Welfare Cost of Inflation, when Credit Card Transaction Services Are Included among Monetary Services[1]

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We investigate the welfare cost of anticipated inflation, when the volume of credit card transactions is included in measured monetary service flows. We use the credit-card-augmented Divisia monetary aggregates in a nonlinear dynamic stochastic general equilibrium (DSGE) New Keynesian model and calculate the welfare costs of inflation. The welfare costs of inflation with credit card services included are greater than without them in the New Keynesian DSGE model. Because of the complexity of the model's dynamical structure, we are not aware of a simple explanation for the increased welfare sensitivity to inflation.

Keyword: Welfare Cost, Divisia, Credit-Card-Augmented Divisia, Monetary Aggregates, Money Demand, Inflation, nonlinear dynamics.

JEL Codes: E31, E41, E51, E52

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1. Introduction

This paper investigates the welfare cost of inflation, when credit-card deferred-payment transaction services are included among monetary services within the economy. We do so relative to a New Keynesian nonlinear dynamic DSGE model. Inclusion of credit card services in the economy is potentially important in welfare economics, since credit card services are growing in importance, as illustrated in Figure 1.

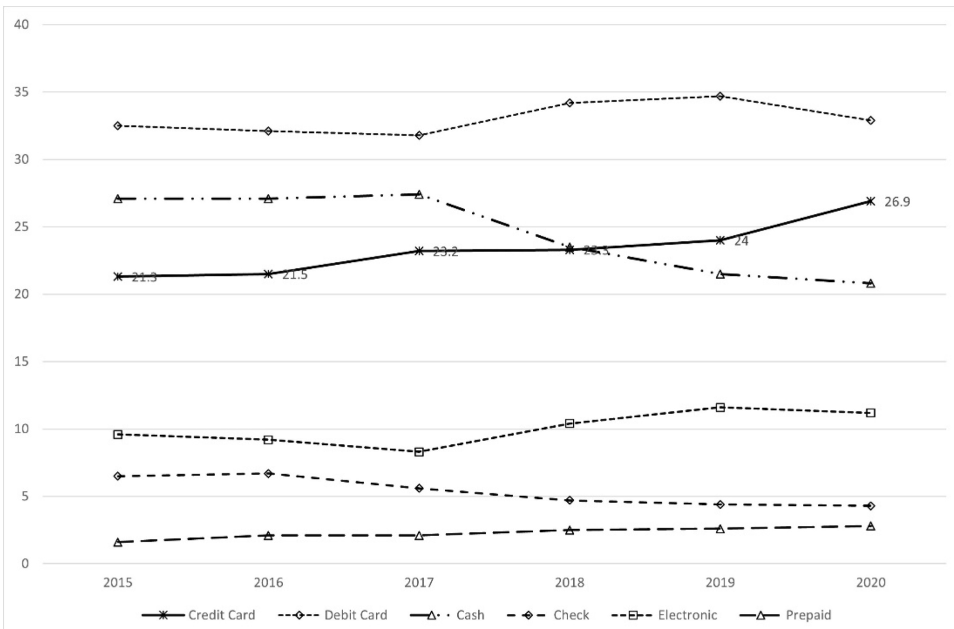

FIGURE 1. U.S. SHARES OF PAYMENT INSTRUMENTS (%)
SOURCE: 2020 SCPC TABLES, FEDERAL RESERVE BANK OF ATLANTA.

Figure 1 displays the share of consumer payment instruments. While the share of cash is decreasing, the share of credit cards has been increasing, has surpassed cash usage, and has reached 26.9% in 2020. But credit card transaction services have never been included in central bank official measures of the money supply, which are based on simple sum aggregation. Accounting conventions do not permit adding liabilities, such as credit card balances, to assets, such as money. However, economic aggregation theory can be used to aggregate over service flows, regardless of whether the monetary services are from assets or liabilities. We use credit-card-augmented Divisia monetary aggregation, including credit card transaction volumes among other monetary services, in accordance with the theory developed by Barnett and Su (2016). For recent research using the credit-card-augmented Divisia monetary aggregates, see, e.g., Barnett and Liu (2019), Liu and Serletis (2020, 2022) and Liu, Dery, and Serletis (2020).

We specify a New Keynesian model with money in the utility function as an extension of Belongia and Ireland (2014). We include credit card transactions services among the monetary services included in the utility function. The New Keynesian model we use is a dynamic stochastic general equilibrium (DSGE) model permitting us to calculate the welfare cost of inflation. The use of Divisia monetary aggregation to measure the welfare costs of inflation has previously been proposed by Lucas (2000) and Ireland (2009), but without inclusion of credit card services in the economy.

Most prior studies of the welfare cost of inflation have used simple sum monetary aggregation, such as Calza and Zaghini (2011). Bailey (1956) treated real money balances as a consumption good and inflation as a tax on real balances. Cooley and Hansen (1989) used a cash-in-advance stochastic optimal growth model with an endogenous labor-leisure decision. They studied the quantitative importance of money and inflation in a Real Business Cycle model in which money, M1, influences real variables through the inflation tax. Cooley and Hansen (1991) found conditions under which the costs of achieving zero inflation exceeded the benefits.

Dotsey and Ireland (1996) calculated the amount of output which had to be given up at an inflation level. They used a general equilibrium monetary model, in which an inflationary tax distorted marginal decisions. They found that a sustained 4 percent inflation rate costs the economy the equivalent of 0.41 percent of output per year, when currency is the relevant definition of money, and over 1 percent of output per year, when simple sum M1 is defined as money. Teo and Yang (2011) studied the welfare cost of inflation in a New Keynesian dynamic stochastic general equilibrium model, in which money was introduced through a cash-in-advance (CIA) constraint. They concluded that the welfare cost of inflation in a New Keynesian framework was much higher than in a Real Business Cycle (RBC) model.

The welfare cost of inflation has also been approached relative to money demand. Phylaktis and Taylor (1993) used the Cagan (1956) model of money demand to evaluate the inflation tax in five Latin American countries during the 1970s and 1980s. Lucas (2000) estimated the welfare cost of inflation in the Sidrauski model and in the McCallum-Goodfriend model using M1 as money. That paper found that the gain from reducing the annual inflation rate from 10 percent to zero is equivalent to an increase in real income of slightly less than 1 percent. With Cagan's semi-log model, Ireland (2009) used post-1980 US data along a stable long-run money demand relationship between the M1/income ratio and the nominal interest rate. Integrating under this money demand curve yielded estimates of very small welfare costs of modest departures from Friedman's zero nominal interest rate rule for his "optimum quantity of money." Dai and Serletis (2019) used a Markov switching approach. The Lucas (2000) approach to measuring the welfare cost of inflation for Canada and the United States was used by Serletis and Yavari (2004).

There also are three papers that estimate the welfare cost of inflation with Divisia monetary aggregates. Cysne (2003) considered economies in which interest-bearing and non-interest-bearing monetary assets are used for transaction purposes and calculated the welfare costs of inflation using the Divisia methodology. Serletis and Virk (2006) investigated the welfare cost of inflation with simple sum, Divisia, and currency equivalent (CE) monetary aggregates. They found that the Divisia monetary aggregates imply a smaller welfare cost than the simple sum or currency equivalent aggregate. Serletis and Libo (2021) calculated the welfare cost of inflation by the Bailey (1956) approach in neoclassical demand theory. They found that raising the rate of inflation from 2% to 4% in the U.S. would produce a welfare cost of 0.30 percent of output, when money is measured by the Divisia M4 monetary aggregate.

