Monetary Policy in a Small Open Economy with Multiple Monetary Assets

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Abstract: This paper revisits the issue of money measurement in the context of a New Keynesian framework with sticky prices and monopolistic competition, where money matters. We adopt the framework for a small open economy with home bias in consumption from Faia and Monacelli (2008) and follow Belongia and Ireland (2014) in their closed economy New Keynesian model by adding commercial banks in the financial sector. We construct various measures of money supply, including the traditional simple-sum and the Divisia index, originated by Barnett (1980). Through simulation of equilibrium dynamics with various shocks and variation of parameters, we find that the Divisia index tracks the movement of money most closely to the aggregation theoretic benchmark, followed by the monetary base, while simple sum often fails to match the correct trend. In a classical model with perfect competition and perfect markets with no rigidities, the tracking advantage of Divisia is derivable, as in Barnett (1980). But in the New Keynesian model, the aggregation theoretic proof is compromised and best investigated empirically. We further analyze the impact of openness on the volatility of macroeconomic variables. We find that as the small economy becomes more open, domestic inflation and nominal interest rates are more volatile, while terms of trade and exchange rates become more stable. In this regard, the Divisia index and the monetary base, without payment of interest on reserves, follow the correct trend, while the simple-sum, again, does not.

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1. Introduction

As the world becomes more and more integrated, economies are increasingly open and most of them are small. Macroeconomic research on small open economies has attracted increasing attention lately. The New Open Economy Macroeconomics literature has grown rapidly in recent years. The New Keynesian literature on optimal monetary policy in an open economy has largely focused on whether exchange rate stability should be part of a central bank's strategy. See, e.g., McCallum (2007). The role of nominal variables such as interest rates, money supply, and exchange rates are emphasized in the recent New Keynesian literature to have impact on real economic variables such as output and growth in the short-run because of the stickiness of prices and wages.

However, recent Dynamic Stochastic General Equilibrium (DSGE) macroeconomic models often ignore the aggregate quantity of money as an instrument of monetary policy. Many DSGE models rely solely on nominal interest rates, such as the federal funds rate, in modeling the impact of monetary policy. In the case of a small open economy, exchange rates are considered as well. The intervention of the central bank is expressed solely in terms of an interest rate rule such the Taylor rule (1993). The support for this view lies in the empirical evidence by Bernanke and Blinder (1988) that demand for simple sum money is more unstable than the demand for credit, such that monetary policy would have better success in stabilizing output if it stabilized interest rates rather than money supply. Moreover, Sims (1980) argues that money loses its predictive power on output when an interest rate is included in the regression.

In response to decreased emphasis on money quantities in the New Keynesian literature, other economists have brought up the issue of money measurement. Up until the 1980s, economists measured money supply using arithmetic measures. Typical measures include simple sum MB (monetary base), M1 (narrow money), M2 (broad money), and M3 (financial liquidity). These simple-sum measures assign the same weight to every monetary asset and implicitly assume they all are perfect substitutes. In our modern financial system, these simple-sum measures clearly are not a proper way to aggregate money and are inconsistent with microeconomic aggregation theory. Chrystal and MacDonald (1994) introduced the term "Barnett critique" to refer to the misleading and potential distorted economic inferences resulting from the use of the simple-sum measures of money.

Barnett (1980) derived the formula to measure the user cost price of monetary services and derived a method to aggregate monetary services from neoclassical microeconomic foundations and index number theory. Accordingly, to aggregate monetary components, we need both their quantities and their user cost prices. Monetary assets are not perishables, but rather durables, having user costs measuring the price of their services during the current time period. If there were perfect rental markets for those services, then the rental prices would equal the user cost prices.

The Divisia measure of money services growth that Barnett derived is a weighted average of component quantity growth rates with the growth rate weights being the user cost evaluated expenditure shares of the component assets. Since the theory of monetary aggregation became available, central banks such as the Federal Reserve (FED) in the US, the Bank of England (BOE) in the UK, the European Central Bank (ECB), the Bank of Japan (BoJ), the National Bank of Poland, and the Bank of Israel, among others, have, at various times and in diverse ways, produced and maintained Divisia indices for monetary aggregation. Replacing the traditional simple-sum measures of money by the Divisia measures, Belongia (1996), Barnett and Chauvet (2010), Belongia and Ireland (2015), and other researchers, have shown in closed economy models that money still shares a strong relationship with aggregate economic activity and the demand for money function has remained stable, unlike results with simple sum money disconnected from economic aggregation and index number theory.

While the New Open Macroeconomics has been receiving increasing attention during the past two decades, the resulting discussion of monetary policy in an open economy context has focused primarily on the matter of stabilizing the exchange rate in models without money. In this paper, we revisit the issue of money measurement in that modern open economy context. We use the microfounded, dynamic and stochastic New Keynesian model to examine the behaviors of different measures of money supply, including the traditional simple-sum measures and Divisia measures, in response to various macroeconomic shocks. To do so, we extend a standard New Keynesian model for a small open economy by introducing a banking sector, in which deposits are created and used as imperfect substitutes for government-issued currency.

This framework allows us to construct and compare the behaviors of different measures of monetary aggregates. It also allows us to look at the impact of openness to the stability of macroeconomic variables in an environment containing New Keynesian market imperfections missing from Barnett's (1980) derivation of the Divisia monetary aggregates. In such an environment, we show that Divisia measures are strictly better than simple-sum measures in tracking the movement of the aggregation theoretic money supply. Our findings re-emphasize the *Barnett critique* that simple sum is misleading, and using it can distort inferences about the economy. We advocate the use of Divisia index in measuring money supply, even in an open economy New Keynesian framework.

The rest of the paper is organized as the following: Section 2 provides details of the theoretical model; section 3 presents the calibration for numerical solution of the model and discusses the results; section 4 concludes. The system of equations as well as all figures can be found in the appendix.

2. Theoretical Model

Our theoretical model is built upon a New Keynesian framework for a small open economy with home bias in consumption from Faia and Monacelli (2008). In that framework, the world consists of a small country (Home) and a rest of the world (Foreign). Home is assumed to be very small compared to Foreign. The small economy (Home) interacts with the rest of the world via the trading of final goods and investing. The total measure of the world's population (or number of households) is normalized to unity, with Home population and Foreign population having measure n and 1-n, respectively. To highlight behaviors of different measures of money in equilibrium dynamics in such a small economy, we follow the approach in Belongia and Ireland (2014) to add private commercial banks alongside the central bank to form the financial sector of a New Keynesian open economy model. There are four sectors in our theoretical model: household sector, foreign sector, production sector, and financial/banking sector. Throughout the equations, subscription t is used to indicate time period in discrete time.

