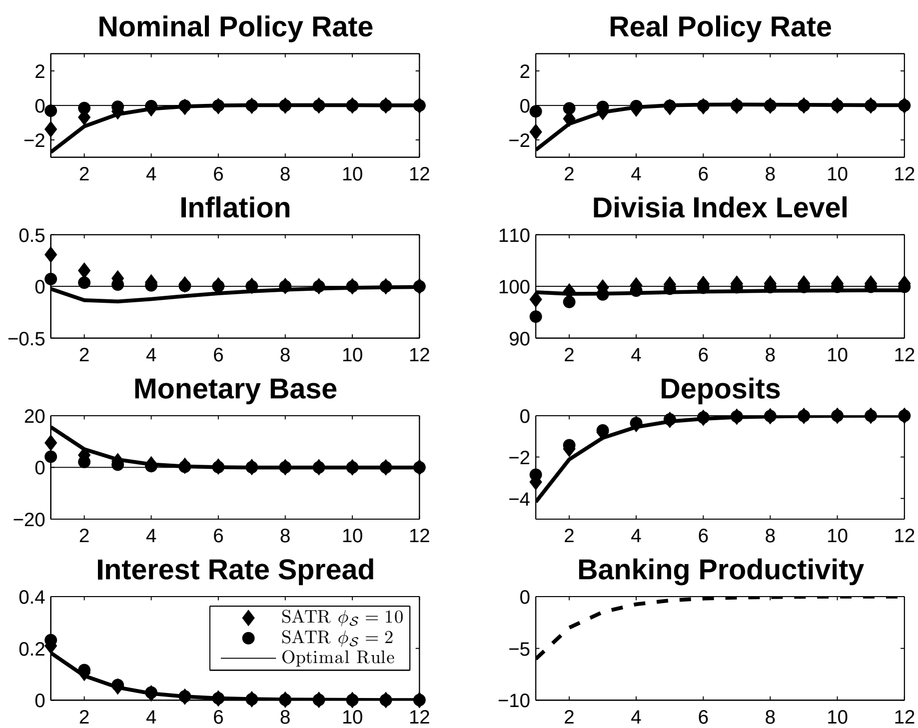

Figure 2: Impulse responses to a negative banking producitivty shock that raises the annualized spread about 25 basis points. All interest rates, the interest rate spread and inflation are in annualized percentage points. All other variables are in % deviations. The diamond line shows the response under the SATR with $\phi_s = 10$ and the dotted line shows the response under the spread adjusted Taylor rule with $\phi_s = 2$ and the solid line shows the response under the optimal simple rule which features a response to inflation and the growth rate of the Divisa index. (See Table 1)

Consider first the interest rate spread rules. A negative banking productivity shock increases the spread between loan and deposit rates resulting in an immediate cut in the policy rate. The real rate will rise by more or less depending on the expected behavior of inflation. Since the rise in the spread has some persistence, the policy rate is expected to remain below steady state and therefore the inflation rate is expected to remain above steady state into the future - causing the real policy rate to fall below the nominal policy rate.[24] This amplification of the real rate results in positive output gaps as long as the spread remains above steady state. More importantly here, the monetary aggregate falls as interest bearing deposits drop, however the expansionary drop in rates is associated with an increase in the monetary base - a liquidity effect.

Compared to the optimal policy rule, this uptick in the monetary base under the SATR is dwarfed. The optimal rule which stabilizes $\tilde{\mu}_t^{Divisia}$, significantly increases the monetary base by lowering the real interest rate by more than even the SATR with $\phi_s = 10$! This increases welfare by providing a substitute for resource intensive financial assets. Moreover, this increase in the monetary base is accomplished with less inflation volatility than the SATR. By committing to adjust interest rates as needed to stabilize the growth rate of the monetary aggregate the central bank is able to create liquidity without large swings in inflation.

Table 7: Inefficient Resource Costs

$PolicyRule^1$	$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\pi}_t^2$	$E_0 \sum_{t=0}^{\infty} \beta^t (\tilde{D}_t + \tilde{x}_t)$
Optimal	.0009	-5.19
SATR $\phi_s = 2$	.0016	-1.57

[1] The optimal rule has policy coefficients of $\phi_\pi = 3.234$ and $\phi_m = 2.366$ (see Table 1).

The welfare from the Divisia rule dominates welfare from the spread rule due to the differences described above. The spread rule results in larger resource costs associated with the creation of costly financial assets by failing to provide the proper liquidty needed to substitute away from deposits. In order to generate a similar rise in the monetary base, the SATR with $\phi_s = 10$, inflation becomes more volatile under the SATR compared to the optimal rule. The result is more resources inefficiently spent on producing financial assets and adjusting prices. Meanwhile the Divisia rule is able to stabilize the liquidity flow, and ultimately reduce the resources allocated to creating financial assets, without excessive inflation costs. These differences are summarized in Table 7 which highlights the intricacy of optimal monetary policy in this financial/sticky-price economy.

5. Comparative Statics, Robustly Optimal Policy and Determinacy

The analysis up to here has been wed to the baseline calibration of the model. Although most of the calibrated values are standard in the literature, the values defining the stochastic processes for the financial sector shocks have not yet been estimated. Therefore in this section we examine the robustness of our results to changes in these key parameters while also examining how the coefficients describing the optimal rule change under such perturbations. First we will

[24] This affect is more amplified when the shock has more serial correlation. In fact, for larger values of ρ_x the nominal rate rises on impact due to the large increase in inflation.

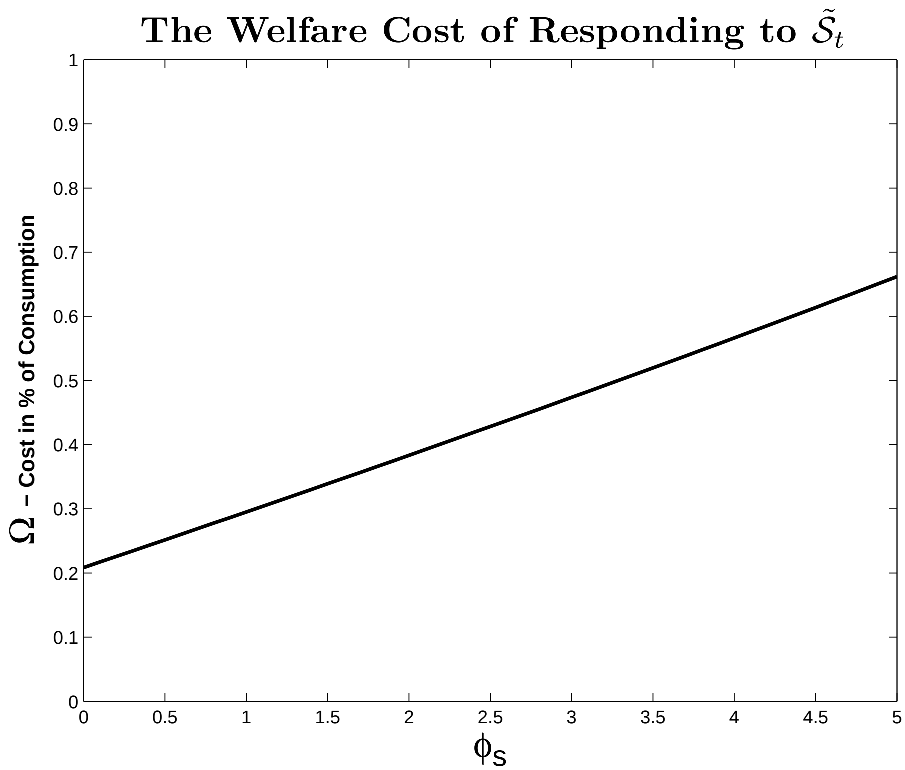

Figure 3: $\mathcal{S}_t = 1 + (r_t^L - r_t^D)$ is the gross interest rate spread between loans and deposits (see Eq. (11)). The welfare cost Ω are relative to the optimal policy (see Table 1). As we vary ϕ_s we keep $\phi_\pi = 1.5$ and $\phi_g = .125$ as in the original Taylor rule (Taylor, 1993).

perform a simple comparative statics exercise by varying the the parameters defining the stochastic shocks and analyzing the change in the optimal policy coefficients. We then offer a robust policy prescription given the uncertainty surrounding the values of these financial disturbances. The robustly optimal policy features again a positive response to only inflation and the growth rate of the Divisia monetary aggregate. We conclude by examining determinacy properties of this rule - a point of practical concern for policy makers. We show the reaction to Divisia compliments the reaction to inflation, so far as determinacy is concerned, resulting in a novel Taylor principle for monetary aggregates.

5.1. Comparative Statics

In much the spirit of Poole (1970) we examine how the optimal policy coefficients change when the shocks driving the economy change. We find that when the standard deviation of financial shocks increase, the optimal response to the monetary aggregate increases as well. This is consistent with Bernanke and Blinder's (1988) suggestions regarding policy when financial and money demand shocks are present; a relative increase in financial shocks should shift the focus to money. Along these same lines, the optimal response to the monetary aggregate decreases when money demand becomes more volatile - the Achilles heel of monetarism. However, the elasticity of ϕ_m^* with respect to σ_v - $\mathcal{E}_{\phi_m^*,\sigma_v}$ - is only 12% (in absolute value) suggesting the tendency to de-emphasize money due to its "instability" has been over stated, at least in regards to its ability to signal movement in the natural rate.

