Hierarchical contagions in the interdependent financial network

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Abstract

We model hierarchical cascades of failures among banks linked through an interdependent network. The interaction among banks include not only direct cross-holding, but also indirect dependency by holding mutual assets outside the banking system. Using data extracted from the European Banking Authority, we present the interdependency network composed of 48 banks and 21 asset classes. Since interbank exposures are not public, we first reconstruct the asset/liability cross-holding network using the aggregated claims. For the robustness, we employ 3 reconstruction methods, called *Anan*, *Hała* and *Maxe*. Then we combine the external portfolio holdings of each bank to compute the interdependency matrix. The interdependency network is much more dense than the direct cross-holding network, showing the complex latent interaction among banks. Finally, we perform macroprudential stress tests for the European banking system, using the adverse scenario in EBA stress test as the initial shock. For different reconstructed networks, we illustrate the hierarchical cascades and show that the failure hierarchies are roughly the same except for a few banks, reflecting the overlapping portfolio holding accounts for the majority of defaults. Understanding the interdependency network and the hierarchy of the cascades should help to improve policy intervention and implement rescue strategy.

Keywords: financial network, interdependent network, contagions, stress test, macroprudential
JEL: G01, G21, G32, G33, D85

1. Introduction

In recent years, network models, systemic stress testing and financial stability have attracted growing interest both among scholars and practitioners (Battiston and Martinez-Jaramillo, 2018). Regular stress tests conducted by authorities, such as the European Banking Authority, aim to evaluate the performance of individual banks in adverse scenarios, which are microprudential. Macroprudential outcomes are not simply the summation of micropridential changes. For example, when financial innovation reduces the cost of diversification, this may trigger a transition from stationary return dynamic to a nonstationary one (Corsi et al., 2016). Therefore, to be truly macroprudential, it is necessary to assess the role of network contagion in potentially amplifying systemic risk (Gai and Kapad, 2019).

There are different interactive channels among financial institutions. Figure 1 illustrates 3 types of financial networks: (a) interbank network, (b) bank-asset bipartite network and (c) interdependent network. The interbank network characterizes direct credit exposures to other banks and risk contagion can be caused by direct cross-holding. Take Dungey et al. (2020) for example, they empirically analyze the transmission of shocks between global banks, domestic banks and the non-financial sector for 11 Eurozone countries. Apart from direct connection, it's apparent that banks are indirectly connected by holding overlapping portfolio outside the banking system as in Figure 1 (b). Barucca et al. (2021) empirically find significant overlapping equity and debt portfolios between different types of financial institution, providing evidence for the existence of a price-mediated channel of contagion between banks.

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The third type of network is much more complex, including not only direct cross-holding, but also indirect dependency by holding mutual assets as in Figure 1 (c). This interdependency has been shown as a realistic source of uncertainty in systemic risk (Roukny et al., 2018). Furthermore, Elliott et al. (2014) study cascading failures in an equilibrium model of interdependent financial network.

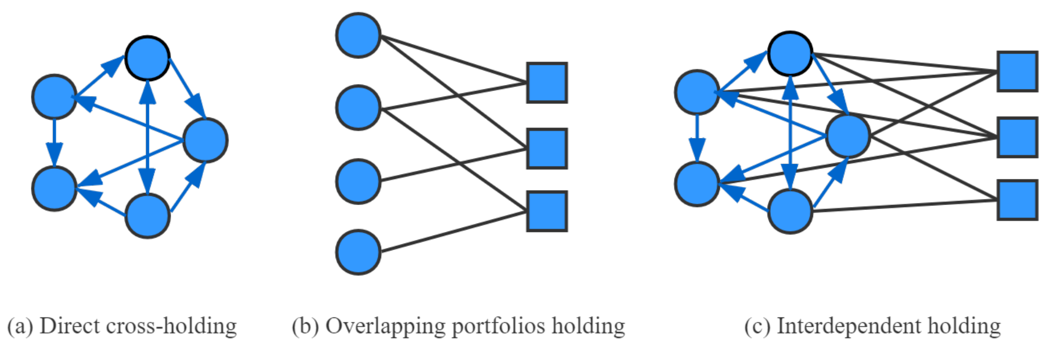


Figure 1: Illustrative examples showing 3 types of financial networks. Circles indicate banks and squares indicate assets.

Since interdependent network model provides two contagion channels and is more realistic, it is worthy of further study. The main goal of this study is to identify cascade hierarchies in the interdependent financial network. Given the available literature our contribution is threefold. First, we slightly revised the model of Elliott et al. (2014) by separating the bank's "value" delivered to final investors outside the banking system to external liabilities and equity value. Such division is in line with the balance sheet and can make clear the bank value in the general sense, although it does not change the derived form of interdependency matrix. This modification also facilitates empirical research for the European banking system, because the European Banking Authority (EBA) dataset does not provide liability items, but only asset items and some equity items such as Tier 1 capital. Second, we integrate microprudential stress test and macroprudential stress test together for the European banking system. Considering that the EBA's stress test is microprudential for individual banks, we perform macroprudential stress test by using the adverse scenario in EBA's stress test as the initial shock. Third, since granular data on interbank credit exposures is not public, we employ 3 reconstruction methods to form the cross-holding network and then study contagion hierarchies comparatively.