2.1. Household Sector

Intra-temporal Optimization Problem

To describe the intra-temporal optimization problem of households, the following equations from (1) to (8) are directly taken from Faia and Monacelli (2008). Accordingly, households consume a composite final goods C, which is a combination of domestic goods C_H and foreign goods C_F. These goods can be substituted with a positive elasticity of substitution η,

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where $0 \leq \gamma \leq 1$ is the weight of foreign goods in the per capita composite consumption bundle of the "representative consumer."

Each consumption bundle C_H, C_F itself is composed of imperfectly substituted varieties of goods with positive elasticity of substitution ε,

$$C_{H,t} = \left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \left(\int_0^n C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

$$C_{F,t} = \left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \left(\int_n^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where n is the relative measure of Home population compared with Foreign. In section 2.2 about the foreign sector, we further analyze the implication of *small economy with home bias in consumption* by discussing how we set the value of the coefficient γ and take the limit of certain variables as $n \to 0$.

Taking the price of domestic goods P_H and price of foreign goods P_F as given, households choose the optimal allocation of expenditure between domestic and foreign goods, C_H and C_F, depending on the relative prices, respectively,

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \tag{4}$$

$$C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t. \tag{5}$$

The optimal allocation within each variety of goods depends on the relative price of each good within that variety,

$$C_{H,t}(j) = \frac{1}{n}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \tag{6}$$

$$C_{F,t}(j) = \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}. \tag{7}$$

The general price level in Home can be expressed as a weighted index of the domestic price level P_H and the foreign price level P_F,

$$P_t = \left[(1-\gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{8}$$

Inter-temporal Maximization Problem

In the second optimization problem, the representative household chooses to allocate its resources to maximize their lifetime expected discounted utility,

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t\, h_t\, U\left(C_t, N_t, \frac{M_t}{P_t}\right), \tag{9}$$

where $0<\beta<1$ is the discount factor and h_t is a preference shock, which is assumed to follow an AR(1) process

$$\ln h_t = \rho_h \ln h_{t-1} + e_{h,t}, \tag{10}$$

with $0\leq\rho_h\leq 1$ and $e_{h,t} \sim$ i.i.d $N(0,\sigma_h^2)$. The first two components in the household's utility function are consumption C and labor N.

We choose a common specification for the utility function, that is increasing in (log) consumption and decreasing in labor,

$$U_t = \ln C_t - \psi_N\left(\frac{N_t^{1+\xi}}{1+\xi}\right) + \psi_M \ln\left(\frac{M_t}{P_t}\right), \tag{11}$$

with the inverse elasticity of labor $\xi>0$, and coefficients $\psi_M,\psi_N>0$. Other than consumption and labor, real balance M/P also enters the utility function in log form. Our model features multiple types of monetary assets by introducing financial institutions such as commercial banks, whose behavior will be discussed in more detail in section (2.4). In this financial environment, households allocate their monetary assets between cash, Ca_t, issued by the central bank and deposits, D_t, in commercial banks. Hence, our money in the economy depends on aggregation over cash and deposit.

Assuming classical perfect markets, the microeconomic theory of monetary aggregation has been thoroughly studied by Barnett with his various publications in the 1980s and 1990s. Belongia and Ireland were the pioneers to incorporate this aggregation in various genres of macroeconomic closed economy models, including Real Business Cycle and New Keynesian models. In our model, we use the Barnett's argument that cash and deposits can be substituted but they are not perfect substitutes. We assume a CES theoretical relation between them,

$$M_t = \left[\nu^{\frac{1}{\omega}} Ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} D_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{12}$$

where $0 \leq \nu \leq 1$ is the weight of cash among monetary assets and ω is the elasticity of substitution between cash and deposit. The above theoretical relation provides the true monetary service flow in this economy conditional upon knowing the values of the aggregator function's parameters. For the purpose of comparing the behavior of different money measures based on simulated data in this model, we later use the above true money as our benchmark.

Given multiple types of monetary assets, the household's budget constraints in our model are also slightly different from those common in the literature. We adopt the technique used in Keating and Smith (2019) to set up the budget constraints in two sub-periods in a manner consistent with our New Keynesian open economy model. Under this setting, households enter each period with a portfolio of maturing bonds B_{t-1} and monetary assets A_{t-1}. In the first sub-period, they receive a lump-sum transfer T from the central bank, and they can also take loans L from commercial banks to finance themselves. Households then allocate their monetary assets between cash C and deposits D and invest in bonds B, such that

$$\frac{B_t}{1+r_t} + Ca_t + D_t = A_{t-1} + B_{t-1} + T_t + L_t, \tag{13}$$

where r_t is the rate of return on holding bonds for one period. In the second sub-period, in addition to receiving wages from work, households collect interest from their deposits, r_t^D, and pay interest on their loan, r_t^L, at commercial banks. At the end of each period, households receive nominal profits, F_t, from holding shares of intermediate firms. After paying for consumption, households carry their monetary asset A_t into the next period,

$$P_t C_t + A_t + (1+r_t^L) L_t = Ca_t + W_t N_t + F_t + (1+r_t^D) D_t. \tag{14}$$

Households choose the optimal sequences $\{C_t, N_t, B_t, A_t, M_t, Ca_t, D_t, L_t\}_{t=0}^{\infty}$ to maximize the expected discounted utility in equation (9), with the utility specification in equation (11) subject to the constraints in equations (12), (13), and (14). Letting Λ_t^1 , Λ_t^2, and Λ_t^3 be the Lagrange multipliers on each constraint respectively, we solve for the first order conditions to obtain the Euler equation,

$$\Lambda_t^3 = \beta \ \mathrm{E}_t \Lambda_{t+1}^3 (1+r_{t+1}) \left(\frac{P_t}{P_{t+1}} \right), \tag{15}$$

with

$$\Lambda_t^3 = \frac{h_t}{C_t}. \tag{16}$$

The other two equations with Lagrange multipliers can be found in Appendix A1.