This paper includes credit card services in a New Keynesian DSGE model. We include credit card services in the banking sector, in the monetary aggregates, and in households' utility functions. Extending Belongia and Ireland (2014), we incorporate credit card services in the economy and compare the welfare cost of inflation with the welfare cost of inflation in the benchmark model without credit card services.

Our paper is organized as follows. Section 2 discusses the quantitative model and welfare cost calculation. Section 3 explains the calibration. Section 4 describes the numerical results, and we conclude in Section 5.

2. Quantitative Model

In this section, we define the dynamic stochastic general equilibrium model with credit card transaction volumes in the monetary services measure. Our model is an extension of Belongia and Ireland (2014), who considered Divisia monetary aggregation with noninterest-earning currency and interest-earning deposits in a closed economy, New Keynesian DSGE model.

We adopt a similar closed economy New Keynesian model with Divisia monetary aggregation, but with monetary services from currency, deposits, and the volume of credit card transactions. Credit card transaction volumes produce deferred payment transaction services not provided by currency or bank deposits. We use credit card transaction volumes, not credit card balances. Credit card balances data would produce double counting of deferred payment services, since credit card balances include carried forward rotating balances used for transactions in prior periods.

2.1. The Representative Household

We use a money in the utility function model, such as Sidrauski (1967), to reflect the fact that money produces services, such as transaction and liquidity services. As is well known in general equilibrium theory, there always exists a derived utility function containing money, if money has positive value (shadow price) in equilibrium. Lucas (2000) similarly employed the general equilibrium model of Sidrauski with utility containing consumption of goods and monetary real balance services.

The representative household's preferences at time t are

$$E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, \ell_t, \frac{M_t^{cc}}{P_t}\right) \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, C_t is consumption, ℓ_t is leisure of the household, and $\frac{M_t^{cc}}{P_t}$ is real money holdings, including credit card transactions volumes. We assume that the current period utility function of the representative agent is given by

$$U\left(C_t, \ell_t, \frac{M_t^{cc}}{P_t}\right) = \zeta \ln C_t + \eta \ln \ell_t + \theta \ln\left(\frac{M_t^{cc}}{P_t}\right), \tag{2}$$

where $\zeta > 0$, $\eta > 0$, and $\theta > 0$ are utility weights on log of consumption, leisure, and real money balances, respectively. The utility function is log Cobb-Douglas, where $\zeta + \eta + \theta = 1$.

Aggregate nominal monetary services, M_t^{cc}, include currency, N_t, total nominal value of the household's deposits, D_t, and credit card transactions volume, CC_t. The nominal monetary aggregator function is CES as follow:[2]

$$M_t^{cc} = \left[\nu^{\frac{1}{\omega}} N_t^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} D_t^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}} CC_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{3}$$

where $\omega > 0$ is the elasticity of substitution among currency, deposits, and real balances in producing the monetary aggregate, M_t^{cc}, while $\nu, \phi \in (0,1)$ denote the weights placed on currency and deposits. This equation is converted to real terms by dividing by the nominal price level, P_t. Substituting equation (3) into (2), an aggregator function over monetary and credit card services exists within the decision problem, since the aggregator function can be factored out of the decision problem as a subfunction (category utility function). The

[2] See Barnett (1980) and Belongia and Ireland (2014, 2019). Belongia and Ireland (2019) adopt the CES form, because Lucas (2000) explains that the CES form makes the model consistent with stationary balanced growth. In addition, the CES function satisfies the homogeneity assumption necessary for quantity aggregation.

monetary aggregator function therefore exists as a blockwise weakly separable subfunction of economic structure.[3]

Equation (4) below is the direct extension of Belongia and Ireland's (2014) equation (1) to include credit card borrowing for purchases. At the start of each period $t = 0,1,2,\cdots$, the resources of the household are M_{t-1} units of currency, B_{t-1} units of bonds, τ_t lump-sum transfer from the monetary authority, L_t loan dollars from the representative bank, CC_t credit card new borrowing for period *t* transactions, and $s_{t-1}(i)$ shares in each intermediate-goods producing firm $i \in [0,1]$. The household purchases B_t units of new bonds at the price of $1/r_t$ dollars per bond, where r_t is the gross nominal interest rate, and also purchases $s_t(i)$ shares in each intermediate-goods producing firm $i \in [0,1]$ at the price of $Q_t(i)$ dollars per share.[4] The household sets aside N_t dollars of currency to be used in buying goods and services and puts the rest in the bank as deposits.

The household's deposits, D_t, satisfy the following constraint equation (4), which can be written in real terms as equation (5):

$$M_{t-1} + B_{t-1} + \tau_t + \int_0^1 Q_t(i)s_{t-1}(i)di - \frac{B_t}{r_t} - \int_0^1 Q_t(i)s_{i,t}(i)di - N_t + L_t + CC_t = D_t \,, \tag{4}$$

$$\frac{M_{t-1} + B_{t-1} + \tau_t - B_t/r_t - N_t + L_t + CC_t}{P_t} + \int_0^1 \frac{Q_i(t)}{P_t}(s_{t-1}(i) - s_t(i))di = \frac{D_t}{P_t} \,. \tag{5}$$

Also, the household holds M_t units of currency into the next period at the end of each period. The household can also make CC_t credit card transactions at the start of the period and repay the credit card company $e_t CC_t$ dollars at the end of the period, where e_t is

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[3] For an index number to be able to track the aggregator function, the aggregator function must exist within the structure of the economy. The existence condition is blockwise weak separability. In fact, in our case the aggregator function is strongly separable within economic structure, since Cobb-Douglas utility is strongly separable in all of its arguments.

[4] See Belongia and Ireland (2014,2019).

the credit card gross interest rate.[5] Since CC_t is the volume of credit card transactions this period, not the credit card balance, we do not face the double-counting problem by which credit card purchases last period would appear again this period, as rotating balances carried forward from last period. As in Belongia (2014)'s equation (5), extended to include credit card transaction volumes, M_t satisfies the following constraint

$$N_t + r_t^D D_t + \int_0^1 F_t(i)s_t(i)di + W_t(1-\ell_t) - P_t C_t - r_t^L L_t - e_t CC_t \geq M_t\,, \tag{6}$$

where r_t^D denotes the gross own yield on deposits, C_t is consumption excluding credit card use, $F_t(i)$ is nominal dividends the household receives for each share in each intermediate-goods producing firm, $i \in (0,1)$, and r_t^L is the gross nominal interest rate on loans. The nominal wage rate is W_t. Since ℓ_t is leisure of the household, $1-\ell_t$ is total units of labor, consisting of the household's labor supplied to the intermediate-goods-producing firm. The household earns $W_t(1-\ell_t)$ of labor income.

Equation (6) can also be written in real terms as follows:

$$\frac{N_t + r_t^D D_t + W_t(1-\ell_t)}{P_t} + \int_0^1 \frac{F_t(i)s_t(i)}{P_t}di \geq C_t + \frac{r_t^L L_t + e_t CC_t + M_t}{P_t}\,. \tag{7}$$

Finally, the representative household maximizes the utility function, (2), subject to constraints (3), (5), and (7). The Lagrangian for the problem and the first-order conditions are provided in Appendix A.