Table 8: *Elasticities*[1] of Optimal Policy Coefficients with respect to structural shock parameters

ϕ_π^*	ϕ_m^*
$\rho_v = 75$	$\rho_v = -31$
$\rho_a = -193$	$\rho_a = -30$
$\rho_\tau = -11$	$\rho_\tau = -18$
$\rho_x = -103$	$\rho_x = -119$
$\sigma_v = 20$	$\sigma_v = -12$
$\sigma_a = 12$	$\sigma_a = 1$
$\sigma_z = 75$	$\sigma_z = -40$
$\sigma_\tau = -6$	$\sigma_\tau = 8$
$\sigma_x = -104$	$\sigma_x = 45$

[1] All elasticities are arc-elasticities in % computed at the baseline values and a 10% change from baseline values.

With regards to the parameters defining the serial correlation of the shocks, we find that increasing the persistence of the financial shocks calls for a reduction in ϕ_m^*. This may be surprising at first, however looking to the natural rate of interest (see Eq. (30)) notice the more persistent the financial shocks become the less they influence the natural rate. However, to the extent that monetary aggregates provide information regarding this variable, we show in the next section the optimal response to the Divisia aggregate is not null, even if the financial shocks are very persistent. Returning momentarily to the money demand shocks notice increasing the persistence of such shocks results in the expected outcome, a decrease in ϕ_m^*. Again though, this effect is relatively small as $\mathcal{E}_{\phi_m^*,\sigma_v}$ is only about 30%.

5.2. Robustly Optimal Policy

The previous section highlights the fact that the exact coefficients that define the optimal policy rule depend on the underlying stochastic processes. Hence, one may wonder then, given our uncertainty over such parameters what is the best policy to follow? We will provide a first attempt to answer this question using a simple 2 player game zero-sum game. In particular, following Giannoni (2002), we set up the zero-sum game Γ between the central bank (CB) and nature (N). The central bank would like to choose policy rule parameters - $\phi \in \Phi$

- to maximize welfare defined by $\mathcal{W}_0$ (see Eq. (31)). At the same time, nature is malevolent and would like to choose shock parameters - $\sigma \in \Sigma$ - which minimize $\mathcal{W}_0$. The Nash equilibrium represents the worst set of structural parameters possible, given policy makers choose the best possible policy given the bad state of the world. Intuitively, the Nash equilibrium of this game provides a policy prescription that does the best under the worst case scenario. The nature of zero-sum games allows us to consider solving the minimax problem to find the Nash equilibrium

$$\min_{\phi\in\Phi}\left\{\max_{\sigma\in\Sigma}\{-\mathcal{W}_0\}\right\}$$

We numerically approach a solution by iterating over natures problem and over the policy makers problem until all parameters and the objective function converge.

Before solving this problem we must set bounds on the choice space of nature. For the money demand, preference and technology process we assume all parameters lie within 2 standard errors using the estimates from Ireland (2004a) with an upper bound of .99 for the persistence parameters[25]. Unfortunately, the parameters driving the financial shocks have no comparable estimates to help determine an appropriately bounded set. Therefore we assume that $\rho_x \in [0, .99]$ and $\rho_\tau \in [0, .99]$. As for the standard deviations of these shocks we continue to rely on the observation that large financial disruptions have occurred about every 80 years, however we relax our normality assumption. In particular, to determine a lower bound for the standard deviations of the financial shocks, we allow for the possibility that financial shocks follow a distribution with significantly "fatter-tails" than a normal distribution [26]. For concreteness, suppose that ε_t^τ and ε_t^x are distributed according to a student's t-distribution with 3 degrees of freedom[27] implying we scale our normal standard errors by $\sqrt{\frac{d.o.f.}{d.o.f.-2}} = \sqrt{3}$. The resulting bounds are given by $\sigma_x \in [.0993,\ .1720]$ and $\sigma_\tau \in (\sqrt{\Sigma_{i=0}^{11}\rho_\tau^{2i}})^{-1}[.4965,\ .8600]$ where the upper bounds are the standard deviations under the normality assumption and the lower bounds are the upper bounds scaled by $\sqrt{3}$. The resulting Nash Equilibrium, or equivalently, the worst state of the world and the best policy in this state are given in Table 9 below.

Nature chooses to maximize the persistence and volatility of all the shocks in the economy, leading to the maximum long-run variance possible in these stochastic processes. In particular nature maximizes the persistence of the financial shocks in the model. How does this minimize welfare? Recall again the

[25]As with the rest of the paper, we continue to assume a unit-root in the technology shock
[26]See for example Mandelbrot (1963) or Fama (1965).
[27]The choice of the degrees of freedom is arbitrary but 3 is the smallest integer for which the student's t-distribution has a finite standard error.

Table 9: Nash Equilibrium in Γ

Nash Equilibrium	
Nature's Strategy	Central Bank's Strategy
$\rho_v^* = .99$	$\rho_r^* = 0$
$\rho_a^* = .99$	$\phi_\pi^* = .8940$
$\rho_\tau^* = .99$	$\phi_y^* = 0$
$\rho_x^* = .99$	$\phi_m^* = .5206$
$\sigma_v^* = .0102$	$\phi_s^* = 0$
$\sigma_a^* = .0352$	
$\sigma_z^* = .0116$	
$\sigma_\tau^* = .2621$	
$\sigma_x^* = .1720$	

equation for the natural rate of interest - reproduced here for convenience.

$$\tilde{r}_t^n = (1-\rho_a)\tilde{a}_t - (1-\rho_z)\tilde{Z}_t - \Psi_x^{r^n}(1-\rho_x)\tilde{x}_t - \Psi_\tau^{r^n}(1-\rho_\tau)\tilde{\tau}_t$$

By pushing the persistence of the financial shocks to their maximum, nature is trying to mitigate the financial shocks role in driving the natural rate. Why? When financial shocks affect the natural rate policy makers reacting to the Divisia aggregate are able to correlate the real rate with the natural rate. However, by limiting the share of variance in the natural rate explained by financial shocks, while at the same time maximizing the long-run variance of money demand shocks, nature is effectively maximizing the noise to signal ratio of Divisia in signaling movements in the natural rate. Interestingly though welfare is still improved by including a positive - but weakened - reaction to the Divisia aggregate. Therefore, much in line with Brainard's (1967) seminal work on policy under uncertainty we find (in contrast to Giannoni (2002)) uncertainty calls for an attenuated reaction to *both* inflation and the monetary aggregate. However the Divisia response is significant - even in this extreme state of the world.

5.3. Determinacy

Thus far we have found the optimal simple *and* robust rules feature a positive response to only inflation and the Divisia monetary aggregate. Clearly if the aforementioned policy rules are followed "to the letter" then the outcome is determinate in the sense that there is a unique path for all nominal and real variables. However, a practical concern for policy makers faced with the policy prescription in this economy is whether reacting to the Divisia monetary aggregate induces undesirable equilibrium outcomes that an inflation only rule would preclude. In other words, if policy deviates slightly from the prescribed rule is the equilibrium outcome still unique? To answer this question we examine the determinacy properties of the optimal rule in this economy. Unfortunately, analytical solutions are not available as in Bullard and Mitra (2002) so instead we examine the determinacy properties of this rule numerically. We examine determinacy by searching over the values $\phi_\pi \in [0,5]$ and $\phi_m \in [0,5]$ by .01 increments and and $\rho_r \in [0,1)$ by .1 increments around both the steady state of the competitive equilibrium and the Ramsey steady state and examining the necessary and sufficient condition for existence and uniqueness as laid out by Blanchard and Kahn (1980). The resulting determinacy region is featured in Figure 5.3.

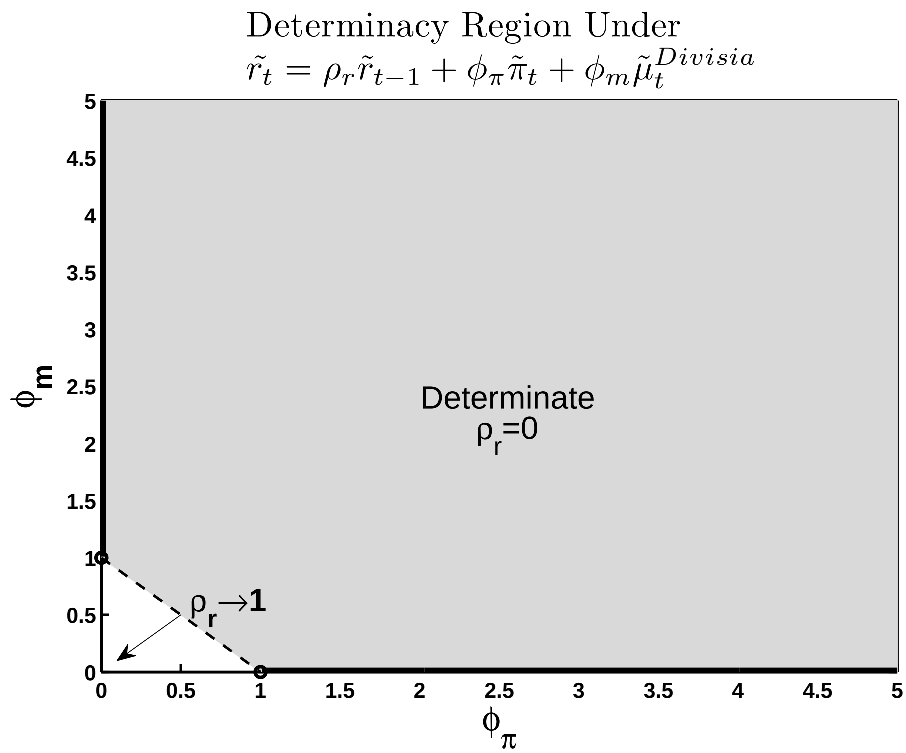

Figure 4: The shadded area to the northeast of the dotted line, defined by $\phi_m + \phi_\pi > 1$, is determinate when $\rho_r = 0$. Moreover, if $\rho_r > 0$ then the condition for determinacy under this rule generalizes to $\frac{\phi_m + \phi_\pi}{1-\rho_r} > 1$, this is a genralized Taylor principle for Divisia monetary aggregates in interest rate rules.