2.2. Foreign Sector

In our model, Home and Foreign interact by trading consumption goods (not intermediate goods), and they are open for capital mobility. Variables in Foreign are analogous to those in Home but distinguished by asterisks. We adopt equations from (17) to (27) from Faia and Monacelli (2008) to describe the behaviors of the foreign sector. The final composite consumption bundle in Foreign, C^*, is specified in a similar manner to that in Home, such that

$$C_t^* = \left[(1-\gamma^*)^{\frac{1}{\eta}} C^{*\frac{\eta-1}{\eta}}_{F,t} + \gamma^{*\frac{1}{\eta}} C^{*\frac{\eta-1}{\eta}}_{H,t} \right]^{\frac{\eta}{\eta-1}}, \tag{17}$$

with γ^* being the weight of final goods exported from Home to Foreign. Let α and α^* in (0,1) measure openness of Home and Foreign respectively, such that γ=(1–n)α and γ^*=$n\alpha^*$. If α=0, the small economy is closed to trade. Home bias in consumption requires the weight on goods produced in one economy must be strictly greater than the weight on them from the other economy,

$$1-\gamma = 1-(1-n)\alpha > \gamma^* = n\alpha^*. \tag{18}$$

We further assume asymmetric open level when α^*=α, and let Home be relatively very small to Foreign, as $n \rightarrow 0$, so that the left hand side in (18) can further become γ=α.

Similar to that in Home, the general price level P^* in Foreign is also a weighted index of the price level in Foreign, P_F^*, and the price level in Home, P_H^*, such that

$$P_t^* = \left[(1-\gamma^*)(P_{F,t}^*)^{1-\eta} + \gamma^*(P_{H,t}^*)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{19}$$

Law of One Price and Exchange Rates

Notice that Home and Foreign are open for free-trade, where final consumption goods can be exported and imported using their own currencies. Let *NE* be the nominal exchange rate measured by the number of units of home currency per one unit of foreign currency. Assuming no barriers for trade, the *law of one price* tells us that goods must be sold at the same price everywhere, after being converted into the same unit of currency either in Home or Foreign, such that

$$P_{H,t}(j) = NE_t P^*_{H,t}(j) \quad \text{and} \quad P_{F,t}(j) = NE_t P^*_{F,t}(j) \quad \text{for all} \quad j \in [0,1]. \tag{20}$$

Given the nominal exchange rate, the real exchange rate illustrates the relative price level between one unit of goods produced in Foreign and Home, with the foreign price converted into domestic currency using the nominal exchange rate. In order to find the real exchange rate, first we define the term of trade S as the relative price between Foreign and Home, such that

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}. \tag{21}$$

Computing the corresponding price ratios for Home and Foreign, we see that they depend on the terms of trade,

$$\frac{P_t}{P_{H,t}} = \left[(1-\gamma) + \gamma S_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv q(S_t), \tag{22}$$

$$\frac{P^*_t}{P^*_{F,t}} = \left[(1-\gamma^*) + \gamma^* \left(\frac{1}{S_t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv q^*(S_t). \tag{23}$$

Our real exchange rate RE now can be written as

$$RE_t \equiv NE_t \frac{P^*_t}{P_t} = S_t \frac{P^*_t}{P^*_{F,t}} \left(\frac{P_t}{P_{H,t}}\right)^{-1}. \tag{24}$$

With further algebra, we see that real exchange rate depends on the terms of trade and other parameters that characterize the open economy,

$$RE_t = S_t \frac{q^*(S_t)}{q(S_t)} = S_t \frac{\left[(1-\gamma^*) + \gamma^* S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\left[(1-\gamma) + \gamma S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}}. \tag{25}$$

Notice that,

$$\frac{\partial q(S_t)}{\partial S_t} > 0, \quad \frac{\partial q^*(S_t)}{\partial S_t} < 0, \quad \text{and} \quad \frac{\partial RE_t}{\partial S_t} > 0.$$

Because of our focus on the macroeconomic dynamics in a small economy, we are not going to set up a general equilibrium model for two countries but rather follow Faia and Monacelli's framework to set the variables in Foreign as exogenous (or given). To do so, we simplify equations for Foreign by taking their values in the limit, as $n \to 0$, to imply that the small economy is relatively very small compared to rest of the world. Given this assumption, the equilibrium dynamics of our system will resemble those in a standard closed economy. As $n \to 0$, we have $P_F^* \to P^*$ and $q^*(S_t) \to 1$. Therefore, in the limit, we find that

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \to NE_t \frac{P_t^*}{P_{H,t}} \text{ as } n \to 0, \text{ and} \tag{26}$$

$$RE_t \to \frac{S_t}{q(S_t)} \text{ as } n \to 0. \tag{27}$$

Interest Rate Parity

In our theoretical model, besides trading, Home and Foreign also interact in the financial market. Capital is allowed to move freely between the two economies for investing. This perfect capital mobility implies that both domestic residents and foreigners can invest in the bond market, and if the nominal interest rates are different in the two economies, the nominal exchange rate adjusts to reflect that difference. From the inter-temporal maximization problem of households, recall that the optimal condition for bond holdings in Home is the Euler equation (15). The parallel optimal condition for bond holdings in Foreign is

$$\Lambda_t^{3*} = \beta\, \mathrm{E}_t \Lambda_{t+1}^{3*}(1+r_{t+1}^*)\left(\frac{P_t^*}{P_{t+1}^*}\right). \tag{28}$$

Since there is no arbitrage opportunity for this investment, the rate of return on bonds must be the same in Home and Foreign, so that

$$1 + r_{t+1} = (1+r_{t+1}^*)\left(\frac{NE_{t+1}}{NE_t}\right). \tag{29}$$

Now use equation (15) and (28) to rewrite (29) as

$$\beta\, \mathrm{E}_t\left(\frac{\Lambda_{t+1}^3}{\Lambda_t^3}\frac{P_t}{P_{t+1}}\right) = \beta\, \mathrm{E}_t\left(\frac{\Lambda_{t+1}^{3*}}{\Lambda_t^{3*}}\frac{P_t^*}{P_{t+1}^*}\frac{NE_t}{NE_{t+1}}\right). \tag{30}$$

Continue to rearrange, we get

$$NE_t \frac{P_t^*}{P_t} = \frac{\Lambda_t^{3*}}{\Lambda_t^3}\mathrm{E}_t\left(\frac{\Lambda_{t+1}^3}{\Lambda_{t+1}^{3*}}\frac{P_{t+1}^*}{P_{t+1}}NE_{t+1}\right). \tag{31}$$

With our specification of the utility function, from the households' first order conditions we know that $\Lambda_t^3 = h_t/C_t$. We further assume $\Lambda_t^{3*} = 1/C_t^*$ along with other initial conditions for the two economies, so that we can iterate the expectation in (31) and rewrite the interest parity as

$$RE_t = \kappa \frac{h_t C_t}{C_t^*}, \tag{32}$$

Where Foreign consumption is exogenous and the coefficient κ is assumed to be unity for simplification. In our model, we assume an AR(1) process for the foreign consumption,

$$\ln\left(C_t^*\right) = \rho_c^* \ln\left(C_{t-1}^*\right) + e_{c,t}^*, \tag{33}$$

with $0<\rho_c^*<1$ and $e_{c,t}^* \sim$ i.i.d $N(0,\sigma_c^{*2})$.

