EXPECTED UTILITY MAXIMIZATION UNDER WEAKENED ASSUMPTIONS CONSISTENT WITH BEHAVIORAL ECONOMICS

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Dedicated to the memory of Charalambos Aliprantis on the occasion of his 80^{th} birthday

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Abstract. While expected utility maximization and its foundations in the Savage Axioms play a major role in normative economics and Bayesian statistics, the axiomatic foundations of expected utility maximization have been the subject of extensive criticism over the years in terms of their descriptive ability to explain actual behavior in laboratory experiments. As a result, behavioral economists do not accept expected utility maximization as descriptive of observed consumer behavior. But the Savage Axioms have been substantially weakened and rendered more widely descriptive of observed behavior by replacing the usual Riemann integral with the Choquet [14] integral. In addition, the observed behavior under the weakened assumptions is relevant to behavior under uncertainty in the Frank Knight [47] sense, rather than the more restrictive context of behavior under risk with known probabilities.

The behavioral implications of expected utility maximization with Choquet integration reduce to the more restrictive axiomatic foundations for Riemann integration only if probabilities always sum to exactly 1.0. By permitting probabilities to sum to more than or less than 1.0, called "nonadditive probabilities," Choquet integration is consistent with far more general observed behavior than is consistent with the Savage Axioms, as has been recognized in Tversky and Kahneman's Prospect Theory [64]. But the mathematical foundations for Choquet integration and its uses in modeling behavior under uncertainty are based on sophisticated mathematics on Riesz space.

Cerreia-Vioglio et al. [8] provided a general integral representation for nonadditive probabilities defined on an Archimedean Riesz space, based on the fundamental work of Aliprantis et al [1, 2, 3]. Aliprantis introduced Riesz space into the field of economics and established the relevancy of Riesz space to Choquet integration and thereby to behavioral economics We shall show that Aliprantis's deep theoretical research in this area has had formidable consequences for advancing behavioral economics in many applications in a rigorous but very practical manner. This approach provides a formal mathematical improvement that is compatible with Allais' [4] and Ellsberg's [29] paradoxes, which Savage's [61] theorem fails to explain.

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1. Introduction

Capacity and Choquet Integral were introduced by Choquet [14] and initially applied in statistical mechanics and potential theory. As an elegant generalization of probability measure, the

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Choquet integral has attracted increasing interest from economists, especially decision theorists. Schmeidler [62, 63] rediscovered the Choquet Integral to put forward a generalized axiomatic model of choice with non-additive beliefs.

This survey introduces the Choquet integral and discusses its pivotal role in the development of decision theory under uncertainty. Some existing applications are described. Many more are possible and likely to appear in the future. In this survey, we use the term "uncertainty" [47], as opposed to "risk", to describe the situation where objective probabilities are unknown.

In section 2, we introduce the notion of capacity as a generalization of probability measure and we describe the mathematical intuition of the Choquet integral. Section 3 presents the Savage's Axioms and the subjective utility theorem as the starting point of the theoretical development. Section 4 briefly recalls Ellsberg's [29] paradoxes, which the strictness of Savage's model fails to include and capture. Section 5 further introduces the revolutionary theoretical generalization, Schmeidler's Choquet expected utility theorem, with its weakened assumptions. Some mathematical properties are listed in Section 6. Section 7 presents some relevant theoretical developments. In section 8, we display some existing applications in economics and finance. Section 9 contains the conclusions.

2. Choquet Integral

Let F be algebra of the nonempty finite set Ω. A set function $\mu : F \to [0,1]$ is called a capacity if

Definition 2.1. (Normalized) $\mu(\emptyset) = 0$, $\mu(\Omega) = 1$, and

Definition 2.2. (Monotone) $A \subseteq B \Rightarrow \mu(A) \leq \mu(B)$ for all $A, B \in F$.

As a generalization of probability measure, capacity is not necessarily additive. That is, for disjoint $A, B \in F$, $\mu(A) + \mu(B) \neq \mu(A \cup B)$ in general. The capacity, μ, is supermodular is $\mu(A \cup B) + \mu(A \cap B) \geq \mu(A) + \mu(B)$, and submodular is $\mu(A \cup B) + \mu(A \cap B) \leq \mu(A) + \mu(B)$ for all $A, B \in F$. If capacity μ is both supermodular and submodular, it is restricted to a classical probability measure.