The remainder of the paper is organized as follows. Section 2 presents the literature review. Section 3 introduces the model and method of identifying cascade hierarchies. Section 4 shows the data and the empirical analyses. Section 5 concludes the paper.

2. Literature review

As in Figure 1, we review existing literature about network contagion according to the network structures adopted.

2.1. Interbank network contagions

This kind of model shows that contagion can be caused by direct credit exposures among banks. Rogers and Veraart (2013) model financial market as a directed graph of interbank obligations and study the occurrence of systemic risk. Gai and Kapadia (2010) develop an analytical network contagion model and suggest that financial systems exhibit a robust-yet-fragile tendency. That is, while the probability of contagion may be low, the influences can be extremely widespread when problems occur. Similarly, Acemoglu et al. (2015) argue that the extent of financial contagion exhibits a form of phase transition. In addition, many studies focus on how interbank network topology creates instability (Bardoscia et al., 2017; Eboli, 2019). Zhang et al. (2021) find that network connectedness of banks strengthens the relationship between liquidity creation and systemic risk. Brunetti et al. (2019) study the interbank market around the 2008 financial crisis and find that the correlation network and the physical credit network behavior different. During the crisis, the correlation network displays an increase in connection, while the physical credit network shows a marked decrease in connection.

2.2. *Overlapping portfolio contagions*

When a bank suffers a negative shock to its equity, a natural way to return to target leverage is to sell assets. Greenwood et al. (2015) present a model in which fire sales propagate shocks across banks. Huang et al. (2013) build a bipartite banking network model composed of banks and assets and present a cascading failure describing the risk propagation process during crises. Similarly, Caccioli et al. (2014) show the amplification of financial contagion due to the combination of overlapping portfolios and leverage, in terms of a generalized branching process. Furthermore, for quantifying the potential exposure to indirect contagion arising from deleveraging of assets in stress scenarios, Cont and Schaanning (2019) propose two indicators. Vodenska et al. (2021) build a bipartite network with weighted links between banks and assets based on sovereign debt holdings, and then model the systemic risk propagation.

2.3. *Interdependent network contagions*

This kind of model investigates how these two channels (the interbank channel and the overlapping channel) propagate individual defaults to systemic cascading failures. Caccioli et al. (2015) argue that neither channel of contagion results in large effects on its own. In contrast, when both channels are active, defaults are much more common and have large systemic effects. Aldasoro et al. (2017) likewise suggest that contagion occurs through deleveraging and interbank connection. The interdependent network models are also applied to characterize contagions in reinsurance and derivatives markets (Klages-Mundt and Minca, 2020; Paddrik et al., 2020).

Elliott et al. (2014) study cascading failures in an interdependent financial network. They show that discontinuous changes in asset values trigger further failures. Furthermore, when banks face potentially correlated risks from outside the financial system, the interbank connections can share these risks, but they also create the channels by which shocks can be propagated (Elliott et al., 2021). In addition, some studies find that the overlapping portfolio holding by banks accounts for the majority of defaults. Chen et al. (2016) confirm that the market liquidity effect has a great potential to cause systemic contagion. Dungey et al. (2020) show that deleveraging speed and concentration of illiquid assets play a critical role in cascades. Ma et al. (2021) further prove that illiquidity is a critical factor in triggering risk contagion and that higher interbank leverage can cause larger losses for both the banks and the external assets. Our results are consistent with these literature, in the sense that the general contagion hierarchies are mainly determined by the overlapping channel, while the structure of interbank network is also important for some specific banks.

3. Methodology

3.1. *The model*

The model follows Elliott et al. (2014), but separates the “value” in their paper, that any bank delivers to final investors outside the system of cross-holding, to external liabilities and equity value. Concretely, for every bank, its assets are divided into external assets and interbank assets, and its liabilities are divided into external liabilities and interbank liabilities. The equity value is the difference between its total assets and its total liabilities. Table 1 illustrates a balance sheet based on this.

Table 1: Balance Sheet of Bank i.