[5] See Barnett, Chauvet, Leiva-Leon, and Su (2016). There are two categories of consumers using credit cards: consumers who pay interest to the credit card issuing banks on rotating balances, and consumers who do not maintain rotating balances and therefore pay no interest on their credit card balances. The "representative consumer" pays interest averaged over both categories of consumers. This paper adopts this averaged credit card interest rate as e_t.

2.2. Production Sector

This paper adopts the production sector from Belongia and Ireland (2014). In this model, the production sector consists of the representative firm producing finished goods and the representative firm producing intermediate goods, respectively.

2.2.1. The Representative Finished-Goods-Producing Firm

The representative finished-goods producing firm uses $Y_t(i)$ units of the intermediate good produced by firm i to manufacture Y_t units of the finished good. As in Belongia (2014, eq. 8), the production function is in the constant returns to scale CES form given by

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

where $\sigma > 1$ is the elasticity of substitution among the various intermediate goods in producing the finished goods. The finished-goods producing firm chooses $Y_t(i)$ for all $i \in (0,1)$ to maximize profits,

$$P_t \left[\int_0^1 Y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} - \int_0^1 P_t(i) Y_t(i) di ,$$

where $P_t(i)$ is the nominal price of intermediate good i for all $t = 0,1,2,\cdots$.

The first-order condition is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\sigma} Y_t \tag{9}$$

for all $i \in (0,1)$ at $t = 0,1,2,\cdots$.

We assume the finished goods market is competitive. The profit of the representative finished-goods producing firm in its competitive market is zero. Then the following condition is satisfied in equilibrium for all $t = 0,1,2,...$:

$$P_t = \left[\int_0^1 P_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}.$$

2.2.2. The Representative Intermediate-Goods Producing Firm

In this section, we define h_t to be labor of the household such that $\ell_t + h_t = 1$. The representative intermediate-goods producing firm produces $Y_t(i)$ units of intermediate good i by employing $h_t(i)$ units of labor from the representative household, so that

$$h_t = \int_0^1 h_t(i) di .$$

As in Belongia (2014, eq. 10), constant returns to scale technology is given by

$$Y_t(i) = Z_t h_t , \tag{10}$$

where Z_t is the aggregate technology shock. It follows a random walk process with positive drift

$$\ln Z_t = \ln z + \ln Z_{t-1} + \varepsilon_{z,t} , \tag{11}$$

where $z > 1$ and $\varepsilon_{z,t} \sim (0, \sigma_z^2)$.

The representative intermediate-goods producing firm sells output in a monopolistically competitive market. The firm sets the nominal price $P_t(i)$ for its output $Y_t(i)$ subject to the representative finished-goods producing firm's demand, equation (9). Furthermore, as in Belongia and Ireland (2014), the intermediate-goods producing firm has a quadratic cost of adjusting its nominal price measured in units of the finished good, as following:

$$\frac{\gamma}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2 Y_t,$$

where $\gamma \geq 0$ is the magnitude of the price adjustment costs, and $\pi > 1$ is the gross steady-state inflation rate.

As in Belongia and Ireland (2014, appendix A.2), we solve the intermediate-goods producing firm problem. Let $\Lambda_{3,t}$ be a Lagrange multiplier defined in our Appendix A on a constraint in the household's optimization problem. From the Euler equation (53) in Appendix A, the intermediate-goods producing firm maximizes its real market value as follows:

$$E_t \sum_{t=0}^{\infty} \beta^t \Lambda_{3,t}\left[\frac{F_t(i)}{P_t}\right]$$

subject to

$$\frac{F_t(i)}{P_t}=\left[\frac{P_t(i)}{P_t}\right]^{1-\sigma} Y_t-\left[\frac{P_t(i)}{P_t}\right]^{-\sigma}\left(\frac{W_t Y_t}{P_t Z_t}\right)-\frac{\gamma}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2 Y_t. \qquad (12)$$

Then the firm's first-order condition is

$$\begin{aligned}&(1-\sigma)\Lambda_{3,t}\left[\frac{P_t(i)}{P_t}\right]^{-\sigma} Y_t+\sigma\Lambda_{3,t}\left[\frac{P_t(i)}{P_t}\right]^{-\sigma-1}\left(\frac{W_t Y_t}{P_t Z_t}\right)-\gamma\Lambda_{3,t}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]\left[\frac{Y_t P_t}{\pi P_{t-1}(i)}\right]\\&+\beta\gamma E_t\left\{\Lambda_{3,t+1}\left[\frac{P_{t+1}(i)}{\pi P_t(i)}-1\right]\left[\frac{Y_{t+1}P_{t+1}(i)P_t}{\pi P_t(i)^2}\right]\right\}=0\end{aligned} \qquad (13)$$

for all $t=0,1,\ldots$.

2.3. Social Planner's Allocations

As a preliminary step in describing the central bank's behavior, Belongia and Ireland (2014, section 2.6 and Appendix A.3) consider a benevolent social planner decision. The social planner allocates the representative household's labor $h_t^*(i)$ to produce each intermediate good i, $Y_t^*(i)$, and to attain the most efficient level of the finished good, Y_t^*, by maximizing the following preferences at time *t*:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\zeta \ln Y_t^* + \eta \ln\left(1 - \int_0^1 h_t^*(i)di\right) + \theta \ln\left(\frac{M_t^{cc}}{P_t}\right)\right].$$

The most efficient level of the finished good Y_t^* is defined by

$$Y_t^* = \frac{\zeta}{\eta} Z_t (1 - h_t), \tag{14}$$

and the output gap g_t^* is defined by

$$g_t^* = \frac{Y_t}{Y_t^*} = \frac{\eta Y_t}{\zeta Z_t (1 - h_t)}. \tag{15}$$ [6]

2.4. The Representative Bank

The representative bank not only makes loans worth L_t to the representative household and accepts deposits worth D_t dollars but also offers credit card services. The following is the direct extension of Belongia and Ireland (2014, section 2.5) to include credit card services. The bank's loans and deposits have the following relationship:

$$L_t = (1 - k_t) D_t, \tag{16}$$

[6] See Belongia and Ireland (2014) equation 17.

where k_t is the required reserve ratio, assumed to follow the AR(1) process:

$$\ln k_t = (1-\rho_k)\ln k + \rho_k \ln k_{t-1} + \varepsilon_{k,t} \ , \tag{17}$$

as in Belongia and Ireland (2014, eq. 12). Here $k_t \in (0,1)$, $\rho_k \in [0,1)$, and $\varepsilon_{k,t} \sim (0,\sigma_k^2)$. Also, the credit card services that the bank offers allow the household to make deferred payment transactions within the credit limit. For simplicity, we assume the credit card transactions volume of the household is constrained by the household's credit limit, which is a function of its deposits as follows:[7]

$$CC_t \leq a_t D_t \ . \tag{18}$$

The credit-deposit ratio, a_t, follows the AR(1) process:

$$\ln a_t = (1-\rho_a)\ln a + \rho_a \ln a_{t-1} + \varepsilon_{a,t} \ , \tag{19}$$

where $a_t \in (0,1)$, $\rho_a > 0$, and $\varepsilon_{a,t} \sim (0,\sigma_a^2)$.[8]

With its technology, the bank creates $x_t \dfrac{D_t}{P_t}$ units of output using total real value of deposits, $\dfrac{D_t}{P_t}$, and $x_t \dfrac{CC_t}{P_t}$ units of output with total real value of credit card transaction

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[7] A relationship between credit card use and demand deposit balances has often been observed, as in Duca and Whitesell (1995), although the time series relationship differs from the cross section relationship.