2.3. Production Sector

Cost Minimization Problem

The production sector in our model is standard in the New Keynesian literature. Monopolistic intermediate production firms use labor to produce homogeneous goods under a constant return to scale technology. Each monopolistic firm j uses labor $N(j)$ to produce homogeneous output $Y(j)$ with a uniform linear technology Z to meet the total demand for their product from the world

$$Y_t(j) = Z_t N_t(j), \tag{34}$$

where the level of technology Z_t follows a random walk process,

$$\ln Z_t = \ln z + \ln Z_{t-1} + e_{z,t}, \tag{35}$$

with $e_{z,t} \sim$ i.i.d $N(0,\sigma_z^2)$ and z being the steady state value of technology level (or productivity). The homogeneous output $Y(j)$ is later used by competitive firms to produce final goods for domestic consumption and exporting to foreign consumers. The demand for each firm's product depends on its relative price and the total output Y,

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t. \tag{36}$$

The optimization problem for intermediate firms is to choose labor input to minimize production cost, given wages, technology, and demand for their output. The resulting Lagrangian is

$$\mathcal{L} = -W_t N_t(j) + \varphi_t(j)\left[Z_t N_t(j) - \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t\right], \tag{37}$$

where $\varphi(j)$ is the Lagrange multiplier on the constraint for each monopolistic intermediate firm. We solve for first order conditions of the cost minimization problem,

$$\varphi_t(j) = \frac{W_t}{Z_t} \quad \text{for all} \quad j \in [0,1], \tag{38}$$

where the Lagrangian multiplier $\varphi(j)$ can be interpreted as nominal marginal cost. Hence, the real marginal cost MC is expressed as

$$MC_t(j) = \frac{W_t}{Z_t P_{H,t}} \quad \text{for all} \quad j \in [0,1]. \tag{39}$$

Price Setting Mechanism

Another optimization problem for monopolistic intermediate firms is to set their price to maximize their profits. In this sticky price environment, intermediate firms can choose to keep their price at the previous period's level or absorb some cost to adjust the price. We set the price-adjustment cost according to Rotemberg (1982). Accordingly, all production firms face the same quadratic cost of adjusting their nominal prices. This cost, measured in term of units of domestic final goods, is

$$\frac{\phi}{2}\left(\frac{P_{H,t}(j)}{(1+\pi_H)P_{H,t-1}(j)}-1\right)^2 Y_t.$$

In our small open economy model, this price-adjustment cost depends on the steady state of gross Home producer's inflation $1+\pi_H$ and the value of the coefficient ϕ. If $\phi=0$, prices are absolutely flexible with no adjustment costs. Given $\phi>0$, each intermediate firm chooses to reset its price $P_{H,t}(j)$ in each period to maximize its expected discounted profit,

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^j \Lambda^3_{t+i} F_{t+i},$$

with nominal profit being

$$F_t(j) = P_{H,t}(j)Y_t(j) - W_t N_t(j) - \frac{\phi}{2}\left(\frac{P_{H,t}(j)}{(1+\pi_H)P_{H,t-1}(j)}-1\right)^2 P_{H,t}Y_t, \tag{40}$$

subject to the resource constraints,

$$Y_t(j) \le \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t \quad \text{and} \quad Y_t(j) = Z_t N_t(j). \tag{41}$$

2.4. Financial Sector

Commercial Banks

Having established behaviors of agents in the real sectors of the small open economy, we continue in this section to establish behaviors in the financial sector. To create a financial environment with multiple monetary assets, we follow Belongia and Ireland's approach to add private financial firms like commercial banks into the system. Hence, besides currency, i.e. cash issued by the central bank and injected into the economy via lump sum transfer, we now have another form of money in the economy: deposits at commercial banks. To describe how commercial banks work, we directly take equations (42), (43), and (44) from Belongia and Ireland (2014). Accordingly, during each period, commercial banks receive deposits D from households, and pay interest rate r^D on those deposits. Banks also provide loans L to households and charge interest rate r^L on the loans. Banks must meet the requirement on reserves set by the central bank. The relationship of loans to deposits depends on the actual reserve ratio τ across commercial banks, where $0 \leq \tau \leq 1$, and

$$L_t = (1-\tau)D_t. \tag{42}$$

For simplicity, we assume commercial banks' real operating costs depend on their real revenue $\upsilon D_t/P_t$ linearly at each period with positive coefficient υ. Commercial banks' nominal profit F^b can then be written as

$$F_t^b = r_t^L L_t - r_t^D D_t - P_t \upsilon \frac{D_t}{P_t}. \tag{43}$$

Competition among commercial banks drives their profits to zero. The profit maximization condition (with respect to D_t) requires that

$$r_t^D = r_t^L (1-\tau) - \upsilon. \tag{44}$$

Central Bank

The central bank injects money into or withdraws money from the economy via the lump sum transfer T to households. The central bank's budget constraint is

$$T_t = A_t - A_{t-1}. \tag{45}$$

Suppose the goal of the central bank is to stabilize the price level, and the central bank conducts monetary policy via targeting nominal interest rates. In this model, we employ a Taylor rule for monetary policy, where r is the steady state interest rate, such that

$$r_t = (1-\rho_r)r + \rho_r r_{t-1} + (1-\rho_r)\rho_\pi(\pi_{H,t} - \pi_H) + e_{r,t}. \tag{46}$$

with $e_{r,t} \sim$ i.i.d $N(0,\sigma_r^2)$. The coefficient ρ_r should be positive, and the coefficient ρ_π must be large enough to avoid indeterminacy. The inflation target in Home, π_H, is to be chosen by the central bank.