Looking first along the horizontal axis where $\phi_m = 0$, the well known Taylor principle continues to hold. That is policy makers can guarantee a unique equilibrium by satisfying

$$\phi_\pi > 1$$

Moreover, looking at the vertical axis where $\phi_\pi = 0$, we find that policy makers can guarantee a unique equilibrium outcome by adjusting the policy rate more than one for one with changes in the growth rate of the Divisia monetary aggregate[28]. That is, the condition that

$$\phi_m > 1$$

is sufficient for determinacy. However, our optimal simple and robust rules feature a positive response to both inflation and the Divisia monetary aggregate. Therefore, focusing our attention on the interior Figure 4, where both responses are positive, we find that the above two conditions are sufficient but clearly not necessary as determinacy is more generally possible by satisfying the condition

$$\phi_\pi + \phi_m > 1 \tag{33}$$

Reacting to the growth rate of Divisia actually allows policy makers to achieve determinacy with even weaker inflation responses so long as the reaction to Divisia is increased one for one. Equation (33) provides an extension of the well-known Taylor principle to Taylor rules featuring a response to the Divisia monetary aggregate - a *modified Taylor principle* for Divisia monetary aggregates. Most generally, if policy makers introduce lagged interest rates into the policy rule we see that condition (33) generalizes to

$$\frac{\phi_\pi + \phi_m}{1 - \rho_r} > 1 \tag{34}$$

Equations (33) and (34) show that there are no more concerns regarding determinacy under optimal rules in this model than there are regarding determinacy in standard NKSP where $\phi_m = 0$ and in fact determinacy is more likely when reacting to the Divisia monetary aggregate.

6. Conclusion

We have analyzed optimal monetary policy in a NKSP augmented to include a financial sector. We showed financial disturbances which decrease banks ability and willingness to produce financial assets lowers the natural rate of interest calling for countercyclical monetary policy. The resulting optimal policy deviates from the common inflation only type of Taylor rule common in this literature. Instead, reacting to the growth rate of the Divisia monetary aggregate, in addition to inflation, performs significantly better than reacting to inflation alone. The resulting policy rule produces a real interest rate which is highly correlated with the natural rate of interest. We showed the spread adjusted Taylor rule as advocated by Taylor (2008) actually performs worse than inflation only rules despite its high correlation with the natural rate of interest. This puzzling result is reconciled by the increased inflation and output gap volatility induced by the spread adjusted Taylor rules which is not present in

[28]We emphasize this result is not generally true for *any* monetary aggregate. In fact, we show in the appendix embedding the simple-sum monetary aggregate (see Eq. (22)) tends to result in indeterminacy and no such modified Taylor principle holds.

the Divisia growth rules. The optimality of reacting positively to the Divisia growth rate and the inflation rate is shown to hold under different assumption regarding policy maker's information sets and preferences over monetary assets. Finally, we offer a *robust* policy prescription under parametric uncertainty using the minimax approach proposed in Giannoni (2002). The resulting policy rule features an attenuated - but positive and economically significant - response to the growth rate of Divisia and inflation. Moreover, we offer some insight regarding the ability of such rules to bring about a unique equilibrium outcome by examining their determinacy region. The result is a *modified Taylor principle* for Divisia monetary aggregate which is sufficient to guarantee determinacy.

As much as the seminal work of Taylor (1993) marked the beginning of interest rate rules dominating monetary policy we hope this paper can be part of the beginning of re-examining the usefulness of monetary aggregates in real time policy making. Given the interest in *simple* rules and macro policy under financial distress considerable work lies ahead in better understanding how policy makers can better use the information available in monetary aggregates and other variables available in real time. Specifically, one may wonder in our environment if Divisia level targeting is superior to reacting to the growth rate of the Divisia monetary aggregate. Also, expanding the literature on statistical index number theory to properly track aggregate quantities of credit seems a worthwhile task given the usefulness of properly constructed monetary aggregates in this model. Furthermore, the zero-lower bound on nominal interest rates suggests the usefulness of monetary aggregates may extend beyond informational variables to policy instruments as called for by Taylor (2009). However, an optimal policy prescription for monetary instrument rules is seriously lacking in this modern framework due to the exclusive focus on interest rates. This calls for future work to understand how different aggregates perform as instruments from their determinacy properties to their welfare performance.

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AppendixA. The Equilibrium System

In this portion of the appendix we derive the model's equilibrium conditions and stationarize the model allowing us to define a recursive (imperfectly) competitive equilibrium. We also derive expressions for the natural rate level of output and the natural interest rate.

AppendixA.1. The Representative Household's FOCS

Non-Separable Preferences: First consider the case when monetary assets are not additively separable from consumption. In particular, recall from section 2.1.3 that the household faces the following problem when preferences over monetary assets are defined according to a shopping time friction;

$$\max_{\{C_t,h_t,M_t^A,N_t,D_t,L_t,B_{t+1},M_{t+1},s_{t+1}(i)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[ln(C_t) - \eta(h_t + h_t^s)\right]$$

subject to (1), (2), (3) and (4) taking B_0, M_0 and $s_0(i)$ as given. After substituting (4) into the objective function, we can form the Bellman equation as follows where all the constraints are expressed in real terms by dividing through by P_t,

$$\begin{aligned} V(B_{t-1}, M_{t-1}, s_{t-1}(i)) = max \Bigg\{ a_t \Bigg[ln(C_t) - \eta h_t - \frac{\eta}{\chi}\left(\frac{\upsilon_t P_t C_t}{M_t^A}\right)^{\chi} \Bigg] \\ -\Lambda_t^1 \left(\frac{D_t - M_{t-1} - B_{t-1} + \int_0^1 Q_t(i)(s_t(i) - s_{t-1}(i))di + B_t/r_t + N_t - L_t}{P_t} \right) \\ -\Lambda_t^2 \left(\frac{M_t^A}{P_t} - \left[\nu^{\frac{1}{\omega}} (\frac{N_t}{P_t})^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (\frac{D_t}{P_t})^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right) \\ -\Lambda_t^3 \left(\frac{M_t - N_t + P_t C_t - W_t h_t - \int_0^1 F_t(i) s_t(i) di - r_t^D D_t + r_t^L L_t}{P_t} \right) \\ +\beta E_t[V(B_t, M_t, s_t(i))] \Bigg\} \end{aligned}$$

The first order necessary conditions are given by the following equations. The system of equations (A.1)-(A.9) is under-determined in the sense that we have introduced various derivatives of the value function. However, we can complement these first order necessary conditions with the Bienveniste-Scheinkman Envelope Conditions to eliminate the value function from the system above. These envelope conditions are given in equations (A.10)-(A.12) below.

$$a_t\left[1-\eta\left(\frac{\upsilon_t P_t C_t}{M_t^A}\right)^{\chi}\right]-\Lambda_t^3 C_t=0 \tag{A.1}$$

$$-a_t\eta+\Lambda_t^3\frac{W_t}{P_t}=0 \tag{A.2}$$

$$\eta a_t\left(\frac{\upsilon_t P_t C_t}{M_t^A}\right)^{\chi}-\frac{\Lambda_t^2 M_t^A}{P_t}=0 \tag{A.3}$$

$$\left(\frac{D_t}{P_t}\right)-(1-\nu)\left(\frac{M_t^A}{P_t}\right)\left[\frac{\Lambda_t^2}{\Lambda_t^1-\Lambda_t^3 r_t^D}\right]^{\omega}=0 \tag{A.4}$$

$$\left(\frac{N_t}{P_t}\right)-\nu\left(\frac{M_t^A}{P_t}\right)\left[\frac{\Lambda_t^2}{\Lambda_t^1-\Lambda_t^3}\right]^{\omega}=0 \tag{A.5}$$

$$\Lambda_t^1-\Lambda_t^3 r_t^L=0 \tag{A.6}$$

$$\frac{-\Lambda_t^1}{r_t}+P_t\beta E_t[V_{B_t}^{'}(B_t,M_t,s_t(i))]=0 \tag{A.7}$$

$$-\Lambda_t^3+P_t\beta E_t[V_{M_t}^{'}(B_t,M_t,s_t(i))]=0 \tag{A.8}$$

$$-\Lambda_t^1 Q_t(i)+\Lambda_t^3 F_t(i)+P_t\beta E_t[V_{s_t(i)}^{'}(B_t,M_t,s_t(i))]=0 \tag{A.9}$$