Assets		Liabilities	
External assets	$\sum_k D_{ik} p_k$	External liabilities	$l_i^{(e)} V_i$
Interbank assets	$a_i \equiv \sum_j C_{ij} V_j$	Interbank liabilities	$l_i \equiv \sum_j C_{ji} V_i$
		Net worth	v_i

Assume that there are N banks and M external assets. The current value of asset k is denoted p_k. Let $D_{ik} \geq 0$ be the fraction of the value of asset k held directly by bank i and let $\mathbf{D}$ donate the matrix whose entry is equal to D_{ik}. A bank can also hold shares of other banks. Let $C_{ij} \geq 0$ is the fraction of bank j owned by bank i, where $C_{ii} = 0$ for each i. The cross-holding matrix $\mathbf{C}$ can be viewed as a network in which there is a directed link from j to i if cash flows in that direction, in other words, if i owns a positive share of j.

Let V_i be the total asset value of bank i. This is equal to the value of external assets holding by bank i plus the value of its claims on other banks:

$$V_i = \sum_k D_{ik} p_k + \sum_j C_{ij} V_j. \tag{1}$$

Equation (1) can be written in matrix notation as

$$\mathbf{V} = \mathbf{Dp} + \mathbf{CV} \tag{2}$$

and solved to yield

$$\mathbf{V} = (\mathbf{I} - \mathbf{C})^{-1} \mathbf{Dp}. \tag{3}$$

On the other hands, the total value of bank i is also equal to its total liabilities plus its equity value v_i. Its total liabilities constitute of interbank liabilities $\sum_j C_{ji} V_i$ and external liabilities $l_i^{(e)} V_i$, where $l_i^{(e)}$ is the ratio of external liabilities to total assets. Hence, the equity value of bank i:

$$v_i = \sum_j C_{ij} V_j - \sum_j C_{ji} V_i + \sum_k D_{ik} p_k - l_i^{(e)} V_i. \tag{4}$$

Now we denote the capital ratio (i.e. ratio of equity value to total value) of bank i as $\widehat{C}_{ii}$, then

$$\widehat{C}_{ii} \equiv 1 - l_i^{(e)} - \sum_{j \in N} C_{ji}. \tag{5}$$

Note that the off-diagonal entries of the matrix $\widehat{\mathbf{C}}$ are defined to be 0. Hence, Equation (4) can be written in matrix notation as

$$\mathbf{v} = \mathbf{CV} - (\mathbf{I} - \widehat{\mathbf{C}})\mathbf{V} + \mathbf{Dp} = (\mathbf{C} - (\mathbf{I} - \widehat{\mathbf{C}}))\mathbf{V} + \mathbf{Dp}. \tag{6}$$

Substituting for the total asset value V from (3), this becomes

$$\begin{aligned} \mathbf{v} &= (\mathbf{C} - \mathbf{I} + \widehat{\mathbf{C}})(\mathbf{I} - \mathbf{C})^{-1}\mathbf{Dp} + \mathbf{Dp} = (\mathbf{C} - \mathbf{I} + \widehat{\mathbf{C}} + (\mathbf{I} - \mathbf{C}))(\mathbf{I} - \mathbf{C})^{-1}\mathbf{Dp} \\ &= \widehat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}\mathbf{Dp} = \mathbf{ADp}. \end{aligned} \tag{7}$$

Here we refer to $\mathbf{A} = \widehat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}$ as the interdependency matrix.

As in Elliott et al. (2014), banks will lose some value in discontinuous ways if their values fall below certain critical thresholds. In fact, it's these discontinuities that lead to cascading failures. If the equity value v_i of a bank i falls below some threshold level $\underline{v}_i$, then the bank is said to fail and incurs failure costs $\beta_i \underline{v}_i$. In many situations, a natural cap for β_i is 1. That is, the maximum loss that can result from the failure of bank i is its value at the time of failure.

The valuations in (3) and (7) are similar when we include the discontinuous failure costs, and so the total value of bank i becomes

$$V_i = \sum_{j \neq i} C_{ij} V_j + \sum_k D_{ik} p_k - \beta_i \underline{v}_i I_{v_i < \underline{v}_i}, \tag{8}$$

where $I_{v_i < \underline{v}_i}$ is an indicator variable taking value 1 if $v_i < \underline{v}_i$ and value 0 otherwise.

This leads to a new version of (3):

$$\mathbf{V} = (\mathbf{I} - \mathbf{C})^{-1}(\mathbf{Dp} - \mathbf{b}(\mathbf{v})), \tag{9}$$

where $b_i(\mathbf{v}) = \beta_i \underline{v}_i I_{v_i < \underline{v}_i}$. Correspondingly, (7) is re-expressed as

$$\mathbf{v} = \widehat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}(\mathbf{Dp} - \mathbf{b}(\mathbf{v})) = \mathbf{A}(\mathbf{Dp} - \mathbf{b}(\mathbf{v})). \tag{10}$$

An entry A_{ij} of the interdependency matrix describes the proportion of j's costs that i pays when j fails as well as i's claims on the external assets that j directly holds.