Monetary Aggregation

In such an environment of multiple monetary assets including cash and deposits, producing different interest rates, possessing different levels of liquidity, and having different levels of risk, the issue of how to properly measure money becomes important. Recall the functional form of the true monetary services aggregate is assumed to be known from equation (12),

$$M_t = \left[\nu^{\frac{1}{\omega}} Ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} D_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}.$$

The level of the monetary aggregate matters less than its growth rate, g^M, defined by

$$1 + g_t^M = \frac{M_t}{M_{t-1}}. \tag{47}$$

We cannot directly observe this true monetary aggregate from data, since the aggregator function contains unknown parameters and the functional form of the aggregator function is unknown to economic agents. How do we measure money supply, if the functional form of the economic aggregator function, (12), and its parameters, are unknown? There are multiple approaches to measure money supply in the literature. The most common way is the official simple-sum measure, *SM*, that is often reported by central banks throughout the world. It is the unweighted arithmetic summation of the components of money,

$$SM_t = Ca_t + D_t. \tag{48}$$

The growth rate of that simple-sum measure, g^S, is

$$1 + g_t^S = \frac{SM_t}{SM_{t-1}}. \tag{49}$$

A second popular measure is the monetary base, which is also frequently found in many central banks' reports. In our model, the monetary base, *MB*, is

$$MB_t = Ca_t + \tau D_t. \tag{50}$$

The growth rate of the monetary base, g^B, is

$$1 + g_t^B = \frac{MB_t}{MB_{t-1}}. \tag{51}$$

The main criticism of the use of the two above traditional measures of money supply is that they resemble unweighted addition of roller skates to subway trains to measure transportation services. In the late 1970s to early 1980s, Barnett proposed and developed another approach derived directly from economic aggregation and index number theory. The resulting Divisia measure requires data not only on component quantities but also on their interest rates, to compute the user cost price of the services of each component monetary asset. We have enough information to construct the Divisia index using the formulas from (52) to (56), acquired from Barnett (1980).

First, we compute the user cost price of cash u^{Ca} as

$$u_t^{Ca} = \frac{r_t - 0}{1 + r_t} = \frac{r_t}{1 + r_t}, \tag{52}$$

which follows from the fact that cash does not produce any interest rate, so that $r^{Ca}=0$. Similarly, the user cost price of deposits, u^D, is

$$u_t^D = \frac{r_t - r_t^D}{1 + r_t}. \tag{53}$$

Then we compute the expenditure shares of cash s^{Ca} and deposits s^D as

$$s_t^{Ca} = \frac{u_t^{Ca} Ca_t}{u_t^{Ca} Ca_t + u_t^D D_t}, \tag{54}$$

$$s_t^D = \frac{u_t^D D_t}{u_t^{Ca} Ca_t + u_t^D D_t}. \tag{55}$$

The growth rate of the Divisia quantity index g^{Dv} is the share weighted average growth rate of all of the components of money,

$$1+g_t^{Dv}=\left(\frac{Ca_t}{Ca_{t-1}}\right)^{\frac{s_t^{Ca}+s_{t-1}^{Ca}}{2}}\left(\frac{D_t}{D_{t-1}}\right)^{\frac{s_t^{D}+s_{t-1}^{D}}{2}}. \tag{56}$$

2.5. Market Clearing Conditions

Symmetric equilibrium implies that all intermediate firms end up setting the same price, while producing the same level of output and using the same amount of labor in equilibrium, $P_{H,t}(j)=P_{H,t}$, $N_t(j)=N_t$, and $Y_t(j)=Y_t$ for all j and t. Define $1+\pi_{H,t}=(P_{H,t}/P_{H,t-1})$ and use it to rewrite the nominal profit of intermediate firms in equilibrium

$$F_t=P_{H,t}Y_t-W_tN_t-\frac{\phi}{2}\left(\frac{1+\pi_{H,t}}{1+\pi_H}-1\right)^2P_{H,t}Y_t. \tag{57}$$

Deriving the first order conditions for optimal price setting, we acquire

$$\begin{aligned}&(1-\varepsilon)\Lambda_t^3Y_t+\varepsilon\,\Lambda_t^3\frac{W_tY_t}{Z_tP_{H,t}}-\phi\left(\frac{1+\pi_{H,t}}{1+\pi_H}-1\right)\left(\frac{1+\pi_{H,t}}{1+\pi_H}\right)Y_t\\&+\beta\,\phi\,E_t\Lambda_{t+1}^3\left(\frac{1+\pi_{H,t+1}}{1+\pi_H}-1\right)\left(\frac{1+\pi_{H,t+1}}{1+\pi_H}\right)Y_{t+1}=0.\end{aligned} \tag{58}$$

The market clearing condition for bonds requires B_t=0 for all t. Since the economy is open with trade, the aggregate resource constraint meets the domestic and foreign demands and covers the costs from the production sector and the private financial sector as follows:

$$Y_t=nC_{H,t}+(1-n)C_{H,t}^*+\frac{\phi}{2}\left(\frac{1+\pi_{H,t}}{1+\pi_H}-1\right)^2Y_t+\upsilon\frac{D_t}{P_{H,t}}. \tag{59}$$

The above condition can be written in term of S_t and $q(S_t)$ as

$$Y_t=(1-\alpha)\left[q(S_t)\right]^{\eta}C_t+\alpha S_t^{\eta}C_t^*+\frac{\phi}{2}\left(\frac{1+\pi_{H,t}}{1+\pi_H}-1\right)^2Y_t+\upsilon\frac{D_t}{P_{H,t}}. \tag{60}$$