Envelope Conditions:

$$V_{B_{t-1}}^{'}(B_{t-1},M_{t-1},s_t(i))=\frac{\Lambda_t^1}{P_t} \tag{A.10}$$

$$V_{M_{t-1}}^{'}(B_{t-1},M_{t-1},s_t(i))=\frac{\Lambda_t^1}{P_t} \tag{A.11}$$

$$V_{s_{t-1}(i)}^{'}(B_{t-1},M_{t-1},s_t(i))=\frac{\Lambda_t^1 Q_t(i)}{P_t} \tag{A.12}$$

Now update (A.10)-(A.12) and substitute the resulting equations into (A.6)-(A.8) yielding:

$$\frac{-\Lambda_t^1}{r_t}+\beta E_t\left[\frac{P_t\Lambda_{t+1}^1}{P_{t+1}}\right]=0 \tag{A.13}$$

$$-\Lambda_t^3+\beta E_t\left[\frac{P_t\Lambda_{t+1}^1}{P_{t+1}}\right]=0 \tag{A.14}$$

$$-\Lambda_t^1\frac{Q_t(i)}{P_t}+\Lambda_t^3\frac{F_t(i)}{P_t}+\beta E_t\left[\frac{\Lambda_{t+1}^1 Q_{t+1}(i)}{P_{t+1}}\right]=0 \tag{A.15}$$

The conditions (A.1)-(A.6) and (A.13)-(A.15) define the consumers optimal behavior.

Separable Preferences: Under additively separable preferences over monetary assets the household's first order conditions are identical to the above conditions after replacing (A.1) and (A.3) with

$$\frac{a_t}{C_t} - \Lambda_t^3 = 0 \tag{A.16}$$

$$\eta_m a_t \upsilon_t - \Lambda_t^2 \frac{M_t^A}{P_t} = 0 \tag{A.17}$$

respectively.

AppendixA.2. Deriving the User Costs of Monetary Assets

As shown in an infinite planning horizon by Barnett and Singleton (1987), we can derive the user costs of all monetary assets in the model from the household's first order necessary conditions. Specifically, the user costs appear naturally as the price of monetary assets according to the familiar optimality condition from microeconomics which dictates, at an optimum, equating the marginal rate of substitution of currency for deposits to the ratio of the price of currency to the price of deposits.

$$\frac{\frac{\partial u_t}{\partial N_t}}{\frac{\partial u_t}{\partial D_t}} = \frac{\frac{\partial u_t}{\partial M_t^A}\frac{\partial M_t^A}{\partial N_t}}{\frac{\partial u_t}{\partial M_t^A}\frac{\partial M_t^A}{\partial D_t}} = \frac{\frac{\Lambda_t^1 - \Lambda_t^3}{\Lambda_t^2}}{\frac{\Lambda_t^1 - \Lambda_t^3 r_t^D}{\Lambda_t^2}} = \frac{\Lambda_t^3 r_t^L - \Lambda_t^3}{\Lambda_t^3 r_t^L - \Lambda_t^3 r_t^D} = \frac{r_t^L - 1}{r_t^L - r_t^D} \equiv \frac{u_t^N}{u_t^D}$$

The second equality follows from equations (A.4) and (A.5), the household's first order conditions for N_t and D_t. The third equality follows from equation (A.6), the household's first order condition for L_t. The resulting ratio defines the relative user costs.

We can also derive the exact price dual to the true quantity aggregator in a similar fashion. Instead of considering the price of each monetary assets individually, consider the optimality condition the monetary aggregate must satisfy. For simplicity, consider the marginal rate of substitution of the monetary aggregate for consumption. Clearly the price of the consumption good is P_t, hence the numerator must denote the price of the monetary aggregate.

$$\frac{\frac{\partial u_t}{\partial M_t^A}}{\frac{\partial u_t}{\partial C_t}} = \frac{\frac{\Lambda_t^2}{P_t}}{\Lambda_t^3} = \frac{\frac{\Lambda_t^2}{\Lambda_t^3}}{P_t} = \frac{\left[\nu(r_t - 1)^{(1-\omega)} + (1-\nu)(r_t - r_t^D)^{(1-\omega)}\right]^{\frac{1}{1-\omega}}}{P_t} \equiv \frac{r_t^L - r_t^A}{P_t}$$

$$u_t^A = r_t^L - r_t^A = \left[\nu(r_t - 1)^{(1-\omega)} + (1-\nu)(r_t - r_t^D)^{(1-\omega)}\right]^{\frac{1}{1-\omega}} \tag{A.18}$$

The first equality follows from equations (A.1) and (A.3), the household's first order conditions for C_t and M_t^A respectively. The third equality follows from solving equation (A.4) for N_t, equation (A.5) for D_t and substituting the resulting expressions into equation (3). The resulting expression can be solved for $\frac{\Lambda_t^2}{\Lambda_t^3}$ yielding the numerator the follows the third equality. For the last equality we define this expression to be $r_t - r_t^A$, or the opportunity cost of holding the aggregate monetary asset M_t^A. The resulting aggregate user cost (A.18) as defined by Barnett (1978) is of the same form as the individual component user costs in equations (20) and (21). It can be verified that (A.18) is in fact the true price dual to the true monetary aggregate in equation (3) since it satisfies Fisher's factor reversal test.

$$M_t^A u_t^A = u_t^N N_t + u_t^D D_t \tag{A.19}$$

Equation (A.19) states that the true quantity index times the true price index equals total expenditures, in this sense, (A.18) is the exact price dual to (3).

AppendixA.3. Equilibrium Quantity of the Monetary Aggregate

In this section we derive a condition relating the monetary aggregate to output, a vector of interest rates and financial shocks. This expression is useful for deriving the natural rates of output and interest later and also because it highlights the difference between broad money which depends on a vector of interest rates and the financial sector's productive ability versus typical "money demand" specifications which assume the equilibrium quantity of money depends only agents demand for monetary assets featuring a single interest rate. All variables with a tilde denote the real detrended variable in log-deviations from steady state (with the exception of interest rates which are in nominal terms). Combine (A.2), (A.3) (or (A.16), (A.17)) and (A.18) to arrive at (up to additive constants)

$$\tilde{M}_t^A = \gamma_m^c \tilde{C}_t + \gamma_m^w \tilde{W}_t - \gamma_m^{u^A} \tilde{u}_t^A + \gamma_m^v \tilde{v}_t \tag{A.20}$$

where under additively separable preferences $\gamma_m^c = \gamma_m^{u^A} = \gamma_m^v = 1, \, and \, \gamma_m^w = 0$ and under non-additively separable preferences $\gamma_m^c = \gamma_m^v = \frac{\chi}{1+\chi}, \, and \, \gamma_m^w = \gamma_m^{u^A} = \frac{1}{1+\chi}$. Now log-linearize the household's first order condition relating the real wage to the marginal utilities of consumption and leisure and we have that

$$\tilde{W}_t = (1+\gamma_m)\tilde{C}_t - \gamma_m \tilde{M}_t^A + \gamma_m \tilde{v}_t \tag{A.21}$$

where $\gamma_m = \frac{u''_{M^A,C}}{u'_C}$ is zero for additively separable preferences. Now substitute (A.21) into (A.20) to eliminate the real wage and arrive at

$$\tilde{M}_t^A = \frac{[\gamma_m^c + \gamma_m^w(1+\gamma_m)]}{[1+\gamma_m^w \gamma_m]}\tilde{C}_t - \frac{\gamma_m^{u^A}}{[1+\gamma_m^w \gamma_m]}\tilde{u}_t^A + \frac{[\gamma_m^w \gamma_m + \gamma_m^v]}{[1+\gamma_m^w \gamma_m]}\tilde{v}_t \tag{A.22}$$

We can express consumption in terms of output and the monetary aggregate by substituting equation (A.5) into equation (24) and log-linearizing to arrive at

$$\tilde{C}_t = \frac{1}{s_c}\left[\tilde{Y}_t - (1-s_c)\left[\tilde{x}_t + \tilde{M}_t^A + \omega\left(\tilde{u}_t^A - \tilde{u}_t^D\right)\right]\right] \tag{A.23}$$

where $s_c = \frac{\bar{C}}{\bar{Y}}$. Our equilibrium condition for the monetary aggregate is obtained by substituting (A.23) into (A.22) and noting that under either specification of preferences $\gamma_m^c + \gamma_m^w = 1$

$$\tilde{M}_t^A = \tilde{Y}_t - \eta_{u^A}\tilde{u}_t^A + \eta_{u^D}\tilde{u}_t^D + \eta_v\tilde{v}_t - \eta_x\tilde{x}_t \tag{A.24}$$

where

$$\begin{aligned}
\eta_{u^A} &= (1-s_c)\omega + \frac{\gamma_m^{u^A} s_c}{(1+\gamma_m\gamma_m^w)} \\
\eta_{u^D} &= \omega(1-s_c) \\
\eta_v &= s_c\left(1 - \frac{1-\gamma_m^v}{1+\gamma_m\gamma_m^w}\right) \\
\eta_x &= (1-s_c)
\end{aligned}$$

and all the coefficients are positive.