3.2. Identifying cascade hierarchies

Based on the interdependent model, we can trace the propagation path initiated by a specific shock. At step t, let Z_t be the set of failed banks. Initialize $Z_0 = \emptyset$ and $\underline{\mathbf{v}} = \theta \mathbf{v}_0$. Assume an adverse scenario that causes prices of mutual assets to decline. Then the cascade hierarchies can be identified as following. At each step $t \geq 1$:

1. Let $\widetilde{\mathbf{b}}_{t-1}$ be a vector with element $\widetilde{b}_i = \beta_i \underline{v}_i$ if $i \in Z_{t-1}$ and 0 otherwise.
2. Let Z_t be the set of all k such that entry k of the following vector is negative:

$$\mathbf{A}\left[\mathbf{Dp} - \widetilde{\mathbf{b}}_{t-1}\right] - \underline{\mathbf{v}}. \tag{11}$$

3. Terminate if $Z_t = Z_{t-1}$. Otherwise return to step 1.

When this algorithm terminates at step T, the sets $Z_1, Z_2, ..., Z_T$ correspond to the failure hierarchies.

4. Empirical analyses for European banking system

4.1. Data

We use data collected by the European Banking Authority (EBA) for the 2018 EU-wide stress test. This public dataset covers a sample of 48 banks in 15 countries in the European Union and European Economic Area at the highest level of consolidation. Table 2 lists 48 banks and countries they belong to. This dataset not only provides the actual balance sheet figures and their International Financial Reporting Standard (IFRS) 9 restated figures, but also covers a three-year horizon baseline and adverse scenarios, which take the end-2017 data as the starting point[1].

The actual and restated figures give the exposure values in various asset classes. Table 3 lists 21 asset classes that we collect from the EBA dataset and provides corresponding EBA items and EBA exposure codes for each type of asset. Among them, type 2100 indicates the aggregated claims on other credit institutions that one bank holds. However, granular exposure data on banking networks is not public. The credit exposure networks can be reconstructed by some inference methods using only aggregated relational data (Anand et al., 2018). The other 20 classes are the external assets mutually holding by 48 banks.

The adverse scenario gives the corresponding exposure values of various asset classes under some assumed macroeconomic shocks, including a growth in gross domestic product (GDP) in the EU of -1.2%, -2.2% and 0.7% as of 2018, 2019 and 2020 respectively. This adverse scenario can be viewed as a ideal initial shock with which the proposed hierarchical contagion model will be tested.

4.2. Reconstruction of interbank network

In order to test the reliability of contagious hierarchies identified by the proposed model, we employ 3 network reconstruction methods to build the asset/liability cross-holding network. We call these 3 methods *Anan* (Anand et al., 2015), *Hała* (Hałaj and Kok, 2013) and *Maxe* (Upper and Worms, 2004). Either of 3 methods can reconstruct interbank networks with aggregated assets and liabilities. However, the EBA dataset only provides asset exposures, no liability data. We refer to some empirical studies based on these data assuming that for bank i, the aggregated interbank assets $\sum_j C_{ij} V_j$ equal to the aggregated interbank liabilities $\sum_j C_{ji} V_i$ (Chen et al., 2016; Glasserman and Young, 2015). We now give a brief description for these 3 methods.

[1]https://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2018

Table 2: Bank list.

Country code	Country	Bank	Bank abbr.
AT	Austria	Raiffeisen Bank International AG	RBI
AT	Austria	Erste Group Bank AG	EBS
BE	Belgium	KBC Group NV	KBC
BE	Belgium	Belfius Banque SA	Belfius
DE	Germany	DZ BANK AG Deutsche Zentral-Genossenschaftsbank	DZ Bank
DE	Germany	Landesbank Baden-Wurttemberg	LBBW
DE	Germany	Deutsche Bank AG	DBK
DE	Germany	Commerzbank AG	CBK
DE	Germany	Norddeutsche Landesbank - Girozentrale -	NORD/LB
DE	Germany	Bayerische Landesbank	BayernLB
DE	Germany	Landesbank Hessen-Thuringen Girozentrale AdoR	Helaba
DE	Germany	NRW.BANK	NRW
DK	Denmark	Danske Bank	Danske
DK	Denmark	Jyske Bank	JYSK
DK	Denmark	Nykredit Realkredit	Nykredit
ES	Spain	Banco Santander S.A.	SAN
ES	Spain	Banco Bilbao Vizcaya Argentaria S.A.	BBVA
ES	Spain	CaixaBank, S.A.	CABK
ES	Spain	Banco de Sabadell S.A.	SAB
FI	Finland	OP Financial Group	OP
FR	France	BNP Paribas	BNP
FR	France	Groupe Credit Agricole	ACA
FR	France	Societe Generale S.A.	GLE
FR	France	Groupe Credit Mutuel	GCM
FR	France	Groupe BPCE	BPCE
FR	France	La Banque Postale	LABP
GB	United Kingdom	Barclays Plc	BARC
GB	United Kingdom	Lloyds Banking Group Plc	LLOY
GB	United Kingdom	HSBC Holdings Plc	HSBC
GB	United Kingdom	The Royal Bank of Scotland Group Plc	RBS
HU	Hungary	OTP Bank Nyrt.	OTP
IE	Ireland	Bank of Ireland Group plc	BIR
IE	Ireland	Allied Irish Banks Group plc	AIB
IT	Italy	UniCredit S.p.A.	UNCRY
IT	Italy	Intesa Sanpaolo S.p.A.	ISP
IT	Italy	Banco BPM S.p.A.	BPM
IT	Italy	Unione di Banche Italiane Societa Per Azioni	UBI
NL	Netherlands	N.V. Bank Nederlandse Gemeenten	BNG
NL	Netherlands	ABN AMRO Group N.V.	ABN
NL	Netherlands	ING Groep N.V.	ING
NL	Netherlands	Cooperatieve Rabobank U.A.	Rabobank
NO	Norway	DNB Bank Group	DNB
PL	Poland	Powszechna Kasa Oszczednosci Bank Polski SA	PKO
PL	Poland	Bank Polska Kasa Opieki SA	PEO
SE	Sweden	Skandinaviska Enskilda Banken - group	SEB
SE	Sweden	Nordea Bank - group	Nordea
SE	Sweden	Swedbank - group	SWDB
SE	Sweden	Svenska Handelsbanken - group	SHB