3. Calibration and Results

3.1. Calibration

Before solving the model numerically, we consolidate 60 equations in the previous sections into a system of equations in 31 endogenous variables: A_t, C_t, Ca_t, D_t, L_t, N_t, NE_t, M_t, $P_{H,t}$, P_t, r_t, r_t^D, r_t^L, S_t, $q(S_t)$, RE_t, W_t, Y_t, Λ_t^1, Λ_t^2, Λ_t^3, $\pi_{H,t}$, SM_t, g_t^S, g_t^B, g_t^{Dv}, g_t^M, s_t^{Ca}, s_t^D, u_t^{Ca}, u_t^D, and 4 exogenous variables: h_t, Z_t, C_t^*, P_t^*. We further transform this system into a stationary system with real effective variables by eliminating nominal price and adjusting for technology level. We label the real effective variables with lower capital letters. This system of equations has 30 endogenous variables: $a_t=(A_t/P_t)/Z_{t-1}$, $c_t=C_t/Z_{t-1}$, $ca_t=(Ca_t/P_t)/Z_{t-1}$, $d_t=(D_t/P_t)/Z_{t-1}$, $l_t=(L_t/P_t)/Z_{t-1}$, $m_t=(M_t/P_t)/Z_{t-1}$, $w_t=(W_t/P_t)/Z_{t-1}$, $y_t=Y_t/Z_{t-1}$, $\lambda_t^1= Z_{t-1}\Lambda_t^1$, $\lambda_t^2= Z_{t-1}\Lambda_t^2$, $\lambda_t^3= Z_{t-1}\Lambda_t^3$, $sm_t=(SM_t/P_t)/Z_{t-1}$, N_t, r_t, r_t^D, r_t^L, S_t, $q(S_t)$, RE_t, π_t, $\pi_{H,t}$, g_t^S, g_t^B, g_t^{Dv}, g_t^M, $g_t^{NE}=NE_t/NE_{t-1}$ (growth rate of nominal exchange rate), s_t^{Ca}, s_t^D, u_t^{Ca}, u_t^D; and 5 exogenous variables: h_t, $z_t= Z_t/Z_{t-1}$, $c_t^f=C_t^*/Z_{t-1}^*$, π_t^f, and $z_t^f= Z_{t-1}/Z_{t-1}^*$. Detail of these 35 equations can be found in Appendix A1.

To solve the above system numerically, we first solve for the steady state and choose reasonable values for parameters. The following observations are based on Table 1 below and Appendix A1. The households' discount factor β= 0.98525 indicates one period in the model as a month in real time, and an annual interest rate of 2% in equilibrium. The inverse elasticity of labor, ξ=1 implies that labor enters the utility function in a quadratic form. The coefficient ψ_N=3.5 orients the steady state value of labor to be in the range from 0.3 to 0.5, which can be understood as 8 hours to 12 hours per day. The elasticity of substitution among different varieties of domestic goods reflects the power of firms.

Notice that the steady state of the real marginal cost is equal to the inverse markup, MC=(ε-1)/ε. We choose ε=10, which implies a steady state markup of 11%. The degree of price stickiness ϕ is calibrated with the value of 105 to match a probability of not resetting prices in a given period of 0.75. The literature is quite varied in assigning the value of η, the elasticity of substitution between domestic and foreign goods. In the special case of η=1, domestic and foreign goods are perfect substitutes. Other than that, most of the papers in the literature adopt a value of η above unity. As a benchmark case, we choose η =2, but we also do parameter variation for sensitivity analysis.

In our model, the size of the small economy is assumed to be very small compared to the rest of the world, n ~ 0. Hence the share of imported goods in the composite consumption bundle becomes γ=(1-n)α ~ α. If α=0, the economy is closed. The level of home bias in consumption, 1- γ, has an inverse relationship with the level of openness α and requires that α<1. We choose the benchmark value of α =0.4 and we vary it from 0 to 1 to see the impact of openness in our model. For the elasticity of substitution between cash and deposits, ω, we choose the benchmark value of 2 and set the weight of cash on monetary assets to ν=0.625, in order to match the ratio of currency in circulation (Ca/SM) of about 10%. Other values of ω, 1.16 and 3, are considered to resemble other scenarios in the economy with 25% and 3% currency in circulation. The financial sector cost, υ=0.005, is measured in units of final goods. It implies that banking activity accounts for about 0.5% of total output in the steady state. The reserve ratio is set at τ=0.02 to resemble the average reserve ratio in a small economy. Other parameters, κ=1 and ψ_M=0.01, are set for simplicity.

Table 1. Calibrated Values of Selected Parameters

Case	omega	eta	alpha	ca/sm	N
1	2	2	0.4	0.11	0.4305
2	1.16	2	0.4	0.27	0.1430
3	3	5	0.4	0.03	0.3438
4	2	5	0.1	0.11	0.4022
5	3	2	0.8	0.03	0.4089

Column 5 and 6 show the steady state values of the ratio of currency in circulation to labor with respect to each set of parameters.

We assume the central bank's ultimate goal is to stabilize the price level, so the exogenous inflation target in Home country is set at π_H=0. Similarly, we assume the central bank in Foreign succeeds in controlling inflation so that exogenous Foreign inflation is π^*=0. The calibrated value z=1.005 implies a growth of productivity (technology) of 6% per year in the model. The relative productivity of Home versus Foreign, $z_t^f = Z_{t-1}/Z_{t-1}^*$, is assumed to be z^f=1, implying the same level of productivity all over the world in steady state.

For the coefficients of the monetary policy rule (a version of the Taylor rule), we followed Belongia and Ireland (2014) to set the coefficient on the reaction to the interest rate to ρ_r=0.75. We need to set the reaction to inflation ρ_π large enough to avoid indeterminacy, i.e, to meet Blanchard and Kahn’s (1980) condition for a unique equilibrium solution of the rational expected model. Given ρ_r=0.75, we set ρ_π=1.5 to meet this condition. Other coefficients are ρ_h=0.9, ρ_c^f=0.95, ρ_z^f =0.95, and ρ_π^f=0.95. All standard deviations are set at 0.01, which implies 1% shocks.

3.2. Results and Discussion

As the benchmark case, we choose the elasticity of substitution between cash and deposit, ω=2, the elasticity of substitution between domestic and foreign goods, η=2, and the openness, α=0.4, which implies a weight of 0.6 on domestic goods in consumption. With this setting of parameters, we are looking at a small economy with 10% currency in circulation in steady state. We shock this economy with various shocks including a preference shock, a monetary policy shock, a home-productivity shock, a shock to foreign demand, a shock to foreign productivity, and a shock to foreign inflation. The impulse responses of the growth rates of different money measures, including the true monetary aggregator function, the Divisia quantity aggregate, the monetary base, and the simple-sum measure are reported in Figure 1 and Figure 2.