The Choquet Integral of a measurable function f with respect to capacity μ is defined as

$$\int f\,d\mu = \int_0^\infty \mu(\{s \mid f(s) > t\})\,dt + \int_{-\infty}^0 \mu(\{s \mid f(s) > t\}) - 1\,dt,$$

where $f : \Omega \to \mathbb{R}$ is an $\mathscr{F}$-measurable function. That is, $\{s \mid f(s) > t\}$ and $\{s \mid f(s) \geq t\}$ for any $t \in \mathbb{R}$. If we restrict f to be nonnegative, the Choquet integral is simplified as

$$\int f\,d\mu = \int_0^\infty \mu(\{s \mid f(s) > t\})\,dt.$$

The Choquet integral integrates rectangles horizontally, as opposed to the vertical Riemann integral on the right-hand side. To see this, let $u : \Omega \to \mathbb{R}$ denote a function with finite range. If we permute the range of u in an ascending order such that $range(u) = \{u_1, u_2, \ldots, u_n\}$, where $0 = u_0 \leq u_1 < u_2 < \ldots < u_n$, then the Choquet integral of a nonnegative function u with respect to μ is

$$\int u\,d\mu = \sum_{i=1}^{n} (u_i - u_{i-1})\mu(u^{-1}(\{u_i, \ldots, u_n\})).$$

To interpret this, the heights of rectangles are the marginal increments in u; the widths are the capacities of subsets containing all elements in Ω that are mapped to a real number greater than or equal to u_i. The vertical representation immediately follows:

$$\int u\, d\mu = \sum_{i=1}^{n} u_i[\mu(u^{-1}(\{u_{i-1}\dots,u_n\})-\mu(u^{-1}(\{u_i,\dots,u_n\}))].$$

The Choquet Integral was rediscovered by Schmeidler [62, 63] to put forward an axiomatic model of decision-making under uncertainty. In Schmeidler's approach, subjective probabilities that reflect the willingness to bet need not be additive, when the objective probabilities are unknown to the economic agent. This approach provides a formal mathematical improvement that is compatible with Allais' [4] and Ellsberg's [29] paradoxes, which Savage's [61] theorem fails to explain. Schmeidler's theorem weakens the assumptions of Savage's subjective probability and proposes an elegant generalization to the classic expected utility theorem.

3. Savage's Subjective Expected Utility Model

First, we present the framework of Savage's subjective expected utility model. For axioms, we use names given in Machina and Schmeidler [50] for better intuitive indications.

Let S be a set of states of the world, and let C be a set of consequences (or outcomes). An act is a function $f: S \to C$, that yields the decision-maker a consequence given each state. Let F denote the set of acts. Let $\mathscr{A}$ be the collection of events, that is, an algebra of subsets of S.

Axiom 1 (Ordering)**.** The preference relation is complete, reflexive, and transitive.

Axiom 2 (Sure-Thing Principle)**.** For all events E and acts f, f^*, g and h,

$$\begin{bmatrix} f^*(s) & \text{if } s \in E \\ g(s) & \text{if } s \notin E \end{bmatrix} \succeq \begin{bmatrix} f(s) & \text{if } s \in E \\ g(s) & \text{if } s \notin E \end{bmatrix} \Rightarrow \begin{bmatrix} f^*(s) & \text{if } s \in E \\ h(s) & \text{if } s \notin E \end{bmatrix} \succeq \begin{bmatrix} f(s) & \text{if } s \in E \\ h(s) & \text{if } s \notin E \end{bmatrix}.$$

Axiom 3 (Eventwise Monotonicity)**.** For all events E, consequences x and y, and act f,

$$\begin{bmatrix} x & \text{if } s \in E \\ f(s) & \text{if } s \notin E \end{bmatrix} \succeq \begin{bmatrix} y & \text{if } s \in E \\ f(s) & \text{if } s \notin E \end{bmatrix} \Leftrightarrow x \succeq y.$$

Axiom 4 (Weak Comparative Probability)**.** For all events A, B, and consequences $x^* \succ x$, $y^* \succ y$,

$$\begin{bmatrix} x^* & \text{if } s \in A \\ x & \text{if } s \notin A \end{bmatrix} \succeq \begin{bmatrix} x^* & \text{if } s \in B \\ x & \text{if } s \notin B \end{bmatrix} \Rightarrow \begin{bmatrix} y^* & \text{if } s \in A \\ y & \text{if } s \notin A \end{bmatrix} \succeq \begin{bmatrix} y^* & \text{if } s \in B \\ y & \text{if } s \notin B \end{bmatrix}.$$

Axiom 5 (Nondegeneracy)**.** There exists consequences x and y, such that $x \succ y$.