Table 3: Asset classes and their EBA data reference codes.

EBA Item	EBA Exposure	Asset classes
183203, 183303	1100	Central banks and central governments
	1200	Regional governments or local authorities
	1300	Public sector entities
	1400	Multilateral Development Banks
	1500	International Organisations
183904, 183905	1700	General governments
	2100	**Credit institutions**
	2200	Other financial corporations
183203, 183303	3000	Corporates (Credit Risk) / Non Financial corporations (NPE- Forbearance)
	4110	Retail - Secured by real estate property - SME
	4120	Retail - Secured by real estate property - Non SME
	4200	Retail - Qualifying Revolving
	4310	Retail - Other - SME
	4320	Retail - Other - Non SME
	4500	Retail - SME
183904, 183905	4700	Households
183203, 183303	5000	Secured by mortgages on immovable property
	6400	Items associated with particularly high risk
	6500	Covered bonds
	6600	Claims on institutions and corporates with a ST credit assessment
	6700	Collective investments undertakings (CIU)

4.2.1. Anan

Anand et al. (2015) propose a method combining information-theoretic arguments with economic incentives to keep the realistic features of interbank network. The authors argue that the Minimum Density (MD) method is suitable for sparse networks such as financial markets, and is able to minimize the cost of additional linkages to reconstruct the network.

Based on this method, c is defined as the fixed cost of establishing a link, N represents the number of banks. C notes the matrix of aggregated exposure values. The aggregated interbank assets of bank i are $\sum_{j=1}^{N} C_{ij}$, and its aggregated liabilities are $\sum_{j=1}^{N} C_{ji}$. Then, the MD method is formulated as:

$$
\begin{aligned}
&\min c \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbf{1}\{C_{ij} \geq 0\}, \quad s.t. \\
&\sum_{j=1}^{N} C_{ij} = a_i \quad \forall i = 1, 2, ..., N \\
&\sum_{i=1}^{N} C_{ij} = a_j \quad \forall j = 1, 2, ..., N \\
&C_{ij} \geq 0 \quad \forall i, j,
\end{aligned}
\tag{12}
$$

where the integer function $\mathbf{1}$ is equal to one, only if bank i lends to bank j, and zero otherwise. Here, the authors design a heuristic to solve this computationally expensive problem.

4.2.2. *Hała*

Hałaj and Kok (2013) propose an iterative algorithm to randomly generate a series of interbank networks. At the initial network, assume that the possibility of all links is the same that all entries in the matrix C^0 are equal to zero, and the unmatched interbank assets and liabilities are initiated as $a^0 = a$ and $l^0 = l$. When iterating to the $k+1$ step, a pair of banks (i, j) are randomly selected. Next, extract the random number f from the unit interval to re-scale the matrix to update the weight C_{ij}^{k+1} as follows:

$$C_{ij}^{k+1} = C_{ij}^{k} + f^{k+1} \min\{a_i^k, l_j^k\} \tag{13}$$

and the unmatched assets and liabilities are:

$$a_i^{k+1} = a_i^k - \sum_{j=1}^{N} C_{ij}^{k+1} \ \ and \ \ l_j^{k+1} = l_j^k - \sum_{i=1}^{N} C_{ij}^{k+1} \tag{14}$$

The iteration is repeated until no more interbank assets are left to be assigned.