[8] Earlier research on the credit-card-augmented Divisia monetary aggregates is based on classical microeconomic theory with all interior solutions. It was implicitly assumed that credit limits are never binding on the representative consumer's credit cards volumes. But in the spirit and context of the Belongia and Ireland (2014) New Keynesian model, we here permit corner solutions at the credit limit. Strictly speaking, for this analysis to be fully consistent with the earlier research on credit-card-augmented Divisia monetary aggregates, we would need to introduce a shadow user-cost price for credit card services, so that the corner solutions would be consistent with interior solutions at the shadow price. But we doubt that such a shadow price would be significantly different from the market user cost price for the representative consumer, since over 50% of consumers do not consume at their credit limits. It should also be observed that the credit-card-augmented Divisia monetary aggregates available from the Center for Financial Stability do not, and need not, impose equation (18) in construction of the aggregates. In general computation of the credit card augmented Divisia monetary aggregates, we do not advocate imposition of the unnecessary constraint (18), but only in the context of this paper's New Keynesian DSGE model. Even in this New Keynesian context, it could be useful in future research to remove equation (18) to investigate robustness of our welfare inferences to that non-classical assumption permitting corner solutions.

volume $\frac{CC_t}{P_t}$. In accordance with Belongia and Ireland (2014, eq. 14), the financial-sector cost shock is x_t following the process:

$$\ln x_t = (1-\rho_x)\ln x + \rho_x \ln x_{t-1} + \varepsilon_{x,t}\,, \tag{20}$$

where $x > 0$, $\rho_x \in (0,1)$, and $\varepsilon_{x,t} \sim (0, \sigma_x^2)$. Then the representative bank maximizes its profits,

$$\Pi_t = (r_t^L - 1)L_t + (e_t - 1)CC_t - (r_t^D - 1)D_t - P_t x_t \frac{D_t}{P_t} - P_t x_t \frac{CC_t}{P_t}.$$

The representative bank is in a competitive market and its economic profits are zero. Setting $\prod_t = 0$, we get the following equation, which is the extension of Belongia and Ireland (2014, eq. 15) to include credit card services:

$$r_t^D = (r_t^L - 1)(1 - k_t) + (e_t - x_t - 1)a_t - x_t + 1\,. \tag{21}$$

Equation (21) implies that the financial-sector cost shock, x_t, the required reserve ratio, k_t, and credit card interest rate, e_t, impact directly on the gross rate of deposits, r_t^D.

2.5. The Monetary Authority

The monetary base, M_t, is defined as the sum of currency, N_t, and commercial bank deposits in reserves at the central bank, N_t^υ, which equals $k_t D_t$, where k_t is the required reserve ratio and D_t is the amount of deposits, so that

$$M_t = N_t + N_t^\upsilon\,, \tag{22}$$

$$N_t^\upsilon = k_t D_t\,. \tag{23}$$

We assume the monetary authority follows a simple Taylor-type class of rules similar in form to Annicchiarico and Rossi (2013) and Belongia and Ireland (2014). Let π_t and g_t^y be the gross rate of inflation and output growth, as follows:

$$\pi_t = \frac{P_t}{P_{t-1}} \ , \tag{24}$$

$$g_t^y = \frac{Y_t}{Y_{t-1}} \ . \tag{25}$$

Then the gross nominal interest rate, r_t, is determined by the following equation, as in Belongia and Ireland (2014, eq. 21):

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r} \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{g_t^*}{g^*}\right)^{\rho_{g^*}} \left(\frac{g_t^y}{g^y}\right)^{\rho_{g^y}} \exp(\varepsilon_{r,t}) \ , \tag{26}$$

where ρ_r, ρ_π, ρ_{g^*}, and ρ_{g^y} are policy parameters and $\varepsilon_{r,t} \sim (0, \sigma_r^2)$.

2.6. Monetary Aggregation

Belongia and Ireland (2014) consider the "true aggregate" of monetary services, the simple sum aggregate, and the Divisia aggregate, where the true aggregate is the microeconomic aggregator function, weakly separable within economic structure, and the Divisia aggregate is the nonparametric Divisia index approximation to the aggregator function. Keating and Smith (2019) also include the monetary base among the competing monetary aggregates. We include the credit-card-augmented Divisia monetary aggregate among the competing measures.

The simple sum measure is the sum of currency and deposits. Due to accounting conventions, which do not permit adding liabilities to asset, credit cards have never been included in measures of the simple sum money supply[9]. Therefore, we define the simple sum monetary aggregate only without inclusion of the volume of the credit card transactions:

$$M_t^{sim} = N_t + D_t . \tag{27}$$

Barnett, Offenbacher, and Spindt (1984), among others, have shown that the Divisia monetary aggregates are strictly preferable to the simple sum monetary aggregates, when the component monetary assets are not perfect substitutes. The Divisia index requires both quantities and prices, where the prices of component services are user costs reflecting the opportunity costs of holding a monetary asset. Divisia measures the aggregate's growth rate as the share weighted average of the component growth rates, with user cost pricing in the shares.

The credit card quantities included in the augmented Divisia index formula are credit card transactions volumes, not credit card balances, which also include rotating balances from previous periods. The credit-card-augmented Divisia monetary aggregates in this model aggregate over the services of currency, interest-bearing deposits, and credit card transaction volumes. The real user cost price (equivalent rental price) of the services of currency is

$$u_t^N = \frac{r_t - 1}{r_t} , \tag{28}$$

where r_t is the maximum expected gross holding-period yield available in the economy. In equilibrium, r_t is the risk-free gross rate of return on a completely illiquid benchmark asset (pure capital). Since the nominal interest rate on capital is the opportunity cost of holding non-interest-bearing money, the user cost of currency is the price of holding currency.

The user cost of deposits is:

$$u_t^D = \frac{r_t - r_t^D}{r_t} , \tag{29}$$

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[9] See Barnett, Chauvet, Leiva-Leon, and Su (2016).

where r_t^D is the gross own yield on deposits. Equations (28) and (29) were first derived by Barnett (1978, 1980) in terms of interest rates instead of gross rates of return, which are interest rates plus 1.0. In terms of interest rates, equation (28) adds 1 to the numerator and denominator, and equation (29) adds 1 to the denominator. Barnett's proof is derived from the flow of funds identities, which are accounting identities, and therefore must apply to all valid models, including New Keynesian models.

Based on the formula derived in Barnett and Su (2016), the real user cost of credit card services is

$$u_t^{cc} = \frac{e_t - r_t}{r_t} \quad , \tag{30}$$

where e_t is the credit card gross interest rate.[10] The derivation again is based on flow of funds accounting identities and hence is relevant to all models, including the New Keynesian model in this paper, generalizing Belongia and Ireland (2014) to inclusion of credit card services. With the user cost formulas, (28), (29), and (30), we can calculate the expenditure shares of currency, deposits, and credit card services, respectively.

$$s_t^N = \frac{u_t^N N_t}{u_t^N N_t + u_t^D D_t + u_t^{cc} CC_t}, \tag{31}$$

$$s_t^D = \frac{u_t^D D_t}{u_t^N N_t + u_t^D D_t + u_t^{cc} CC_t} \quad , \tag{32}$$

$$s_t^{cc} = \frac{u_t^{cc} CC_t}{u_t^N N_t + u_t^D D_t + u_t^{cc} CC_t} . \tag{33}$$

[10] Barnett, Chauvet, Leiva-Leon, and Su (2016) introduce the credit card interest rate, e_{jt}, and consider two categories of consumers using their credit cards. They consider consumers who do pay interest to the credit card issuing banks and those consumers who do not pay interest. The interest rate e_{jt} is averaged over both those consumer groups. The subscript *j* is the credit card type, where CFS considers four such credit card companies, Visa, MasterCard, Discover, and American Express. We assume that there is only one credit card company (banks). Since the derivation in Barnett and Su (2016) does not use gross rates of return, the formula in that paper adds 1 to the denominator.