AppendixA.4. The Representative Intermediate Goods Firm's FOCS

Recall from section 2.4, the representative intermediate goods producing firm $i \in [0,1]$ maximizes its share price in every period $t = 0,1,2,3,....$ Mathematically, we have

$$\max_{\{h_t(i),P_t(i)\}_{t=0}^{\infty}} \frac{Q_t(i)}{P_t} \; subject\, to\, (14)\, and\, (15).$$

To derive the first order necessary conditions for this problem use the equity pricing relationship (A.15) from the representative household's first order condition to solve for period t. Solving (A.15) forward and assuming no bubbles yields

$$\frac{Q_t(i)}{P_t} = E_t\left[\sum_{j=0}^{\infty}\beta^j\frac{\Lambda_{t+j}^3 F_{t+j}(i)}{\Lambda_t^1 P_{t+j}}\right]. \tag{A.25}$$

Equation (A.25) is a familiar asset pricing relationship which states that the market share price of the representative intermediate goods producing firm is proportional to the expected future stream of dividends adjusted for risk. In particular, the stochastic discount factor is given by $\frac{\beta^j\Lambda_{t+j}^3}{\Lambda_t^1}$. In any period $t = 0,1,2,3,...$, intermediate goods producing firm $i \in [0,1]$ pays out all profits as dividends. In real terms, the period t real dividend is given by

$$\frac{F_t(i)}{P_t} = \frac{P_t(i)}{P_t}Y_t(i) - \frac{W_t}{P_t}h_t(i) - \frac{\phi}{2}\left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1\right]^2 Y_t. \tag{A.26}$$

The first term of the above profit function is period t real revenue, the second term in the firm's period t real wage bill and the third term will be 0 unless the intermediate goods producing firm changes its price from period $t-1$ to period

t an amount different than the steady-state gross rate of inflation rate, π. Substituting (A.26) into (A.25), we can restate the intermediate goods producing firm's problem as

$$\max_{\{h_t(i),P_t(i)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t^3}{\Lambda_0^1} \left[\frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} h_t(i) - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right]^2 Y_t \right]$$

subject to

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \; and \; Y_t(i) = z_t h_t(i).$$

The problem can be simplified by substituting the inverse of the technology constraint for $h_t(i)$ and then substituting the factor demand into the resulting expression for $Y_t(i)$ so that now the representative intermediate goods producing firm solves the following recursive problem defined by Bellman's equation

$$\begin{aligned} V(P_{t-1}(i)) = \max_{P_t(i)} \Bigg\{ & \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \left[\left[\frac{P_t(i)}{P_t} \right]^{1-\theta} - \frac{W_t}{P_t z_t} \left[\frac{P_t(i)}{P_t} \right]^{-\theta} - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right]^2 \right] \\ & + \beta E_t[V(P_t)] \Bigg\} \end{aligned}$$

The first order necessary condition for the problem is given by

$$\begin{aligned} (1-\theta) \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \frac{1}{P_t} + \theta \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \frac{W_t}{P_t^2 z_t} \left[\frac{P_t(i)}{P_t} \right]^{-1-\theta} & \\ -\phi \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right] \frac{1}{P_{t-1}(i)\pi} + \beta E_t[V^{'}(P_t)] = 0 & \qquad \text{(A.27)} \end{aligned}$$

Once again invoking the Bienveniste-Scheinkman Envelope Condition we have

$$V^{'}(P_{t-1}(i)) = \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \phi \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right] \frac{P_t(i)}{\left(P_{t-1}(i)\right)^2 \pi}. \qquad \text{(A.28)}$$

Updating (A.28) one period and substituting into (A.27) and then multiplying the resulting equation by $\Lambda_0^1 P_t$ yields

$$(1-\theta)\left[\frac{P_t(i)}{P_t}\right]^{-\theta}+\theta\frac{W_t}{P_t z_t}\left[\frac{P_t(i)}{P_t}\right]^{-1-\theta}-\phi\left[\frac{P_t(i)}{P_{t-1}(i)\pi}-1\right]\frac{P_t}{P_{t-1}(i)\pi}$$
$$+\beta\phi E_t\left[\frac{\Lambda_{t+1}^3 Y_{t+1}}{\Lambda_t^3 Y_t}\left[\frac{P_{t+1}(i)}{P_t(i)\pi}-1\right]\frac{P_t P_{t+1}(i)}{(P_t(i))^2\pi}\right]=0 \tag{A.29}$$

Finally, when (A.29) is log-linearized around a symmetric equilibrium where $P_t(i) = P_t \ \forall \ t = 0, 1, 2, 3, ...$ it takes the form of a New-Keynesian Phillips Curve relating current inflation to the average real marginal cost and expected future inflation.

AppendixA.5. The Efficient Level of Output

To find the efficient level of output used to define the efficient output gap we solve the benevolent social planner's problem

$$max_{C_t,Y_t,h_t,\{Y_t(i),h_t(i)\}_{i\in[0,1]}}a_t\left[ln(C_t)-\eta h_t\right]$$

$$s.t.\,(12),\ (15)\,(23)\,and\,Y_t=C_t$$

We can simplify the above constrained maximization problem, into the unconstrained problem

$$max_{,h_t(i)\}_{i\in[0,1]}}a_t\left[ln\left(Z_t\left[\int_0^1 h_t(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}\right)-\eta\int_0^1 h_t(i)di\right]$$

The first order condition for this problem is given by

$$h_t(i)^{-\frac{1}{\theta}}=\eta\left[\int_0^1 h_t(i)^{\frac{\theta-1}{\theta}}di\right] \tag{A.30}$$

Raising each side of (A.30) to the $(1-\theta)$ power and integrating each side over i, we have

$$1=\eta\left[\int_0^1 h_t(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}} \tag{A.31}$$

Substituting out $h_t(i)$ in (A.31) using (15) and then using the definition of Y_t from (12) we have the social planner's - or efficient - level of output

$$Y_t^e=\frac{1}{\eta}Z_t$$

Then the gross efficient output gap is simply

$$G_t^e = \frac{Y_t}{Y_t^e} = \eta \frac{Y_t}{Z_t} \tag{A.32}$$

AppendixA.6. The Natural Level of Output

Here we derive the natural level of output in this economy as defined in definition 2 in section 2.7. Assuming prices are flexible, or $\phi = 0$, the monopolistically competitive firm will set its price a constant "mark-up" - $\mathcal{M} = \frac{\theta}{\theta - 1}$ - over marginal cost (see Eq. (A.29))

$$P_t = \mathcal{M} \frac{W_t}{Z_t} \tag{A.33}$$

Rearranging a bit and then expressing (A.33) in terms of log-deviations from steady state we have, where all variables with a tilde that follow are real detrended log-deviations from steady state with the exception of interest rates which are in nominal terms.

$$0 = \tilde{W}_t - \tilde{Z}_t$$

Now use the household's first order condition relating the marginal utility of consumption and leisure to the real wage and log-linearizing we have

$$0 = (1 + \gamma_m)\tilde{C}_t - \gamma_m \tilde{M}_t^A + \gamma_m \tilde{v}_t - \tilde{Z}_t \tag{A.34}$$

where $\gamma_m = \frac{u''_{M^A,C}}{u'_C}$ is zero for additively separable preferences. Now substitute equation (A.23), the log-linearized market clearing condition, into(A.34) to obtain

$$\begin{aligned} 0 = &\frac{(1+\gamma_m)}{s_c}\tilde{Y}_t^n - \frac{(1-s_c)+\gamma_m}{s_c}\tilde{M}_t^A - \frac{(1+\gamma_m)(1-s_c)\omega}{s_c}(\tilde{u}_t^A - \tilde{u}_t^D) \\ &+\gamma_m \tilde{v}_t - \frac{(1+\gamma_m)(1-s_c)}{s_c}\tilde{x}_t - \tilde{Z}_t \end{aligned} \tag{A.35}$$

Next, substitute equation (A.24) into (A.35) to eliminate the monetary aggregate and impose the condition that at the natural rate of output $\tilde{u}_t^A = 0$.

$$\begin{aligned} 0 &= \tilde{Y}_t^n - \tilde{Z}_t + \omega(1-s_c)\tilde{u}_t^D - (1-s_c)\tilde{x}_t \\ &+ \left[\frac{s_c\gamma_m(1-\eta_v) - (1-s_c)(1+\gamma_m)\eta_v}{s_c}\right]\tilde{v}_t \\ &\equiv \tilde{Y}_t^n - \tilde{Z}_t + \omega(1-s_c)\tilde{u}_t^D - (1-s_c)\tilde{x}_t + \Psi_v^y \tilde{v}_t \end{aligned} \tag{A.36}$$