Axiom 6 (Small Event Continuity)**.** For any acts f and g, and consequence x, there exists a finite set of events $\{A_1,\dots,A_n\}$ forming a partition of S such that

$$f \succeq \begin{bmatrix} x & \text{if } s \in A_i \\ g(s) & \text{if } s \notin A_i \end{bmatrix} \text{ and } \begin{bmatrix} x & \text{if } s \in A_i \\ f(s) & \text{if } s \notin A_i \end{bmatrix} \succeq g, \text{ for all } i, j \in 1,\dots,n.$$

Axiom 7 (Uniform Monotonicity)**.** For all events E and all acts f and f^*, if

$$\begin{bmatrix} f^*(s) & \text{if } s \in E \\ g(s) & \text{if } s \notin E \end{bmatrix} \succeq (\preceq) \begin{bmatrix} x & \text{if } s \in E \\ g(s) & \text{if } s \notin E \end{bmatrix}$$

for all $g \in F$ and rach $x \in f(E)$, then

$$\begin{bmatrix} f^*(s) & \text{if } s \in E \\ h(s) & \text{if } s \notin E \end{bmatrix} \succeq (\preceq) \begin{bmatrix} x & \text{if } s \in E \\ h(s) & \text{if } s \notin E. \end{bmatrix} \text{ for all } h \in F.$$

Theorem 3.1 (see Savage [61]). *let $\succeq$ be a preference relation on F that satisfies Axioms 1 to 7, then there exists a unique, finitely additive, non-atomic probability measure $p(\cdot)$ on $\mathscr{A}$, and a unique up to positive affine transformation, state-dependent utility function $u(\cdot)$ on C, such that for all acts f and g, $f \succeq g$ is and only of*

$$\int u(f(s))\,dp(s) \geq \int u(g(s))\,dp(s).$$

Savage's Theorem provides axiomatic support for the classic von Neumann and Morgenstern expected utility model. Satisfying the above seven Axioms is equivalent to having a preference relation implemented by maximizing the expectation of a utility function with a unique probability measure on the set of all events.

Many people believed the expected utility model was the only legitimate theory to describe decision-making under uncertainty. However, the model, particularly the Sure Thing Principle, often fails to capture some observed experimental behaviors. The two famous experiments that fall outside this model are Allais' paradox and Ellsberg's Paradox. Here we present the more general case, Ellsberg's Paradox, where the objective probabilities are unknown.

4. Ellsberg's Paradox

Ellsberg's thought experiment considers an urn known to contain 30 red balls and 60 black and yellow balls. The proportion of black and yellow balls is unknown. One ball is drawn at random from the urn. The possible outcomes are displayed in the table below.

Acts	Red	Black	Yellow
f_1	\$100	\$0	\$0
f_2	\$0	\$100	\$0
f_3	\$100	\$0	\$100
f_4	\$0	\$100	\$100

Table 1. Ellsberg's Paradox

An uncertainty-averse decision-maker has the following preferences that violate the Sure Thing Principle: $f_1 \succ f_2$, and $f_4 \succ f_3$. The classical expected utility theorem is also incompatible with this, since $f_1 \succ f_2$ indicates $p(Red) > p(Black)$, and $f_4 \succ f_3$ indicates $p(Red) < p(Black)$, which yields a contradiction.

5. Schmeidler's Generalized Model and the Weakened Axiom

Schmeidler solves Ellsberg's Paradox by sketching the following generalized theorem.

Theorem 5.1 (see Schmeidler [63]). *let $\succeq$ be a preference relation that satisfies weak order, comonotonic independence, continuity, monotonicity and nondegeneracy. Then there exists a* *unique capacity $\mu(\cdot)$ on $\mathscr{A}$, and a state-independent utility function $u(\cdot)$ on C, such that for all acts f and g, $f \succeq g$, if and only if*

$$\int u(f)\,d\mu \geq \int u(g)\,d\mu,$$

where $u(\cdot)$ is unique up to positive affine transformations.