In accordance with Barnett and Su (2016), we have the following for the credit-card-augmented Divisia monetary aggregate, M_t^{CDiv}, having growth rate $\log \mu_t^{CDiv}$ = $\log M_t^{CDiv} - \log M_{t-1}^{CDiv}$:

$$\mu_t^{CDiv} = \mu_t^{N^{(s_t^N + s_{t-1}^N)/2}} \mu_t^{D^{(s_t^D + s_{t-1}^D)/2}} \mu_t^{cc^{(s_t^{cc} + s_{t-1}^{cc})/2}} , \tag{34}$$

where μ_t^N, μ_t^D, and μ_t^{cc} are defined as follows:

$$\mu_t^N = \frac{N_t}{N_{t-1}} , \tag{35}$$

$$\mu_t^D = \frac{D_t}{D_{t-1}} , \tag{36}$$

$$\mu_t^{cc} = \frac{CC_t}{CC_{t-1}} . \tag{37}$$

Taking the log of each side of (34), it follows that the growth rate of the credit card augmented Divisia monetary aggregate is the weighted average of the growth rates of its three components, with the weights being the expenditure shares averaged within the period.[11]

To derive (34) directly from this paper's New Keynesian model in accordance with the proof in Barnett and Su (2016), it would be necessary to assume that contemporary interest rates are known with certainty, although there could be risk in the intertemporal decision regarding future interest rates. Then the household's decision problem could be solved in two stages, with the second stage being conditional allocation of total current period monetary and credit card services optimally over individual current period monetary and credit card services, with the solution for the total allocation having been determined by the prior stage intertemporal allocation. The Barnett and Su (2016) derivation would apply to the second stage conditional decision.

[11] The relationship to Barnett and Su (2016) is more easily seem by taking the log of each side of equation (34). We nevertheless provide the form of equation (34) for easier comparison with equation (30) in Belongia and Ireland (2014).

2.7. Equilibrium Condition

In equilibrium, in accordance with Calvo (1983), $P_{i,t} = P_t$, $h_{i,t} = h_t$, $Y_{i,t} = Y_t$, $F_{i,t} = F_t$, and $Q_{i,t} = Q_t$ for all i. Market clearing conditions are $B_t = B_{t-1} = 0$, $M_t = M_{t-1} + \tau_t$, and $s_{i,t} = s_{i,t-1} = 1$ for all time, t.

Applying these equilibrium conditions and solving the optimization problem under stationarity, we collect together (3), (5), (7), (10)-(23), (25)-(37), and (47)-(57).[12] Appendix B explains the stationary process of the system of equations.

2.8. Welfare Cost Calculation

This section investigates the consumption equivalent variation (EV) approach to calculating welfare costs. We approach the welfare cost of steady-state inflation through the change in steady-state utility of the representative household, as in Cooley and Hansen (1989,1991). We calculate the change in consumption to attain the target rate of inflation by the conventional approach to equivalent variation in consumption. By that approach, welfare cost is measured as the amount of consumption the household has to give up to achieve indifferent utility at the steady-state.

Let ψ be the welfare cost satisfying the following equation after applying the stationary equation system in Appendix A:

$$U\left[c_\pi, \ell_\pi, m_\pi^{cc}(n_\pi, d_\pi, cc_\pi)\right] = U\left[(1+\psi)c_*, \ell_*, m_*^{cc}(n_*, d_*, cc_*)\right], \qquad (38)$$

where c_π, ℓ_π, $m_\pi^{cc} = M_\pi^{cc} / P_\pi$, n_π, d_π, and cc_π are the consumption, leisure, real money balances, currency, deposits, and credit card transaction volumes associated with inflation rate π, while c_*, ℓ_*, $m_*^{cc} = M_*^{cc} / P_*$, n_*, d_*, and cc_* are their values at the steady-state net

[12] See Appendix A and Appendix B.

annual inflation rate (benchmark inflation rate). We thereby compute the welfare costs when the inflation rate π changes to the steady-state inflation rate.

Applying the utility function of the representative household, (2), (3), the welfare cost ψ satisfies equation (38), which can be written as follows:

$$\begin{aligned}&\zeta \ln c_{\pi} + \eta \ln \ell_{\pi} + \theta \ln \left\{ \left[\nu^{\frac{1}{\omega}} n_{\pi}^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} d_{\pi}^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}} cc_{\pi}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right\} \\ &= \zeta \ln\left[(1+\psi) c_{*}\right] + \eta \ln \ell_{*} + \theta \ln \left\{ \left[\nu^{\frac{1}{\omega}} n_{*}^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} d_{*}^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}} cc_{*}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right\}.\end{aligned} \tag{39}$$

3. Calibration

We set the discount factor, β, to equal 0.99, so that the quarterly nominal interest rate is 1% and the annual interest rate is 4%. In the utility function of the representative household, we follow the method of estimation of utility weights on labor and real money balances in Chari, Kehoe, and McGrattan (2000) and Liu and Spiegel (2015). Here ζ, η, and θ are calibrated at 0.944, 0.3, and 0.028 by using U.S. data from July 2006 through May 2021. In the benchmark model without credit card services, we get $\zeta = 0.669$, $\eta = 0.31$, and $\theta = 0.021$ in the same manner.

Belongia and Ireland (2014) and Keating and Smith (2019) have only currency and deposits in their true monetary aggregates and set the weights for currency, ν, and for deposits, ϕ, at 0.225 and 0.275 respectively in those monetary aggregates. We set the weight for currency ν at 0.198, and for the deposits ϕ at 0.480. The weight for credit card transactions volumes, $1-\nu-\phi$, is equal to 0.322. Those parameters were estimated to make the proportion of steady-state currency, deposits, and credit card transactions volumes in the total amount of the three components equal to 0.103, 0.877, and 0.019, respectively. In that computation, we used the U.S. Divisia components from July 2006 through May 2021.[13] The previous literature calculated ν using older U.S. data from 1959 through 2009. In the

[13] The Center for Financial Stability (CFS) provides Divisia indices data. We use the CFS Divisia M2 components data. See http://www.centerforfinancialstability.org/.

benchmark model in which the true money aggregates contains only currency and deposits, we set $\nu = 0.224$, so that the ratio of steady-state currency to simple sum money equals 0.1012, while the ratio of deposits to simple sum money is 0.8988. We set the elasticity of substitution among currency, deposits, and real balances to be $\omega = 1.5$ as in Belongia and Ireland (2014).

As in Annicchiarico and Rossi (2013) and Belongia and Ireland (2014), σ is calibrated to be 6. As in Belongia and Ireland (2014), the setting $z = 1.005$ implies 2% growth per year on average for most of the real variables, while $\gamma = 50$ implies a speed of price adjustment of 3.75 quarters on average.

Since the initial steady-state annual rate of inflation is 2%, the inflation target is $\pi = 1.005$. Based on the method in Belongia and Ireland (2014), the ratio of total reserves to deposits, k, in M2 is calibrated to be 0.04. We regress the U.S. Divisia components data from July 2006 through May 2021 to get $a = 0.0217$ and $\rho_a = 0.80$ in equations (18) and (19). Following Belongia and Ireland (2014), we get $x = 0.01$, $\rho_k = 0.50$, and $\rho_x = 0.50$.