Finally we can eliminate $\tilde{u}_t^D$, and hence $\tilde{r}_t$, from (A.36) by log-linearizing $\tilde{u}_t^D = \tilde{\mathcal{S}}_t = \gamma_{u^D}^r \tilde{r}_t + \gamma_{u^D}^\tau \tilde{\tau}_t + \gamma_{u^D}^x \tilde{x}_t$ and then eliminate $\tilde{r}_t$ from this expression using the condition that $\tilde{u}_t^A = 0$ which implies (using Eq. (A.18))

$$0 = \tilde{u}_t^A = \gamma_{u^N}\gamma_{u^N}^r\tilde{r}_t + \gamma_{u^D}\left[\gamma_{u^D}^r\tilde{r}_t + \gamma_{u^D}^\tau\tilde{\tau}_t + \gamma_{u^D}^x\tilde{x}_t\right]$$

which defines the policy rate in terms of stochastic shocks. Clearly to stabilize the aggregate interest rate spread the policy rate must fall when the loan-deposit spread rises. This is confirmed below.

$$\begin{aligned}\tilde{r}_t &= -\left[\frac{\gamma_{u^D}\gamma_{u^D}^\tau}{\gamma_{u^N}\gamma_{u^N}^r + \gamma_{u^D}\gamma_{u^D}^r}\right]\tilde{\tau}_t - \left[\frac{\gamma_{u^D}\gamma_{u^D}^x}{\gamma_{u^N}\gamma_{u^N}^r + \gamma_{u^D}\gamma_{u^D}^r}\right]\tilde{x}_t \\ &\equiv -s_D^P\frac{\gamma_{u^D}^\tau}{\gamma_{u^D}^r}\tilde{\tau}_t - s_D^P\frac{\gamma_{u^D}^x}{\gamma_{u^D}^r}\tilde{x}_t \end{aligned} \tag{A.37}$$

Using this expression in $\tilde{u}_t^D = \tilde{\mathcal{S}}_t = \gamma_{u^D}^r\tilde{r}_t + \gamma_{u^D}^\tau\tilde{\tau}_t + \gamma_{u^D}^x\tilde{x}_t$ we have that

$$\begin{aligned}\tilde{u}_t^D &= (1 - s_D^P)\gamma_{u^D}^\tau\tilde{\tau}_t + (1 - s_D^P)\gamma_{u^D}^x\tilde{x}_t \\ &\equiv s_N^P\gamma_{u^D}^\tau\tilde{\tau}_t + s_N^P\gamma_{u^D}^x\tilde{x}_t \end{aligned} \tag{A.38}$$

Substituting (A.38) into (A.36) and solving for $\tilde{Y}_t^n$ we have

$$\tilde{Y}_t^n = \tilde{z}_t - \Psi_\tau^y\tilde{\tau}_t - \Psi_x^y\tilde{x}_t - \Psi_\upsilon^y\tilde{\upsilon}_t \tag{A.39}$$

where

$$\begin{aligned}\Psi_\tau^y &= (1 - s_c)\omega\gamma_{u^D}^\tau \\ \Psi_x^y &= (1 - s_c)[\omega s_N^P\gamma_{u^D}^x - 1] \\ \Psi_\upsilon^y &= \left[\frac{s_c\gamma_m(1 - \eta_\upsilon) - (1 - s_c)(1 + \gamma_m)\eta_\upsilon}{s_c}\right]\end{aligned}$$

Although the expression in (A.39) is only accurate to first order, it will be useful to define the equivalent expression in terms of stationary variables in levels for uniformness when we define the detrended equilibrium system below. It can be easily verified the resulting expression is proportional to

$$Y_t^n = \left(\frac{Z_t}{Z_{t-1}}\right)\upsilon_t^{-\Psi_\upsilon^y}x_t^{-\Psi_x^y}\tau_t^{-\Psi_\tau^y} \tag{A.40}$$

AppendixA.7. The Natural Rate of Interest

Combine equations (A.1), (A.6) and (A.13) and log-linearize the resulting expression to obtain the consumption Euler expression which is given by

$$\tilde{r}_t - E_t[\tilde{\pi}_{t+1}] = E_t[(1+\gamma_m)\Delta\tilde{C}_{t+1} - \gamma_m\Delta\tilde{M}^A_{t+1} - \Delta\tilde{r}^L_{t+1}] \\ + E_t[\gamma_m\Delta\tilde{\upsilon}_{t+1} - \Delta\tilde{a}_{t+1}] \quad \text{(A.41)}$$

Following a similar process that was used to arrive at the natural rate of output we can now substitute the market clearing condition (see (A.23)) to eliminate $\tilde{C}_t$ and the equilibrium monetary aggregate condition (see (A.24)) to eliminate $\tilde{M}^A_t$. Then impose the definition of the natural rate of output (see Def. 2, Section 2.7) on the resulting expression using (A.39) for the natural rate of output and almost all terms cancel leaving us with just

$$\tilde{r}^n_t = -E_t[\Delta\tilde{a}_{t+1}] + E_t[\Delta\tilde{Z}_{t+1}] - E_t[\Delta\tilde{r}^L_{t+1}] \quad \text{(A.42)}$$

To eliminate the interest rate from (A.42) use equation (A.38) to arrive at the expression for the natural rate of interest

$$\tilde{r}^n_t = \tilde{a}_t(1-\rho_a) - \tilde{z}_t(1-\rho_z) - \Psi^r_\tau\tilde{\tau}_t(1-\rho_\tau) - \Psi^r_x\tilde{x}_t(1-\rho_x) \quad \text{(A.43)}$$

where

$$\Psi^r_\tau = \left[\frac{\gamma_{u^D}\gamma^\tau_{u^D}}{\gamma_{u^N}\gamma^r_{u^N} + \gamma_{u^D}\gamma^r_{u^D}}\right] = \frac{\gamma^\tau_{u^D}}{\gamma^r_{u^D}}(1-s^p_N)$$

$$\Psi^r_\tau = \left[\frac{\gamma_{u^D}\gamma^x_{u^D}}{\gamma_{u^N}\gamma^r_{u^N} + \gamma_{u^D}\gamma^r_{u^D}}\right] = \frac{\gamma^x_{u^D}}{\gamma^r_{u^D}}(1-s^p_N)$$

Although the expression in (A.43) is only accurate to first order, it will be useful to define the equivalent expression in levels and impose the unit root in the technology process for uniformness when we define the equilibrium system below. It can be easily verified the resulting expression is proportional to

$$r^n_t = a_t^{(1-\rho_a)} Z_t^{(1-\rho_z)} \tau_t^{-\Psi^r_\tau(1-\rho_\tau)} x_t^{-\Psi^r_x(1-\rho_x)} \quad \text{(A.44)}$$

AppendixA.8. Calibrated Values

In Table D.10 we summarize the values assigned to the model's parameters in the baseline model. See section 3.1 for a detailed explanation of the calibration. Also, in section 5.2 we analyze *robustly* optimal policy in which case the values assigned to the parameters driving the model's stochastic shocks (which play a crucial role in shaping optimal policy) are varied over a large range of values.

AppendixA.9. The Detrended Equilibrium System

At this point it is now possible to define a symmetric equilibrium in the model. First, impose the equilibrium conditions (23), (25), (26), (27) and (28) on the models equations. As mentioned in the text, the resulting model, in its current form, is not stationary due to the unit roots in the technology process and the price level. Hence, we must next detrend the non-stationary variables and then define an equilibrium in the stationarized model. The resulting model is described in detail in the Table D.11.

AppendixB. The Simple-Sum Monetary Aggregate

When the Simple-Sum monetary aggregate (see Eq. 22) is embedded in the policy rule (see Eq. (18)) in place of the Divisia monetary aggregate (see Eq. 19) the economy is likely to be indeterminate. In this section we highlight why this occurs. Since issues of local determinacy can be sufficiently analyzed with a linear approximation to the model we linearize the Simple-Sum monetary aggregate to quantify its error in tracking the true aggregate. Begin by substituting (A.4) and (A.5) into the definition of the growth rate of the simple sum aggregate to eliminate N_t and D_t.

$$\begin{aligned} ln(\mu_t^{Simple-Sum}) &= ln\left(\frac{N_t + D_t}{N_{t-1} + D_{t-1}}\right) \\ &= \Delta ln(M_t^A) + \omega \Delta ln(u_t^A) \\ &+ ln\left(\frac{\nu(u_t^N)^{-\omega} + (1-\nu)(u_t^D)^{-\omega}}{\nu(u_{t-1}^N)^{-\omega} + (1-\nu)(u_{t-1}^D)^{-\omega}}\right) \end{aligned} \tag{B.1}$$

The expression for the growth rate of the Simple-Sum aggregate to this point is exact. To gain further insight we now linearize (B.1) around the competitive equilibrium.