4.2.3. *Maxe*

Maxe is the maximum entropy method, the basis of iterative methods (Upper and Worms, 2004). In the initial guess network, the exposure of bank i to bank j is equal to the aggregated exposure of bank i multiplied by the aggregated exposure of bank j, namely, $Q_{ij} = a_i a_j$. Next, the network is re-scaled until the constraints are satisfied. This entails maximizing the entropy function:

$$-\sum_{i,j} C_{i,j} \log(C_{i,j}/Q_{i,j}) \tag{15}$$

Entropy optimization can achieve network reconstruction through an effective iterative algorithm. Paltalidis et al. (2015) employ this method to reconstruct interbank network to study transmission channels of systemic risk.

4.2.4. *Reconstructed European interbank networks*

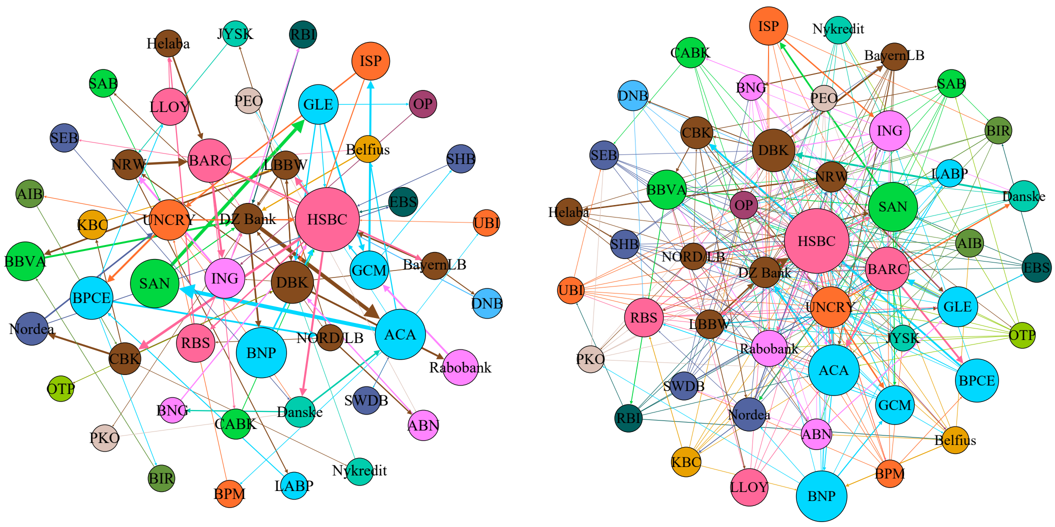
Table 4 reports the network statistics we compute for reconstructed networks using above 3 approaches. It's shown that the reconstructed networks are very different. Network generated by *Maxe* has the largest number of links, the highest density and degree, so as to clustering and core size. This is because *Maxe* network is fully connected. Compared *Anan* with *Hala*, we find that the *Anan* network is more sparse, having lower density and clustering, smaller average degree and core size. The lender/borrower dependency is defined as the average of the market share of the largest borrower or lender, respectively. The HHI (Herfindahl-Hirschman Index) describes the concentration of both assets and liabilities. Due to the sparsity of *Anan* network, it's reasonable that this network has higher dependency and concentration. The assortativity characterizes the preference for a network's nodes to attach to others that are similar. Both *Anan* and *Hala* have negative assortativities, which is consistent with the statistic of the genuine interbank networks computed in Anand et al. (2018).

Figure 2 displays the European interbank network (direct cross-holding matrix **C**) reconstructed by *Anan* and *Hała* respectively. The widths of the arrows are proportional to the sizes of the cross-holdings. The area of bank node is proportional to its equity value. The banks with the same color are belong to the same country. The arrow direction means that the origin bank has claims on the destination bank. Consistent with Table 4, the *Anan* network is more sparse than the *Hała* network. We can also find that in both reconstructed networks, banks from the UK (the pink node), Germany (the brown node) and France (the blue node) are located in more central positions, showing that these banks are connected densely.

Figure 3 displays the interdependent matrix **A** in European banking system reconstructed by *Anan* and *Hała*. The widths of the arrows are proportional to the degrees of inter-dependency. Note that the interdependent matrix **A** not only describes the direct cross-holding among banks, but also the indirect claims on the external assets that other banks hold. Therefore, the interdependent network **A** are more dense than the direct interbank network **C**. This is exactly explain what is interdependency and the difference between interdependency model and simple cross-holding model.

Table 4: Network statistics for reconstructed interbank networks.