In monetary policy rules, the policy parameters ρ_r and ρ_π are calibrated to be 0.75 and 0.3. Following Belongia and Ireland (2014), we set the policy coefficients on the output gap ρ_{g^*} and output growth ρ_{g^y} at zero. The standard deviations of exogenous shocks are $\sigma_z = 0.01$, $\sigma_k = 1$, $\sigma_x = 0.25$, $\sigma_r = 0.0025$, and $\sigma_a = 0.07$.

4. Numerical Results

We have two different model settings. For comparison purposes, one is the benchmark model with only currency and the deposits in the true monetary aggregate (Case 1), which includes no credit card services. We also consider the new true monetary aggregate (Case 2), including not only currency and deposits but also credit card transaction volumes as in equation (3).

Table 1 displays the welfare costs of inflation at various inflation levels in the New Keynesian model defined in section 2. We experiment with the target inflation rates of 0%, 2%, 5%, and 10%. The welfare cost measures the percentage goods consumption burden on the household, when the economy transitions to the steady-state inflation from each of the four experiment inflation rates. Since we assume 2% inflation in the steady state, the welfare cost of inflation at the 2% inflation rate is zero. The current usual view is that zero inflation is not optimal, and the Federal Reserve currently targets 2% inflation rate.[14]

Table 1 indicates higher welfare costs to reach the steady-state inflation rate with the model including credit card services rather than without credit card transactions volumes. When we have no credit card services in the model (upper panel of table), the welfare costs of 0%, 5%, and 10% inflation are 1.0041%, 0.9951%, and 0.9837% of consumer goods output. When we include the credit card transactions of the representative household (lower panel of table), the welfare costs of 0%, 5%, and 10% inflation are 1.0657%, 1.0945%, and 1.1228% of consumer goods output, respectively.

Compared to the results of the benchmark model, our model including credit card services shows larger welfare cost of inflation. Because of the complexity of the model's dynamical structure, we are not aware of a simple explanation for the increased welfare sensitivity to inflation. However, we can consider possible explanations intuitively. When households have credit card transaction services in their utility functions, the increasing credit card balances during inflation increase the debt burden, and the market interest rate on that debt increases as inflation increases.

Table 1: Welfare Cost of Inflation Estimated in New Keynesian Model

	0% inflation	2% inflation (Steady-state)	5% inflation	10% inflation
Case 1: Benchmark model				
Welfare cost (ψ)	1.0041	0	0.9951	0.9837
Case 2: Model including credit card services				
Welfare cost (ψ)	1.0657	0	1.0945	1.1228

Note: Welfare cost is EV percentage change in consumption goods quantities relative to the steady state inflation rate level of consumer goods output.

[14] In contrast, Poole (1999) argued that the target of the Federal Reserve should be zero inflation to maximize the credibility of monetary policy and minimize distortions to the economy. He also concluded that declining inflation would slow real growth and increase unemployment.

5. Conclusion

A substantial literature exists on measuring the welfare costs of inflation. We contribute to that literature by conditioning on a nonlinear dynamic New Keynesian DSGE model, including credit card deferred payment services, and using economic aggregation theory to aggregate over monetary services including those deferred payment services. We compare our results on welfare loss with the benchmark model having no credit card services in the economy. We find that inclusion of credit card services in the economy increases the welfare loss from inflation. Because of the complexity of the model's structure and nonlinear dynamics, we are not aware of a simple explanation for the increased welfare sensitivity to inflation, when credit card deferred payment services are included in the economy. But our results are related to the previously overlooked welfare loss associated with the increased debt burden from the increasing credit card balances as inflation increases, along with the corresponding increased market interest rates charged on that debt.

Possible future research in this modelling context could investigate the welfare cost of other shocks. For example, Evans and Kenc (2003) explore the welfare cost of volatility from monetary and fiscal policy shocks, while Mao, Huang, and Chang (2019) investigate the welfare costs of money growth rate, required reserve ratio, and leverage ratio. It might also be useful to investigate changes in value added in banking associated with changes in inflation. That research could use the supply side theory in Barnett (1987) and Barnett and Su (2018) employing a model of financial firms producing both monetary services and credit card transactions services.

REFERENCES

ANNICCHIARICO, BARBARA, AND LORENZA ROSSI. 2013. "Optimal Monetary Policy in a New Keynesian Model with Endogenous Growth," Journal of Macroeconomics 38, 274-285.

BAILEY, MARTIN J. 1956. "The Welfare Cost of Inflationary Finance," Journal of Political Economy, 64(2), 93-110.

BARNETT, WILLIAM A. 1980. "Economic Monetary Aggregates: An Application of Index Number and Aggregation Theory," Journal of Econometrics 14(1), 11-48. Reprinted in Barnett, W. A. and Serletis, A. (eds.) (2000), The Theory of Monetary Aggregation, North Holland, Amsterdam, 11–48.

BARNETT, WILLIAM A. 1987. "The Microeconomic Theory of Monetary Aggregation," in New Approaches to Monetary Economics, Barnett, W. A. and Singleton, K. (eds.), 115-168. Reprinted in Barnett, W. A. and Serletis, A. (eds.) (2000). The Theory of Monetary Aggregation, North Holland, Amsterdam, 49-99.

BARNETT, WILLIAM A., AND JINAN LIU. 2019. "User Cost of Credit Card Services under Risk with Intertemporal Nonseparability," Journal of Financial Stability, 42, 18-35.

BARNETT, WILLIAM A., AND LITING SU. 2016. "Joint Aggregation over Money and Credit Card Services under Risk," Economics Bulletin, 36(4), 2301-2310.

BARNETT, WILLIAM A., AND LITING SU. 2018. "Financial Firm Production of Inside Monetary and Credit Card Services: An Aggregation Theoretic Approach," Macroeconomic Dynamics 24(1), 1-31.

BARNETT, WILLIAM A., MARCELLE CHAUVET, DANILO LEIVA-LEON, AND LITING SU. 2016. "The Credit-Card-Services Augmented Divisia Monetary Aggregates," MPRA Paper 73245, University Library of Munich, Germany.

BARNETT, WILLIAM A., EDWARD K. OFFENBACHER, AND PAUL A. SPINDT. 1984. "The New Divisia Monetary Aggregates," Journal of Political Economy 92(6), 1049-1085.

BELONGIA, MICHAEL T., AND PETER N. IRELAND. 2014. "The Barnett Critique after Three Decades: A New Keynesian Analysis," Journal of Econometrics 183, 5-21.

BELONGIA, MICHAEL T., AND PETER N. IRELAND. 2019. "The Demand for Divisia Money: Theory and Evidence," Journal of Macroeconomics, 61, 103128.

CAGAN, PHILLIP D. 1956. "The Monetary Dynamics of Hyperinflation," Studies in the Quantity Theory of Money, ed. M. Friedman, 24-117. Chicago: University of Chicago Press.

CALVO, GUILLERMO A. 1983. "Staggered Prices in a Utility-Maximizing Framework", Journal of Monetary Economics 12(3), 383-398.

CALZA, ALESSANDRO., AND ANDREA ZAGHINI. 2011. "Welfare Cost of Inflation and the Circulation of US Currency Abroad", ECB Working Paper, No. 1326.