$$\begin{aligned} ln(\mu_t^{Simple-Sum}) &\approx \Delta ln(M_t^A) + \omega \Delta\left[(\gamma_{u^N}\gamma_{u^N}^r + \gamma_{u^D}\gamma_{u^D}^r)\tilde{r}_t + \gamma_{u^D}(\gamma_{u^D}^\tau \tilde{\tau}_t + \gamma_{u^D}^x \tilde{x}_t)\right] \\ &- \omega \Delta\left[(\gamma_{u^N}^{SS}\gamma_{u^N}^r + \gamma_{u^D}^{SS}\gamma_{u^D}^r)\tilde{r}_t + \gamma_{u^D}^{SS}(\gamma_{u^D}^\tau \tilde{\tau}_t + \gamma_{u^D}^x \tilde{x}_t)\right] \\ &= \Delta ln(M_t^A) + \omega\left[\gamma_{u^N}^r(\gamma_{u^N} - \gamma_{u^N}^{SS}) + \gamma_{u^D}^r(\gamma_{u^D} - \gamma_{u^D}^{SS})\right]\Delta\tilde{r}_t \\ &+ \omega(\gamma_{u^D} - \gamma_{u^D}^{SS})\left[\gamma_{u^D}^\tau \Delta\tilde{\tau}_t + \omega\gamma_{u^D}^x \Delta\tilde{x}_t\right] \\ &= \Delta ln(M_t^A) + \omega(\gamma_{u^N}^r - \gamma_{u^D}^r)(\gamma_{u^N} - \gamma_{u^N}^{SS})\Delta\tilde{r}_t \\ &- \omega(\gamma_{u^N} - \gamma_{u^N}^{SS})\left[\gamma_{u^D}^\tau \Delta\tilde{\tau}_t + \omega\gamma_{u^D}^x \Delta\tilde{x}_t\right] \\ &\equiv \Delta ln(M_t^A) + \Psi_r^{SS}\Delta\tilde{r}_t - \Psi_\tau^{SS}\Delta\tilde{\tau}_t - \Psi_x^{SS}\Delta\tilde{x}_t \end{aligned} \tag{B.2}$$

Its worth noting that $\gamma_{u^N} - \gamma_{u^N}^{SS} = -(\gamma_{u^D} - \gamma_{u^D}^{SS}) = 0$ when $\bar{r}^D = 1$. That is, when currency and deposits are prefect one for one substitutes (i.e. their user costs are identical) the simple sum aggregate tracks the true aggregate without error. However, this environment is at odds with reality (and this model).[29]To gain further insight into the determinacy properties of Taylor rules reacting to the Simple-Sum aggregate, it is useful to evaluate Ψ_r^{SS} at the models steady state. [30] In which case we find that $\Psi_r^{SS} \approx 5$. To see why such a large

[29]Meanwhile, carrying out a similar linearization for $ln(\mu_t^{Divisia})$ one can verify that $\gamma_{u^N} - \gamma_{u^N}^{Divisia} = -(\gamma_{u^D} - \gamma_{u^D}^{Divisia}) = 0$ in general in this model. That is, regardless of whether $\bar{r}^D = 1$, $ln(\mu_t^{Divisia}) \approx \Delta ln(M_t^A)$.

[30]The values of Ψ_τ^{SS} and Ψ_x^{SS} have no bearing on determinacy.

positive value of Ψ_r^{SS} is troublesome examine the policy rule when reacting to the (growth rate) of the Simple-Sum aggregate.

$$\begin{aligned}\tilde{r}_t &= \phi_\pi \tilde{\pi}_t + \phi_m \tilde{\mu}_t^{SS} \\ &= \phi_\pi \tilde{\pi}_t + \phi_m \left[\Delta ln(M_t^A) + \Psi_r^{SS} \Delta \tilde{r}_t\right]\end{aligned} \tag{B.3}$$

Rearranging (B.3) so that it can explicitly expressed as an interest rate rule with a unitary coefficient on $\tilde{r}_t$ on the left hand side we have:

$$\tilde{r}_t = \begin{cases} -\left[\frac{\phi_m \Psi_r^{SS}}{1-\phi_m \Psi_r^{SS}}\right] \tilde{r}_{t-1} + \left[\frac{\phi_\pi}{1-\phi_m \Psi_r^{SS}}\right] \tilde{\pi}_t + \left[\frac{\phi_m}{1-\phi_m \Psi_r^{SS}}\right] \Delta ln(M_t^A) & 0 < \phi_m < 1/\Psi_r^{SS} \\ \left[\frac{\phi_m \Psi_r^{SS}}{\phi_m \Psi_r^{SS}-1}\right] \tilde{r}_{t-1} - \left[\frac{\phi_\pi}{\phi_m \Psi_r^{SS}-1}\right] \tilde{\pi}_t - \left[\frac{\phi_m}{\phi_m \Psi_r^{SS}-1}\right] \Delta ln(M_t^A) & \phi_m > 1/\Psi_r^{SS} \end{cases}$$

Using our baseline calibration, for $\phi_m \gtrapprox .2$ the sign of the policy rule's reaction to inflation is flipped so that higher inflation leads to lower nominal rates. Hence, the sizable error in tracking the true aggregate is positively related to the policy rate which leads to this reversed policy rule. Interestingly, the determinacy region displayed in Figure B.5 shows that the rule is not always indeterminate despite this perverse reaction to inflation. To see why notice that for $\phi_m > 1/\Psi_r^{SS}$ the policy rule is *super-inertial* with a *negative* reaction to inflation. Although the literature on determinacy is scant regarding super-inertial policy rules with a negative inflation reaction (for good reason), numerical analysis in this model appears to bring out a relationship sufficient for determinacy as follows:

$$If\ \phi_\pi < 0\ and\ \rho_r > 1\ then\ \rho_r + \phi_\pi > 1\ is\ determinate. \tag{B.4}$$

This condition however offers little hope for stabilizing Taylor rules which react to the simple sum aggregate since as $\phi_m \to \infty$ the policy rule becomes

$$\tilde{r}_t = \tilde{r}_{t-1} - \Delta ln(M_t^A) \tag{B.5}$$

which is always indeterminate.[31] Instead, the condition in (B.4) only explains why the simple-sum rule is *ever* determinate. Indeed, the small determinacy region in Figure B.5 is described by the condition:

$$If\ \phi_\pi > 0\ and\ 1 > \phi_m > \frac{1}{\Psi_r^{SS}}\ then\ \phi_m + \phi_\pi < 1\ is\ determinate.$$

Not surprisingly, the optimization on this small region of determinacy is not successful hence no policy prescription for policy rules reacting to simple-sum is available. Since reacting to simple-sum limits policy makers reaction to inflation

[31] Interestingly Eq. (B.5) is similar to a constant growth rate of Simple-Sum rule, suggesting using Simple-Sum as an instrument is troublesome as well.

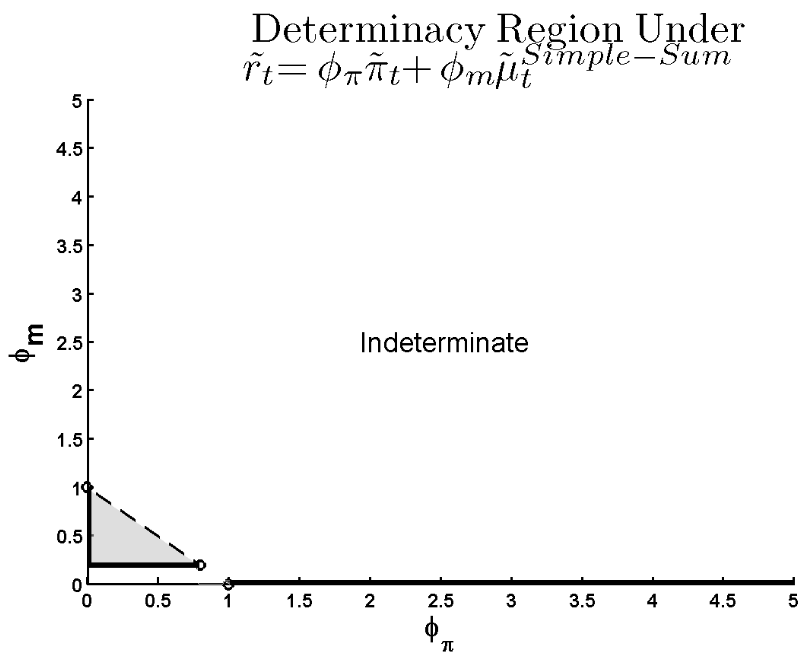

Figure B.5: The shadded area to the southwest of the dotted line, defined by $\phi_m > 1/\Psi_r^{SS}$ and $\phi_m + \phi_\pi < 1$, is determinate. Here ϕ_m denotes the response to the growth rate simple-sum monetary aggregate.