Schmeidler generalizes the probability measure $p(\cdot)$ to the non-additive capacity $\mu(\cdot)$. As integration is with respect to capacity, this way of calculating expected utility is referred to as Choquet Expected Utility. This representation better describes the economic agent's perception of different levels of prizes and attitude toward uncertainty. Under expected utility theory, perception of prizes and attitude toward uncertainty are mixed, and both are indicated by the curvature of the utility function.

Now, we demonstrate how Choquet expected utility is well compatible with the Ellsberg paradox. As given by the setting $\mu(\emptyset) = 0$, $\mu(\{Red\}) = \frac{1}{3}$, $\mu(\{Black, Yellow\}) = \frac{2}{3}$, and $\mu(\{Red, Black, Yellow\}) = 1$. However, the decision maker perceives $\mu(\{Black\}) = \mu(\{Yellow\}) = \frac{1}{6}$ and $\mu(\{Red, Black\}) = \mu(\{Red, Yellow\}) = \frac{1}{2}$. For simplicity, we assume $u(\$x) = x$, then

$$\begin{aligned}
CEU(f_1) &= (u(\$100) - u(\$0))\mu(Red) = 33\frac{1}{3}, \\
CEU(f_2) &= (u(\$100) - u(\$0))\mu(Black) = 16\frac{2}{3}, \\
CEU(f_3) &= (u(\$100) - u(\$0))\mu(Red, Yellow) = 50, \\
CEU(f_1) &= (u(\$100) - u(\$0))\mu(Black, Yellow) = 66\frac{2}{3}.
\end{aligned}$$

The above Choquet expected utilities represent $f_1 \succ f_2$, and $f_4 \succ f_3$. Note the illustrated capacity is not additive, $\mu(\{Black\}) + \mu(\{Yellow\}) = \frac{1}{3} \neq \frac{2}{3} = \mu(\{Black, Yellow\})$. To interpret, the economic agent is informed of the sum of black and yellow balls, but the uncertainty aversion makes neither of them attractive to bet on.

Schmeidler's axiomatic conditions are slightly different from Savage's. Schmeidler's Theorem is under a simpler framework developed by Anscombe and Aumann [5]. Instead of acts (actions), they introduced horse lotteries that map the state to a probability distribution over money amount prizes. Some alternative derivations of this axiomatic model[1] use similar but not quite equivalent approaches. But all of them adopt some form of the Comonotonic Sure-Thing Principle (i.e., Comonotonic Independence in Schmeidler's Theorem):

Axiom 8 (Comonotonic Sure-Thing Principle)**.** For all events E and pairwise comonotonic actions f, f^*, g, and h,

$$\begin{bmatrix} f^*(s) & \text{if } s \in E \\ g(s) & \text{if } s \notin E \end{bmatrix} \succeq \begin{bmatrix} f(s) & \text{if } s \in E \\ g(s) & \text{if } s \notin E \end{bmatrix} \Rightarrow \begin{bmatrix} f^*(s) & \text{if } s \in E \\ h(s) & \text{if } s \notin E \end{bmatrix} \succeq \begin{bmatrix} f(s) & \text{if } s \in E \\ h(s) & \text{if } s \notin E \end{bmatrix}.$$

[1] As listed in Machina [49], see Gilboa [30] and Axiom (ii) of Schmeidler [63], Yutaka Nakamura [56], Rakesh Sarin and Peter P. Wakker [60], Chew and Karni [12], Chew and Wakker [13], Wakker [69], Mohammed Abdellaoui and Wakker [53], and Veronika Köbberling and Wakker [65].

Acts f and g are comonotonic, if there exist no states s and s', such that $f(s) \succ f(s')$ and $g(s) \succ g(s')$. In other words, two acts are said to be comonotonic if they never move in opposite directions. The rankings of consequences are unchanged among comonotonic actions.

Suppose the Choquet expected utility holds. Then the Comonotonic Sure-Thing Principle also holds; to have an additive expected utility representation, the stricter Sure-Thing Principle must hold for all acts in F [69]. Chew and Wakker [13] further studied the comonotonic sure thing principle to form a generalization of all cumulative forms, including rank-dependent expected utility, Choquet expected utility, and the famous cumulative prospect theory.