Notice that the shock in foreign inflation is entirely absorbed by the exchange rate, and all other macroeconomic variables do not response to this shock. This results from the assumption of perfect capital mobility. The small economy chooses to let capital flow in and out freely, so that the domestic interest rate will be determined by the interest rate in the world. To ensure such an environment, the economy must float its exchange rate. As a consequence, the shock to foreign inflation does not impact other macroeconomic variables, except for the exchange rate. For the other five stochastic shocks, while the Divisa quantity aggregate tracks almost perfectly the movement of the true monetary aggregate, the monetary base fails in the home-productivity shock and over-reacts with respect to the monetary policy shock. More problematically, the simple-sum monetary aggregate behaves differently from that of the true monetary aggregator function in three out of the five shocks. For the other two shocks, its responses are not close to the movement of the true monetary aggregator function.

To check for the robustness of our results, we vary the parameters ω, η and α. Figure 3 and Figure 4 show the responses under the variation of the parameter ω. A higher value of the elasticity of substitution between cash and deposit, a smaller ω, implies an economy with less cash in circulation and vice versa. See Table 1 for a numerical image. The behaviors of the Divisia quantity aggregate and the monetary base do not change when we vary ω; however, simple-sum does. With a smaller value of ω, the simple-sum monetary aggregate is less likely to misbehave. But with larger ω, the misbehavior becomes increasingly serious. The intuition behind this result is that as cash and deposits becomes far away from perfect substitutes, people are willing to hold less cash. The simple-sum measure, which implicitly assumes perfect substitution among components and assigns the same weight to different monetary assets, becomes increasingly distorted.

In a similar manner, we vary the elasticity of substitution between domestic and foreign goods in Home's consumption. See Figures 5 and 6. We later vary the openness (or the inverse degree of home-bias in consumption). See Figures 7 and 8 for sensitivity analysis. Our conclusion is the same in every case. The Divisia index tracks the movements of the true monetary aggregator function very closely, the monetary base is good in some cases but fails in some others, and the simple-sum monetary aggregate often behaves very differently from the theoretical benchmark. Regarding the sometimes favorable results with the monetary base, it should be observed that in our model, the central bank does not pay interest of reserves. The Federal Reserve's current policy of paying interest on reserves undermines the usefulness of the supply-side monetary base as a measure of outside money without affecting the relative usefulness of the simple sum or Divisia monetary aggregates.

We further look at the volatility of macroeconomic variables under different values of α from 0 to 1. As α goes from 0 to 1, the economy becomes more open (it is a closed economy when α=0), and consumption becomes less home-biased. We find that nominal interest rates and domestic inflation fluctuate more as the economy is more open, while the growth rate of the exchange rate and true money become more stable. See Figure 9. Once again, the Divisa index follows the correct trend of the theoretical money aggregate, while simple-sum behaves totally differently. This pattern does not change under different financial structures with different values of ω.

4. Conclusion

Since the Divisia monetary aggregate and monetary aggregation theory became available in the 1980s, a number of theoretical and empirical studies have repeatedly shown that the Divisa measure is strictly preferable to its official simple-sum counterpart. However, the availability of the theoretically inadmissible simple-sum aggregates has continued. This paper revisits the issue of money measurement in the context of a small open economy using a recent highly micro founded DSGE model. Our results are consistent with other recent results such as Barnett and Chauvet (2010), Belongia and Ireland (2015), and Keating and Smith (2019). This paper is among the first to introduce the Divisia measure in a small open economy. It introduces a banking sector and the Divisia measure of money in a New Keynesian framework for a small open economy. This paper is the first to analyze the effect of openness (home bias in consumption) to the volatility of macroeconomic variables in a such an economy and the relevancy of monetary aggregates under those circumstances.

In the future work, we plan to continue our analysis of Divisia monetary aggregates in a small open economy. We want to investigate whether money is more informative than interest rates and exchange rates in explaining/predicting output and inflation and to determine whether the relation of the quantity of money to macroeconomic variables remains stable. Furthermore, to find out the optimal monetary policy rule, we plan to evaluate the representative household's welfare under different targets of monetary policy, such as an interest rate rule, a rule for a fixed growth rate for the monetary base, a fixed growth rate for simple sum money, a fixed growth rate for the Divisia monetary aggregate, and a rule for a fixed exchange rate.

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A Appendix

A.1 System of Equations with Real Effective Variables

In this appendix, we gather together relevant equations for convenient reference.

First order conditions for households:

$$\frac{h_t}{c_t} = \lambda_t^3 .$$

Euler equation:

$$\lambda_t^3 = \beta \,\mathrm{E}_t \left(\frac{\lambda_{t+1}^3 (1 + r_{t+1})}{1 + \pi_{H,t+1}} \right) \frac{q(S_t)}{q(S_{t+1}) z_t} .$$

$$\lambda_t^2 - \lambda_t^3 = \lambda_t^1 \left[\nu^{\frac{1}{\omega}} ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}} \nu^{\frac{1}{\omega}} ca_t^{-\frac{1}{\omega}} .$$

$$\lambda_t^2 - (1 + r_t^D) \lambda_t^3 = \lambda_t^1 \left[\nu^{\frac{1}{\omega}} ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}} (1-\nu)^{\frac{1}{\omega}} d_t^{-\frac{1}{\omega}} .$$

$$\lambda_t^2 = \lambda_t^3 (1 + r_t^L) .$$

Interest rate:

$$r_t^L = r_t .$$

Labor supply:

$$\lambda_t^3 w_t = \psi_N h_t N_t^{\xi} .$$

Money demand:

$$\lambda_t^1 m_t = \psi_M h_t .$$

Terms of trade:

$$q(S_t) = \left[(1-\gamma) + \gamma S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} .$$

Nominal exchange rate:

$$\frac{S_t}{S_{t-1}} = \left(\frac{1 + \pi_t^f}{1 + \pi_{H,t}} \right) (1 + g_t^{NE}) .$$

Relative price ratio in Home:

$$\frac{q(S_t)}{q(S_{t-1})} = \frac{1 + \pi_t}{1 + \pi_{H,t}} .$$

Real exchange rate:

$$RE_t = \frac{S_t}{q(S_t)}.$$

Interest rate parity:

$$\frac{S_t}{q(S_t)} = \kappa h_t \frac{c_t}{c_t^f} z_t^f .$$

Production function:

$$y_t = z_t N_t .$$

First order condition for optimal price setting:

$$(1-\varepsilon) + \varepsilon \frac{w_t q(S_t)}{z_t} - \phi \left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1 \right) \left(\frac{1+\pi_{H,t}}{1+\pi_H} \right).$$

$$+\beta\, \phi\, \mathrm{E}_t \frac{\lambda_{t+1}^3 y_{t+1}}{\lambda_t^3 y_t} \left(\frac{1+\pi_{H,t+1}}{1+\pi_H} - 1 \right) \left(\frac{1+\pi_{H,t+1}}{1+\pi_H} \right) = 0 .$$