CHARI, V. V., PATRICK J. KEHOE, AND ELLEN R. MCGRATTAN. 2000. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?", Econometrica 68(5), 1151-1179.

COOLEY, THOMAS F., AND GARY D. HANSEN. 1989. "The Inflation Tax in a Real Business Cycle Model," The American Economic Review, 79(4), 733-748.

COOLEY, THOMAS F., AND GARY D. HANSEN. 1991. "The Welfare Costs of Moderate Inflations," Journal of Money, Credit, and Banking 23(3), 483-503.

CYSNE, RUBENS P. 2003. "Divisia Index, Inflation, and Welfare," Journal of Money, Credit, and Banking 35(2), 221-238.

DAI, WEI, AND APOSTOLOS SERLETIS. 2019. "On the Markov Switching Welfare Cost of Inflation," Journal of Economic Dynamics and Control, 108, 29-47.

DOTSEY, MICHAEL, AND PETER N. IRELAND. 1996. "The Welfare Cost of Inflation in General Equilibrium," Journal of Monetary Economics, 37(1), 29-47.

DUCA, JOHN V. AND W. C. WHITESELL. 1995. "Credit Cards and Money Demand: A Cross-Sectional Study," Journal of Money, Credit and Banking, Vol. 27, No. 2, 604-623.

EVANS, LYNNE AND TURALAY KENC. 2003."Welfare Cost of Monetary and Fiscal Policy Shocks," Macroeconomic Dynamics 7(2), 212-238.

IRELAND, PETER N. 2009. "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand," The American Economic Review, 99(3), 1040-1052.

KEATING, JOHN W., AND ANDRES L. SMITH. 2019. "The Optimal Monetary Instrument and the (Mis)use of Causality Tests," Journal of Financial Stability 42, 90-99.

LIU, JINAN, AND APOSTOLOS SERLETIS. 2020. "Money Growth Variability and Output: Evidence with Credit-Card-Augmented Divisia Monetary Aggregates," Studies in Nonlinear Dynamics and Econometrics, 24(5), 1-11.

LIU, JINAN, AND APOSTOLOS SERLETIS. 2022. "Credit Cards, the Demand for Money, and Monetary Aggregates," Macroeconomic Dynamics, 26(8), 2161-2203.

LIU, JINAN, COSMAS DERY, AND APOSTOLOS SERLETIS. 2020. "Recent Monetary Policy and the Credit Card-Augmented Divisia Monetary Aggregates", Journal of Macroeconomics, 64, 103203.

LIU, ZHENG, AND MARK M. SPIEGEL. 2015. "Optimal Monetary Policy and Capital Account Restrictions in a Small Open Economy," IMF Economic Review, 63(2), 298-324.

LUCAS JR, ROBERT E. 2000. "Inflation and Welfare", Econometrica, 68(2), 247-274.

MAO, SHENG-ZHI, CHIEN-YU HUANG, AND JUIN-JEN CHANG. 2019. "Growth Effects and Welfare Costs in an Innovation-Driven Growth Model of Money and Banking," Journal of Macroeconomics, 62, 103049.

PHYLAKTIS, KATE, AND MARK P. TAYLOR. 1993. "Money Demand, the Cagan Model and the Inflation Tax: Some Latin American Evidence," The Review of Economics and Statistics, 75(1), 32-37.

POOLE, WILLIAM. 1999. "Is Inflation Too Low?", Federal Reserve Bank of St. Louis Review, July/August 1999, 3-10.

SERLETIS, APOSTOLOS, AND LIBO XU. 2021. "The Welfare Cost of Inflation," Journal of Economic Dynamics and Control 128, 104144.

SERLETIS, APOSTOLOS, AND JAGAT J. VIRK. 2006. "Monetary Aggregation, Inflation, and Welfare," Applied Financial Economics, 16(7), 499-512.

SERLETIS, APOSTOLOS AND KAZEM YAVARI. 2004. "The Welfare Cost of Inflation in Canada and the United States," Economics Letters 84(2), August, 199-204.

SIDRAUSKI, MIGUEL. 1967. "Rational Choice and Patterns of Growth in a Monetary Economy," The American Economic Review, 57(2), 534-544.

TEO, WING LEONG, AND PO CHIEH YANG. 2011. "Welfare Cost of Inflation in a New Keynesian Model," Pacific Economic Review, 16(3), 313-338.

Appendix A. Household Optimization

In Section 2.1, the representative household maximizes the utility function (2) subject to the constraints (3), (5), and (7). The Lagrangian for the problem can be written as

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \zeta \ln C_t + \eta \ln \ell_t + \theta \ln\left(\dfrac{M_t^{cc}}{P_t}\right) \\ +\Lambda_{1,t}\left[\left(\nu^{\frac{1}{\omega}}\left(\dfrac{N_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}}\left(\dfrac{D_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}}\left(\dfrac{CC_t}{P_t}\right)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}} - \dfrac{M_t^{cc}}{P_t}\right] \\ +\Lambda_{2,t}\left[\dfrac{M_{t-1} + B_{t-1} + \tau_t - B_t/r_t - N_t + L_t + CC_t}{P_t} + \int_0^1 \dfrac{Q_{i,t}}{P_t}(s_{i,t-1} - s_{i,t})di - \dfrac{D_t}{P_t}\right] \\ +\Lambda_{3,t}\left[\dfrac{N_t + r_t^D D_t + W_t(1-\ell_t)}{P_t} + \int_0^1 \dfrac{F_{i,t}}{P_t} s_{i,t} di - C_t - \dfrac{r_t^L L_t + e_t CC_t + M_t}{P_t}\right] \end{array} \right\}.$$

The first-order conditions with respect to B_t, C_t, L_t, M_t, ℓ_t, M_t^{cc}, $s_{i,t}$, N_t, D_t, and CC_t are as follows, where $\Lambda_{1,t}$, $\Lambda_{2,t}$, and $\Lambda_{3,t}$ denote the Lagrange multipliers on the constraints).

$$\frac{\Lambda_{2,t}}{P_t r_t} = \beta E_t\left[\frac{\Lambda_{2,t+1}}{P_{t+1}}\right] \tag{40}$$

$$\Lambda_{3,t} = \frac{\zeta}{C_t} \tag{41}$$

$$\Lambda_{2,t} = r_t^L \Lambda_{3,t} \tag{42}$$

$$\frac{\Lambda_{3,t}}{P_t} = \beta E_t\left[\frac{\Lambda_{2,t+1}}{P_{t+1}}\right] \tag{43}$$

$$\Lambda_{3,t}\frac{W_t}{P_t} = \frac{\eta}{\ell_t} \tag{44}$$

$$\Lambda_{1,t}\frac{M_t^{cc}}{P_t} = \theta \tag{45}$$

$$\Lambda_{2,t}\frac{Q_{i,t}}{P_t} - \Lambda_{3,t}\frac{F_{i,t}}{P_t} = \beta E_t\left[\Lambda_{2,t+1}\frac{Q_{i,t+1}}{P_{t+1}}\right] \tag{46}$$

$$\Lambda_{2,t} - \Lambda_{3,t} = \Lambda_{1,t}\left[\nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}} \nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{-\frac{1}{\omega}} \tag{47}$$

$$\Lambda_{2,t} - r_t^D \Lambda_{3,t} = \Lambda_{1,t}\left[\nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}} \phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{-\frac{1}{\omega}} \tag{48}$$

$$e_t\Lambda_{3,t} - \Lambda_{2,t} = \Lambda_{1,t}\left[\nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}} (1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{-\frac{1}{\omega}} \tag{49}$$