	Anan	*Hała*	*Maxe*
Number of Links	99	344	2256
Density	4.388	15.248	100.000
Avg Degree	2.063	7.167	47.000
Med Degree	1	7	47
Assortativity	-0.308	-0.321	NaN
Clustering	0.678	21.794	100.000
Lender Dependency	83.718	57.910	10.708
Borrower Dependency	86.334	71.905	10.708
Mean HHI Assets	0.785	0.463	0.045
Median HHI Assets	1.000	0.439	0.045
Mean HHI Liabilities	0.824	0.639	0.045
Median HHI Liabilities	1.000	0.620	0.045
Core Size (% banks)	10.417	18.750	97.917



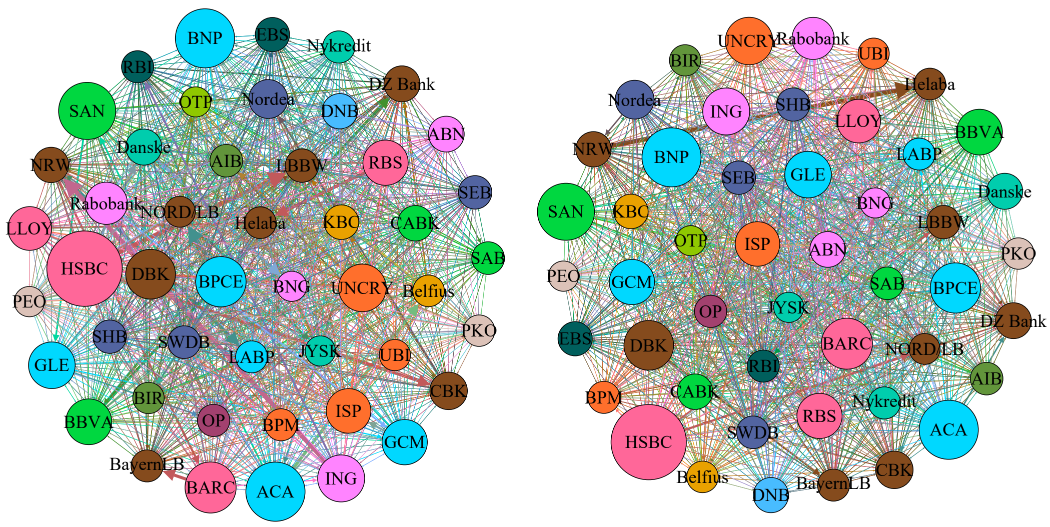
(a) Direct cross-holding **C** reconstructed by *Anan*.

(b) Direct cross-holding **C** reconstructed by *Hała*.

Figure 2: Direct cross-holding matrix **C** in European banking system reconstructed by *Anan* and *Hała*. The widths of the arrows are proportional to the sizes of the cross-holdings. The area of bank node is proportional to its equity value. The banks with the same color are belong to the same country.

4.3. Cascades

To illustrate the hierarchical cascades, we consider the adverse scenario in EBA 2018 EU-wide stress test. The initial shock to the values of 20 types of external assets is extracted from the adverse scenario as of 2020. The failure thresholds $\underline{\mathbf{v}}$ are set to θ times the IFRS 9 restated figures at the end-2017 (which is the actual balance sheet data).



(a) Interdependent matrix **A** reconstructed by *Anan*. (b) Interdependent matrix **A** reconstructed by *Hała*.

Figure 3: Interdependent matrix **A** in European banking system reconstructed by *Anan* and *Hała*. The widths of the arrows are proportional to the degrees of inter-dependency. The area of bank node is proportional to its equity value. The banks with the same color are belong to the same country.

Various levels of θ are chosen to test the cascade process. If a bank fails, then the loss in value is $\beta \underline{v}_i$, where β is set to 0.3 for lower failure cost and 0.8 for higher failure cost.

We examine the results for *Anan* network, *Hała* network and *Maxe* network respectively. In Table 5, Panel A and B display the hierarchies of cascades for *Anan* reconstructed network. In case of $\theta = 0.971$, there are 5 banks hit its failure point under the initial shock. For both levels of failure costs, cascades do not occur. We then raise θ to 0.973 and see how cascades occur. In this case, there are 17 banks failed under the initial shock. Then DZ Bank, BayernLB and ING are triggered by a contagion when $\beta = 0.3$. When failure cost is raised to 0.8, three more banks (EBS, GLE and UNCRY) are failed in this hierarchy. In the next cascading round, when $\beta = 0.3$, LBBW and BBVA are triggered to fail due to their exposures to the former two rounds of failed banks. For example, both LBBW and BBVA have claims on DZ Bank (see Figure 2(a)). Pushing β up to 0.8, there are two more banks (Belfius and OP) failed due to taking higher failure cost.