6. Properties

We list some basic properties of the Choquet Integrals (see Choquet [14], and Grabisch [32, 34]):

Proposition 6.1 (Positive homogeneity). *For all $\alpha \geq 0$,*

$$\int \alpha f \, d\mu = \alpha \int f \, d\mu$$

Proposition 6.2 (Monotonicity with respect to the integrand). *For any capacity μ, and bounded $\mathscr{F}$-measurable function f and f',*

$$f \leq f' \Rightarrow \int f \, d\mu \leq \int f' \, d\mu$$

Proposition 6.3 (Boundaries). *If $int f$ and $sup f$ are attained, then*

$$int f = \int f \, d\mu_{min}, \;\; sup f = \int f \, d\mu_{max},$$

with $\mu_{min}(A) = 0$ for all $A \subset \Omega$, $A \in F$, $\mu_{min}(\Omega) = 1$, and $\mu_{nax}(A) = 1$ for all nonempty $A \in F$.

For any normalized capacity μ,

$$ess \;\; inf_{\mu} f \leq \int f \, d\mu \leq ess \;\; sup_{\mu} f.$$

Proposition 6.4 (Strongly uncertainty averse [63]). *A decision maker is strongly uncertainty averse if, for every pair of acts X and Y, and $\alpha \in [0,1]$*

$$X \succeq Y \Rightarrow \alpha X + (1-\alpha) Y \succeq Y$$

In other words, a decision maker is strongly uncertainty averse if her preferences are convex. For a decision-maker whose preferences are represented by (μ, u), she is strongly uncertainty averse if and only if μ is convex and u is concave (see Chateauneuf, Dana, and Tallon [10]).

The following properties are applied to any set function $\nu : F \to \mathbb{R}$ that satisfies $\nu(\emptyset) = 0$, called a game, a generalization of capacity.

Proposition 6.5 (Comonotonic Additivity). *For all comonotonic functions f and $g : \Omega \to \mathbb{R}$, that is, $((f(s) - f(s'))(g(s) - g(s')) \geq 0$ for all $s, s' \in \Omega$, then*

$$\int f + g \, d\nu = \int f \, d\nu + \int g \, d\nu.$$

Proposition 6.6 (Monotonicity with respect to the game)**.** *If $f \geq 0$, then*

$$\nu \leq \nu' \Rightarrow \int f\, d\nu \leq \int f\, d\nu'.$$

Proposition 6.7 (Linearity with respect to the game)**.** *For all $\alpha \in \mathbb{R}$,*

$$\int f\, d(\nu + \alpha \nu') = \int f\, d\nu + \alpha \int f\, d\nu'.$$

7. Theoretical Development

First, we present the closely related theorems mentioned in the previous section.

Rank-Dependent Expected Utility

Rank-Dependent Expected Utility (RDEU) was developed to explain observed economic behavior under risk (see Quiggin [58]), such as the Allais paradox, where objective probability is known. Here we present Yaari's dual theory, a special case of RDEU.

Let P be a probability measure on Ω and $f : [0,1] \to [0,1]$ be a non-decreasing function such that $f(0) = 0$, and $f(1) = 1$. Then the composition of f and P, i.e. $\mu(A) = f(P(A))$, is called a distorted probability.

Theorem 7.1. *(see Yaari* [73]*) The preference relation is neutral, weak order, continuous, monotonic, and comonotonic independent, if and only if there exists a distortion function f : $[0,1] \to [0,1]$ such that for all acts g and h, $g \succeq h$ if and only if*

$$\int u(g(s))\, df(p(s)) \geq \int u(h(s))\, df(p(s))$$

Details can be found in Wakker [68] and Heilpern's [36] survey.

Maxmin Expected Utility

Gilboa and Schmeridler [31] axiomatized the maxmin decision rule by choosing the act that maximizes the minimal expected utility, where the minimum is taken over the set of prior probabilities. The decision maker is extremely uncertainty-averse under this approach, selecting the worst subjective probability measure, given the objective probability.