Appendix B. The Stationary System

Since most variables will be nonstationary in Section 2, we transform the system to be stationary by defining the new set of variables $c_t = C_t / Z_{t-1}$, $y_t = Y_t / Z_{t-1}$, $y_t^* = Y_t^* / Z_{t-1}$, $f_t = (F_t / P_t) / Z_{t-1}$, $\lambda_{1,t} = Z_{t-1} \Lambda_{1,t}$, $\lambda_{2,t} = Z_{t-1} \Lambda_{2,t}$, $\lambda_{3,t} = Z_{t-1} \Lambda_{3,t}$, $m_t = (M_t / P_t) / Z_{t-1}$, $m_t^{cc} = (M_t^{cc} / P_t) / Z_{t-1}$, $n_t = (N_t / P_t) / Z_{t-1}$, $n_t^{\upsilon} = (N_t^{\upsilon} / P_t) / Z_{t-1}$, $l_t = (L_t / P_t) / Z_{t-1}$, $d_t = (D_t / P_t) / Z_{t-1}$, $cc_t = (CC_t / P_t) / Z_{t-1}$, $w_t = (W_t / P_t) / Z_{t-1}$, $z_t = Z_t / Z_{t-1}$, $\pi_t = P_t / P_{t-1}$, $q_t = (Q_t / P_t) / Z_{t-1}$.

We get equations (3), (5), (7), (10)-(23), (25)-(37), and (47)-(56) in terms of the transformed new variables:

$$m_t^{cc} = \left[\nu^{\frac{1}{\omega}} n_t^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} + (1 - \nu - \phi)^{\frac{1}{\omega}} cc_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \tag{3}$$

$$m_t - n_t + l_t + cc_t = d_t \tag{50}$$

$$n_t + r_t^D d_t + w_t h_t + f_t = c_t + r_t^L l_t + e_t cc_t + m_t \tag{51}$$

$$y_t = z_t h_t \tag{52}$$

$$\ln z_t = \ln z + \varepsilon_{z,t} \tag{53}$$

$$f_t = t_t - \frac{w_t h_t}{z_t} - \frac{\gamma}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 y_t \tag{54}$$

$$(1-\sigma)\lambda_{3,t}y_t+\sigma\lambda_{3,t}\left(\frac{w_t y_t}{z_t}\right)-\gamma\lambda_{3,t}\left(\frac{\pi_t}{\pi}-1\right)\left(\frac{y_t\pi_t}{\pi}\right)+\beta\gamma\mathrm{E}_t\left[\lambda_{3,t+1}\left(\frac{\pi_{t+1}}{\pi}-1\right)\left(\frac{y_{t+1}\pi_{t+1}}{\pi}\right)\right]=0 \tag{55}$$

$$y_t^*=\frac{\zeta}{\eta}z_t(1-h_t) \tag{56}$$

$$g_t^*=\frac{\eta y_t}{\zeta z_t(1-h_t)} \tag{57}$$

$$l_t=(1-k_t)d_t \tag{58}$$

$$\ln k_t=(1-\rho_k)\ln k+\rho_k\ln k_{t-1}+\varepsilon_{k,t} \tag{59}$$

$$cc_t=a_t d_t \tag{60}$$

$$\ln a_t=(1-\rho_a)\ln a+\rho_a\ln a_{t-1}+\varepsilon_{a,t} \tag{61}$$

$$\ln x_t=(1-\rho_x)\ln x+\rho_x\ln x_{t-1}+\varepsilon_{x,t} \tag{62}$$

$$r_t^D=(r_t^L-1)(1-k_t)+(e_t-x_t-1)a_t-x_t+1 \tag{63}$$

$$m_t=n_t+n_t^\upsilon \tag{64}$$

$$n_t^\upsilon=k_t d_t \tag{65}$$

$$g_t^y y_{t-1}=y_t z_{t-1} \tag{66}$$

$$\frac{r_t}{r}=\left(\frac{r_{t-1}}{r}\right)^{\rho_r}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\left(\frac{g_t^*}{g^*}\right)^{\rho_{g^*}}\left(\frac{g_t^y}{g^y}\right)^{\rho_{g^y}}\exp(\varepsilon_{r,t}) \tag{67}$$

$$m_t^{sim}=n_t+d_t \tag{68}$$

$$u_t^N = 1 - \frac{1}{r_t} \tag{69}$$

$$u_t^D = 1 - \frac{r_t^D}{r_t} \tag{70}$$

$$u_t^{cc} = \frac{e_t}{r_t} - 1 \tag{71}$$

$$s_t^N = \frac{u_t^N n_t}{u_t^N n_t + u_t^D d_t + u_t^{cc} cc_t} \tag{72}$$

$$s_t^D = \frac{u_t^D d_t}{u_t^N n_t + u_t^D d_t + u_t^{cc} cc_t} \tag{73}$$

$$s_t^{cc} = \frac{u_t^{cc} cc_t}{u_t^N n_t + u_t^D d_t + u_t^{cc} cc_t} \tag{74}$$

$$\mu_t^{CDiv} = \mu_t^{N^{(s_t^N + s_{t-1}^N)/2}} \mu_t^{D^{(s_t^D + s_{t-1}^D)/2}} \mu_t^{cc^{(s_t^{cc} + s_{t-1}^{cc})/2}} \tag{75}$$

$$n_{t-1}\mu_t^N = n_t z_{t-1}\pi_t \tag{76}$$

$$d_{t-1}\mu_t^D = d_t z_{t-1}\pi_t \tag{77}$$

$$cc_{t-1}\mu_t^{cc} = cc_t z_{t-1}\pi_t \tag{78}$$

$$z_t\lambda_{2,t} = \beta r_t E_t\left[\frac{\lambda_{2,t+1}}{\pi_{t+1}}\right] \tag{79}$$

$$\lambda_{3,t} = \frac{\zeta}{c_t} \tag{80}$$

$$\lambda_{2,t} = r_t^L \lambda_{3,t} \tag{81}$$

$$z_t \lambda_{3,t} = \beta E_t \left[\frac{\lambda_{2,t+1}}{\pi_{t+1}} \right] \tag{82}$$

$$\lambda_{3,t} w_t = \frac{\eta}{\ell_t} \tag{83}$$

$$\lambda_{1,t} m_t^{cc} = \theta \tag{84}$$

$$\lambda_{2,t} q_t - \lambda_{3,t} f_t = \beta E_t \left[\lambda_{2,t+1} q_{t+1} \right] \tag{85}$$

$$\lambda_{2,t} - \lambda_{3,t} = \lambda_{1,t} m_t^{cc^{\frac{1}{\omega}}} \nu^{\frac{1}{\omega}} n_t^{-\frac{1}{\omega}} \tag{86}$$

$$\lambda_{2,t} - r_t^D \lambda_{3,t} = \lambda_{1,t} m_t^{cc^{\frac{1}{\omega}}} \phi^{\frac{1}{\omega}} d_t^{-\frac{1}{\omega}} \tag{87}$$

$$e_t \lambda_{3,t} - \lambda_{2,t} = \lambda_{1,t} m_t^{cc^{\frac{1}{\omega}}} (1 - \nu - \phi)^{\frac{1}{\omega}} cc_t^{-\frac{1}{\omega}} \tag{88}$$

$$\ell_t + h_t = 1 \tag{57}$$