(something central bankers would understandably be adverse to doing), *the most practical policy prescription when reacting to Simple-Sum is to fix* $\phi_m = 0$. This stresses the importance of properly constructing monetary aggregates for use in policy making, extending the point of Belongia (1996) that "measurement matters" to the policy making realm. Indeed when the properly weighted Divisia aggregate is embedded in the policy rule, the determinacy region satisfies a novel Taylor principle for Divisia aggregates.[32]

AppendixC. The Ramsey Problem

In all welfare calculations we approximate the model around the Ramsey planner's steady state. We calculate this steady state using the OLS method described in Schmitt-Grohe and Uribe (2012). Specifically, in terms of detrended variables and equations, define

$$E_t[C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_t, S_{t-1})] = 0 \tag{C.1}$$

$$(S_t - S) = \rho(S_{t-1} - S) + \epsilon_t \tag{C.2}$$

[32] See Figure 5.3 and Eqs. (33) and (34).

where (C.1) denotes the system of equilibrium conditions defined in Table D.11 definition of the stochastic processes which are defined in equation (C.2). and hence $\mathcal{E}$ denotes the (1x17) vector of endogenous variables[33] and the vector S_t denotes the (5x1) vector of exogenous stochastic variables[34] implying ϵ_t is the vector of structural disturbances. The relevant portion of the Ramsey planners Lagrangian is given by

$$\begin{aligned}\mathcal{L} = \ldots - \beta^{t-1}\Gamma_{t-1}^T C(\mathcal{E}_t, \mathcal{E}_{t-1}, S_t, S_{t-1}) \\ + \beta^t u_t(\mathcal{E}_t, S_t) - \beta^t \Gamma_t^T C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_{t+1}, S_t) + \ldots\end{aligned}$$

where $\{\Gamma_t\}_{t=-1}^{\infty}$ denotes the sequence of (16x1) Lagrange multipliers for the 16 structural equations.[35]The relevant first order condition is given by the (1x17) matrix equation[36]

$$\begin{aligned}\frac{\partial u_t(\mathcal{E}_t, S_t)}{\partial \mathcal{E}_t} &= \Gamma_t^T E_t\left[\frac{\partial C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_{t+1}, S_t)}{\partial \mathcal{E}_t}\right] \\ &\quad + \beta^{-1}\Gamma_{t-1}^T \frac{\partial C(\mathcal{E}_t, \mathcal{E}_{t-1}, S_t, S_{t-1})}{\partial \mathcal{E}_t} \end{aligned} \tag{C.3}$$

Finding the Ramsey steady state requires finding values of $(\bar{\mathcal{E}}, \bar{\Gamma})$ that solve

$$\begin{aligned}\left.\frac{\partial u_t(\mathcal{E}_t, S_t)}{\partial \mathcal{E}_t}\right|_{(\bar{\mathcal{E}}, \bar{S})} &= \bar{\Gamma}^T \left.\frac{\partial C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_{t+1}, S_t)}{\partial \mathcal{E}_t}\right|_{(\bar{\mathcal{E}}, \bar{S})} \\ &\quad + \beta^{-1}\bar{\Gamma}^T \left.\frac{\partial C(\mathcal{E}_t, \mathcal{E}_{t-1}, S_t, S_{t-1})}{\partial \mathcal{E}_t}\right|_{(\bar{\mathcal{E}}, \bar{S})} \end{aligned} \tag{C.4}$$

$$C(\bar{\mathcal{E}}, \bar{\mathcal{E}}, \bar{S}, \bar{S}) = 0 \tag{C.5}$$

We guess a value of $\bar{\pi}$ and then solve (C.5) for the remaining variables in $\bar{\mathcal{E}}$. We then use $\bar{\mathcal{E}}$ to solve (C.4) for $\hat{\bar{\Gamma}}_{OLS}$ as proposed in Schmitt-Grohe and Uribe (2012) and define $\delta(\bar{\pi})$ as the resulting residual.[37] We then use MATLAB's fminsearch(-) to minimize $\delta(\bar{\pi})^T\delta(\bar{\pi})$ where the minimizing argument is the Ramsey steady state value of inflation. Under our baseline preferences this value is $\bar{\pi} = 1.028$ and under additively separable preferences this value is $\bar{\pi} = 1.033$.

[33]To be explicit the 17 endogenous variables are given by the detrended versions of: π_t, r_t, W_t,Λ_t^3, Λ_t^1, r_t^L, r_t^D, C_t, Λ_t^2, M_t^A, N_t, D_t, L_t, Y_t, h_t, F_t, Q_t .

[34]To be explicit the 5 exogenous stochastic variables are given by the detrended versions of: υ_t, a_t, x_t, τ_t, Z_t.

[35]To be explicit the 16 structural equations are given by: (3), (8), (10), (15), (24), (A.1)-(A.6), (A.13)-(A.15), (A.26), (A.29).

[36]All derivatives are computed analytically using MATLAB's symbolic toolbox.

[37]More specifically, $\hat{\bar{\Gamma}}_{OLS}$ is the projection of $\left[\beta^{-1}\frac{\partial C_{t-1}}{\partial \mathcal{E}_t} + \frac{\partial C_t}{\partial \mathcal{E}_t}\right]^T$ onto $\left[\frac{\partial U_t}{\partial \mathcal{E}_t}\right]^T$.

AppendixD. Tables

Table D.10: *Baseline*[1,2] Calibration of Parameters

Parameter		Value	Source
Discount Rate:	β	0.99	Kydland and Prescott (1982)
Disutility of Work:	η	2.5	Belongia and Ireland (2012)
Shopping-Time Function:	χ	2	Belongia and Ireland (2012)
CES Monetary Aggregate:	ν	0.225	Belongia and Ireland (2012)
CES Monetary Aggregate:	ω	1.5	Belongia and Ireland (2012)
Final Goods CES:	θ	6	Ireland (2000, 2004a,b)
Cost of Price Adjustment:	ϕ	50	Ireland (2004b)
Steady-State Inflation:	π	1.005	Belongia and Ireland (2012)
Money Demand Shock:	υ	0.4	Belongia and Ireland (2012)
Technology Shock:	Z	1.005	Belongia and Ireland (2012)
Reserves Demand Shock:	τ	0.02	Ireland (2011)
Banking Productivity Shock:	x	0.01	Belongia and Ireland (2006, 2012)
Preference Shock:	ρ_a	0.9579	Ireland (2004a)
Money Demand Shock:	ρ_υ	0.9853	Ireland (2004a)
Reserves Demand Shock:	ρ_τ	0.5	Belongia and Ireland (2012)
Banking Productivity Shock:	ρ_x	0.5	Belongia and Ireland (2012)
Preference Shock:	σ_a	0.0188	Ireland (2004a)
Money Demand Shock:	σ_υ	0.0088	Ireland (2004a)
Reserves Demand Shock:	σ_τ	0.7448	Dellas, Diba, and Loisel (2010)
Banking Productivity Shock:	σ_x	0.1720	Belongia and Ireland (2006) Dellas, Diba, and Loisel (2010)
Technology Shock:	σ_z	0.0098	Ireland (2004a)

[1] Here the baseline calibrations are presented. In section 5.2 we analyze *robustly* optimal policy and vary the calibrated values of the model's shocks.

[2] Under additively separable prefernces we set $\eta_m = .1$ which matches $\frac{N+D}{PC} = 3.3$ in steady state, the average of Simple-Sum M2 to nominal consumption expenditures since 1959.

Table D.11: Equilibrium Variables and System of *Equations*[1]

Variable	Detrended Variable	Equation
υ_t	υ_t	(5)
a_t	a_t	(6)
x_t	x_t	(7)
τ_t	τ_t	(9)
Z_t	Z_t/Z_{t-1}	(16)
P_t	Π_t	(18)
r_t	r_t	(A.13)
W_t	W_t/P_tZ_{t-1}	(A.29)
Λ_t^3	$\Lambda_t^3 Z_{t-1}$	(A.2)
Λ_t^1	$\Lambda_t^1 Z_{t-1}$	(A.14)
r_t^L	r_t^L	(A.6)
r_t^D	r_t^D	(10)
$\mathcal{S}_t$	$\mathcal{S}_t$	(11)
C_t	C_tZ_{t-1}	(A.1)
M_t^A	M_t^A/P_tZ_{t-1}	(3)
Λ_t^2	$\Lambda_t^2 Z_{t-1}$	(A.3)
N_t	N_t/P_tZ_{t-1}	(A.4)
D_t	D_t/P_tZ_{t-1}	(A.5)
L_t	L_t/P_tZ_{t-1}	(8)
Y_t	Y_t/Z_{t-1}	(24)
h_t	h_t	(15)
F_t	F_t/P_tZ_{t-1}	(A.26)
Q_t	Q_t/P_tZ_{t-1}	(A.15)
r_t^A	r_t^A	(A.18)
s_t^N	s_t^N	(20)
s_t^D	s_t^D	(21)
$\mu_t^{Divisia}$	$\mu_t^{Divisia}$	(19)
$\mu_t^{Simple-Sum}$	$\mu_t^{Simple-Sum}$	(22)
G_t^e	G_t^e	(A.32)
–[2]	Y_t^n	(A.40)
–[2]	r_t^n	(A.44)
$u_t = ln(C_t) - \eta h_t$ $-\eta\frac{1}{\chi}(\frac{\upsilon_t P_t C_t}{M_t^A})^\chi$	$u_t = ln(C_t/Z_{t-1}) - \eta h_t$ $-\eta\frac{1}{\chi}(\frac{\upsilon_t C_t/Z_{t-1}}{M_t^A/P_tZ_{t-1}})^\chi$	–[3]

[1] The order of the variables correspond to the order in which to solve the system to find the steady state. The corresponding equation number corresponds to the equation used to find the steady state value of the respective variable. (An exception to this is the steady state value for inflation which is calibrated and thus the policy rule is not needed to solve for its steady state value.)

[2] The natural rate of output and hence the natural rate of interest do not have a non-stationary analogue since these variables are only definded by a linear approximation around the steady state.

[3] The equation defining the steady state value of the utility function is simply the equation in the second column of this table.