Panel C and D in Table 5 display the hierarchies of cascades for *Hała* reconstructed network. The initial failed banks are the same as the *Anan* cases. However, cascades are triggered in case of $\theta = 0.971$, that is, causing UBI to fail. This is due to the fact that cross-holding network reconstructed by *Hała* has higher connection and density compared to the *Anan* network. In the next round, UBI's failure further causes RBI to fail because RBI has claims on UBI (see Figure 2(b)). When failure cost is raised to 0.8, similar with the *Anan* case, there are more banks failed in each cascading round and finally up to four failure hierarchies. Pushing θ up to 0.973 leads to more banks failed and would cause failures at earlier levels, but would not change the ordering. Take $\beta = 0.8$ for example, in case of $\theta = 0.971$, the DZ Bank failed at the third hierarchy, while in case of $\theta = 0.973$, the DZ Bank failed at second hierarchy.

Panel E and F in Table 5 display the hierarchies of cascades for *Maxe* reconstructed network. It is found that the

Table 5: Hierarchies of cascades in macroprudential stress test for the European banking system. Three reconstructure algorithms (i.e. *Anan*, *Hała* and *Maxe*) for the interbank cross-holding network are considered. This table reports the test results with different failure thresholds θ and different failure cost coefficients β.

	β=0.3	β=0.8
Panel A: *Anan*, θ=0.971		
First Failure	JYSK, GCM, Rabobank, DNB, SEB, SHB	JYSK, GCM, Rabobank, DNB, SEB, SHB
Panel B: *Anan*, θ=0.973		
First Failure	RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB	RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB
Second Failure	DZ Bank, BayernLB, ING	EBS, DZ Bank, BayernLB, GLE, UNCRY, ING
Third Failure	LBBW, BBVA	Belfius, LBBW, BBVA, OP
Panel C: *Hała*, θ=0.971		
First Failure	JYSK, GCM, Rabobank, DNB, SEB, SHB	JYSK, GCM, Rabobank, DNB, SEB, SHB
Second Failure	UBI	HSBC, UBI
Third Failure	RBI	RBI, DZ Bank, ACA, AIB
Fourth Failure		LBBW
Panel D: *Hała*, θ=0.973		
First Failure	RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, ING, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB	RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, ING, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB
Second Failure	DZ Bank, UNCRY	Belfius, DZ Bank, LBBW, BBVA, UNCRY
Third Failure	LBBW	Helaba, OP
Panel E: *Maxe*, θ=0.971		
First Failure	JYSK, GCM, Rabobank, DNB, SEB, SHB	JYSK, GCM, Rabobank, DNB, SEB, SHB
Second Failure	UBI	HSBC, UBI
Third Failure	RBI	RBI, DZ Bank, ACA, AIB
Fourth Failure		LBBW
Panel F: *Maxe*, θ=0.973		
First Failure	RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, ING, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB	RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, ING, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB
Second Failure	DZ Bank, UNCRY	Belfius, DZ Bank, LBBW, BBVA, UNCRY
Third Failure	LBBW	Helaba, OP

cascading hierarchies are exactly the same as the *Hała* case. Even compared with the *Anan* case, the failure banks and the cascading hierarchies are roughly the same. It's reasonable since banks' external assets holdings weight more and play a key role in cascading dynamics. However, the structure of cross-holding network is also important for some specific banks. For example, Helaba failed in the cases of *Hała* and *Maxe*, while not in the *Anan* case. Our results are consistent with Chen et al. (2016), who find that the market liquidity effect has a greater potential than the network effect to cause systemic contagion.

5. Concluding remarks

Based on a simple model of interdependent financial networks, we have examined cascades in the European banking system. The interdependency means that the connections between banks include not only direct cross-holding (interbank network) but also indirect dependency by holding mutual assets outside the banking system (bipartite network). Through analyzing bank's balance sheet, an equilibrium matrix is derived to characterize this interdependency.

We use data extracted from the European Banking Authority to illustrate the interdependency. First, we collect 20 classes of external assets mutually holding by 48 banks. For the cross-holding, interbank exposures are not available but the aggregated claims are public. Then we employ 3 network reconstruction methods to build the asset/liability cross-holding network. Finally, we compute the interdependency matrix. The interdependency network is much more dense than the direct cross-holding network, showing the complex latent interaction among banks.

Next we perform macroprudential stress tests for the European banking system, using the adverse scenario in EBA 2018 EU-wide stress test as the initial shock. For different reconstructed networks, we illustrate the hierarchical cascades and show that the failure hierarchies are roughly the same except for a few banks, reflecting the overlapping portfolio holding accounts for the majority of defaults.

Clearly the above tests are based on moderate scenario taken by EBA (recalling that they assume GDP in the EU only decreases -1.2%, -2.2% and even increases 0.7% as of 2018, 2019 and 2020 respectively), so that the default threshold must be set to a very high value (i.e. 0.97) to successfully trigger the initial failures. Nonetheless, we emphasize that understanding the interdependency network and the hierarchy of the cascades can help to improve policy intervention and implement rescue strategy.

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