To illustrate, we show De Castro and Yannelis' [16] interpretation in solving Ellsberg's paradox with maxmin expected utility. The set of probabilities with respect to the objective probability is

$$\mathscr{P}_i \equiv \{\pi \in \Delta : \pi(\{Red\}) = \frac{1}{3}; \pi(\{Black, Yellow\}) = \frac{2}{3}\}.$$

Again, we assume $u(\$x) = x$, then the maxmin decision maker chooses acts by assuming the worst case scenario:

$$MEU(f_1) = \min_{\pi \in \mathscr{P}_1} \int_\Omega u(\$100) \times 1_{\{Red\}} \, d\pi = \min_{\pi \in \mathscr{P}_1} \pi(\{Red\}) = 33\frac{1}{3};$$
$$MEU(f_2) = \min_{\pi \in \mathscr{P}_1} \int_\Omega u(\$100) \times 1_{\{Black\}} \, d\pi = \min_{\pi \in \mathscr{P}_1} \pi(\{Black\}) = 0;$$
$$MEU(f_3) = \min_{\pi \in \mathscr{P}_1} \int_\Omega u(\$100) \times 1_{\{Red,Yellow\}} \, d\pi = \min_{\pi \in \mathscr{P}_1} \pi(\{Red,Yellow\}) = 33\frac{1}{3};$$
$$MEU(f_4) = \min_{\pi \in \mathscr{P}_1} \int_\Omega u(\$100) \times 1_{\{Black,Yellow\}} \, d\pi = \min_{\pi \in \mathscr{P}_1} \pi(\{Black,Yellow\}) = 66\frac{2}{3}.$$

This approach solves the paradox by completing preferences. Again, $f_1 \succ f_2$ and $f_4 \succ f_3$ as desired.

Cumulative Prospect Theory

Cumulative Prospect Theory [64] is the sum of two separate Choquet Integrals with the value of gains and losses. Cumulative prospect theory defines the outcome set X as a set of monetary outcomes for simplicity. The mapping $f : S \to X$ is called an uncertain prospect under the following conditions. The positive part of f, denoted f^+, is defined by letting $f^+ = f(s)$ if $f(s) > 0$. The negative part of f, f^- is defined similarly. Let V be the functional that represents the weak preference $\succeq$. Then

$$V(f) = V(f^+) + V(f^-),$$

where $V(f^+)$ is a Choquet integral, and $V(f^-)$ is a Choquet integral in the opposite direction, with respect to the reference level.

With the support of experimental evidence, prospect theory finds the following common risk attitude: risk aversion for gains and risk seeking for losses of high probability; risk seeking for gains and risk aversion for losses of low probability.

Waegenaere and Wakker [67] studied signed Choquet integrals, which relax the monotonicity condition of regular Choquet integrals. One of their results gives the conditions under which the Choquet pricing functional can be decomposed into a linear part and a non-negative, sub-additive part.

Jaffray et al. [37]-[43] further developed capacities and made the most tractable separation of risk attitudes, ambiguity attitudes, and ambiguity beliefs in their works. Different ways of updating Choquet beliefs are compared in the Eichberger, Grant, and Kelsey [25] paper.

Cerreia-Vioglio et al. [8] gave a general integral representation for nonadditive probabilities defined on an Archimedean Riesz space based on the foundation of Aliprantis's fundamental work [1, 2, 3], which introduced Riesz space into the field of economics and established the relevancy of Riesz Space to Choquet integration and thereby to behavioral economics.

Other important approaches to weakening the Savage Axioms include Wald [70], De Castro and Yannelis [17], De Castro, Pesce, and Yannelis [15], and Gilboa and Schmeidler [31].

8. Application

Choquet Integration is widely used in many areas. Here we list some applications in economics and finance.

Choquet Pricing

Kelsey and Milne [46] used the Choquet integral to extend the arbitrage pricing theorem and showed that the linear representation of price remains true if certain non-expected utility preferences are used. To capture the uncertainty aversion of the economic agent, Chateauneuf, Kast, and Lapied [11] proposed to use Choquet integrals as pricing functions for insurance and finance. Waegenaere, Kast, and Lapied [66] then introduced a general equilibrium model that allows for non-linearity and showed that Choquet pricing is consistent with general equilibrium. Choquet pricing is further studied by Castagnoli et al. [7], who showed if prices in a market are Choquet expectations, the existence of one frictionless asset may force the whole market to be frictionless.

Game Theory

Dow and Werlang [23] defined Nash equilibrium for two-person normal-form games in the presence of Knightian [47] uncertainty. Eichberger and Kelsey [26] introduced the concept of an "equilibrium under uncertainty" in n-player games. By using a class of capacities proposed by Jaffray and Philippe [42]. Eichberger and Kelsey [27] further studied comparative statics of changes in ambiguity-attitude in games with strategic complements. Marinacci [52] introduced ambiguous games, which allow vagueness in players' beliefs over opponents' choice of strategies. Haller [35] studied how the introduction of non-additive capacities affects the solvability of strategic games. Eichberger, Kelsey, and Schipper [28] showed that pessimism has the effect of increasing(decreasing) equilibrium prices under Cournot (Bertrand) competition. Dominiak and Eichberger [19] proposed Context-Dependent Equilibrium Under Ambiguity, for strategic games where players' beliefs are influenced by exogenous information.

Multi-Criteria Decision-Making Problems

Grabisch [33] introduced fuzzy measures, including both the Choquet Integral and Sugeno Integral, for aggregation in multi-criteria decision-making problems. Marichal [51] presented an axiomatic approach to support the Choquet integral as a tool to aggregate interacting criteria. The Choquet integral serves as an extension of the weighted arithmetic mean by taking interaction among criteria into consideration. This approach is further developed by Kojadinovic [48].

Monetary Assets

Barnett, Han, and Zhang [6] found that using the Choquet integral yields boundaries to the user cost of monetary assets. Their findings explain why there are situations in which people are not active in changing their monetary asset portfolios.

Auctions

To explain why experimental submitted bids in first-price sealed-bid auctions exceed Nash equilibrium predictions for risk-neutral bidders, Salo and Weber [59] used the Choquet expected utility theory to attribute the observed bidding behavior to uncertainty aversion.

Search

To present the difference between risk and uncertainty in job searching, Nishimura and Ozaki [57] used the Choquet integral to show that an increase in risk increases the reservation wage, while an increase in uncertainty reduces it.

Wages

Mukerji and Tallon [55] studied optimal wage contracting by assuming agents are uncertainty averse in decisions and found that such agents will choose not to include any indexation coverage in their wage contracts even when inflation is uncertain, under low inflation uncertainty perception.

Portfolio Choice Capital Investment

In optimal investment decisions, Dow and Werlang [24] suggest that maximizing Choquet expected utility may be a good model to explain an uncertainty-averse investor's behavior.

Insurance

In actuarial sciences, Denneberg [18] and Wang [71] proposed the Choquet integral with respect to the concave monotone set function to represent the premium principle relative to risk.

Axiomatic characterization and theorems of Choquet insurance prices can be found in Wang, Young and Panjer [72]. Jeleva [44] showed that the impact of the background risk on the demand for insurance is attributed to the attitude towards wealth, when the insurable and the background risk are comonotonic. It they are anti-comonotonic, the attitude towards uncertainty determines the impact.

Risk Sharing

In the context of optimal risk sharing, Chateauneuf, Dana, and Tallon [9] showed that if the capacity is convex in a Choquet expected utility setting, then the set of Pareto optima is the same as if agents have beliefs with a common vNM probability measure.

Incomplete Contracts

Mukerji [54] used the Choquet integral to show that ambiguity aversion can explain the existence of incomplete contracts, where instructions for some possible events are not included.

Trade

Kajii and Ui [45] initiated the characterization of the existence of an aggregable bet and an aggregable trade by applying convex capacity. Dominiak, Eichberger, and Lefort [20] extended the aggregable trade results in Kajii and Ui [45] by allowing Choquet preferences to be non-convex.

Agreement Theorems

Dominiak and Lefort [21, 22] discussed the impact on "agreeing to disagree" type results by relaxing the expected utility assumption, using the Choquet integral.

9. Conclusion

Much of the research in behavioral economics diverges from formal microeconomic foundations, with the divergences motivated by paradoxes in experimental results that are not fully compatible with implications of rational behavior under risk implied by the Savage Axioms. We have shown that consistency between behavioral economics and formal microeconomic foundations can be reestablished in many applications be replacement of the Riemann integral with the Choquet integral on Riesz space. The needed theory has been provided by Choquet [14], Cerreia-Vioglio et al. [8], and in its most fundamental form by Aliprantis et al [1, 2, 3].

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