Loan and deposits:

$$l_t = (1-\tau) d_t .$$

First order condition for profit maximization of commercial banks:

$$r_t^D = r_t^L (1-\tau) - \upsilon .$$

Aggregate resource constraint:

$$y_t = (1-\alpha) q(S_t)^\eta c_t + \alpha (S_t)^\eta \frac{c_t^f}{z_t^f} + \frac{\phi}{2} \left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1 \right)^2 y_t + \upsilon d_t q(S_t) .$$

Theoretical functional form (aggregator function) of true money:

$$m_t = \left[\nu^{\frac{1}{\omega}} ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} .$$

Growth rate of true money:

$$1 + g_t^M = m_t / m_{t-1} .$$

Simple-sum measure of money:

$$ca_t + d_t = sm_t .$$

Growth rate of the simple-sum measure:

$$1 + g_t^S = sm_t / sm_{t-1} .$$

Monetary base:

$$[mb_t \equiv]\ a_t = ca_t + \tau d_t .$$

Growth rate of monetary base:

$$1+g_t^B = mb_t / mb_{t-1} = a_t / a_{t-1}.$$

User cost price of cash:

$$u_t^{Ca} = \frac{r_t - 0}{1+r_t} = \frac{r_t}{1+r_t}.$$

User cost price of deposits:

$$u_t^D = \frac{r_t - r_t^D}{1+r_t}.$$

Expenditure share of cash:

$$s_t^{Ca} = \frac{u_t^{Ca} ca_t}{u_t^{Ca} ca_t + u_t^D d_t}.$$

Expenditure share of deposits:

$$s_t^D = \frac{u_t^D d_t}{u_t^{Ca} ca_t + u_t^D d_t}.$$

Growth rate of the Divisia monetary quantity index:

$$1+g_t^{Dv} = \left(\frac{ca_t}{ca_{t-1}}\right)^{\frac{s_t^{Ca}+s_{t-1}^{Ca}}{2}} \left(\frac{d_t}{d_{t-1}}\right)^{\frac{s_t^D+s_{t-1}^D}{2}}.$$

Monetary policy rule (as a version of the Taylor rule):

$$r_t = (1-\rho_r)r + \rho_r r_{t-1} + (1-\rho_r)\rho_\pi(\pi_{H,t} - \pi_H) + e_{r,t}.$$

Preference shock in Home:

$$\ln h_t = \rho_h \ln h_{t-1} + e_{h,t}.$$

Technology (productivity) shock in Home:

$$\ln z_t = \ln z + e_{z,t}.$$

Technology (productivity) shock in Foreign:

$$\ln z_t^f = (1-\rho_z^f)\ln z_t^f + \rho_z^f \ln z_{t-1}^f + e_{z,t}^f.$$

Shock to consumption in Foreign:

$$\ln c_t^f = \rho_c^f \ln c_{t-1}^f + e_{c,t}^f.$$

Shock to inflation in Foreign:

$$\pi_t^f = (1-\rho_p^f)\pi_t^f + \rho_p^f \pi_{t-1}^f + e_{p,t}^f.$$

A.2 Figures

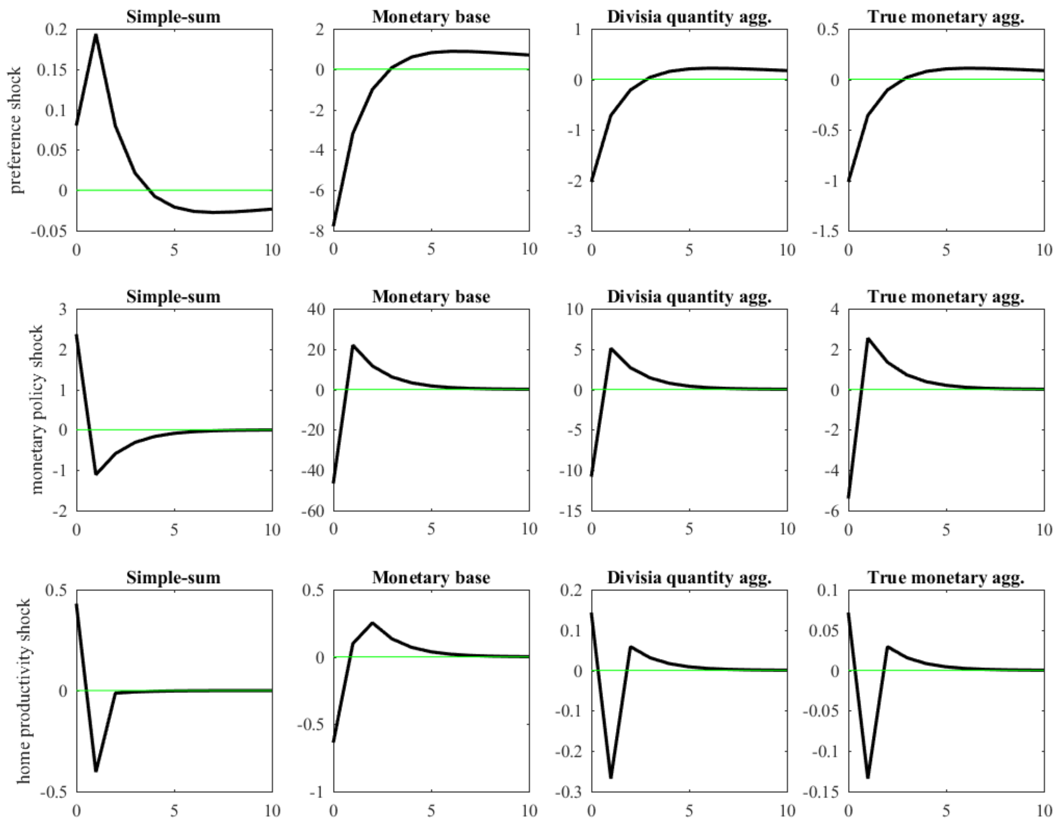

Figure 1. Impulse Responses of the Growth Rate of Different Money Measures with Respect to Domestic Shocks, with Benchmark ω=2, η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

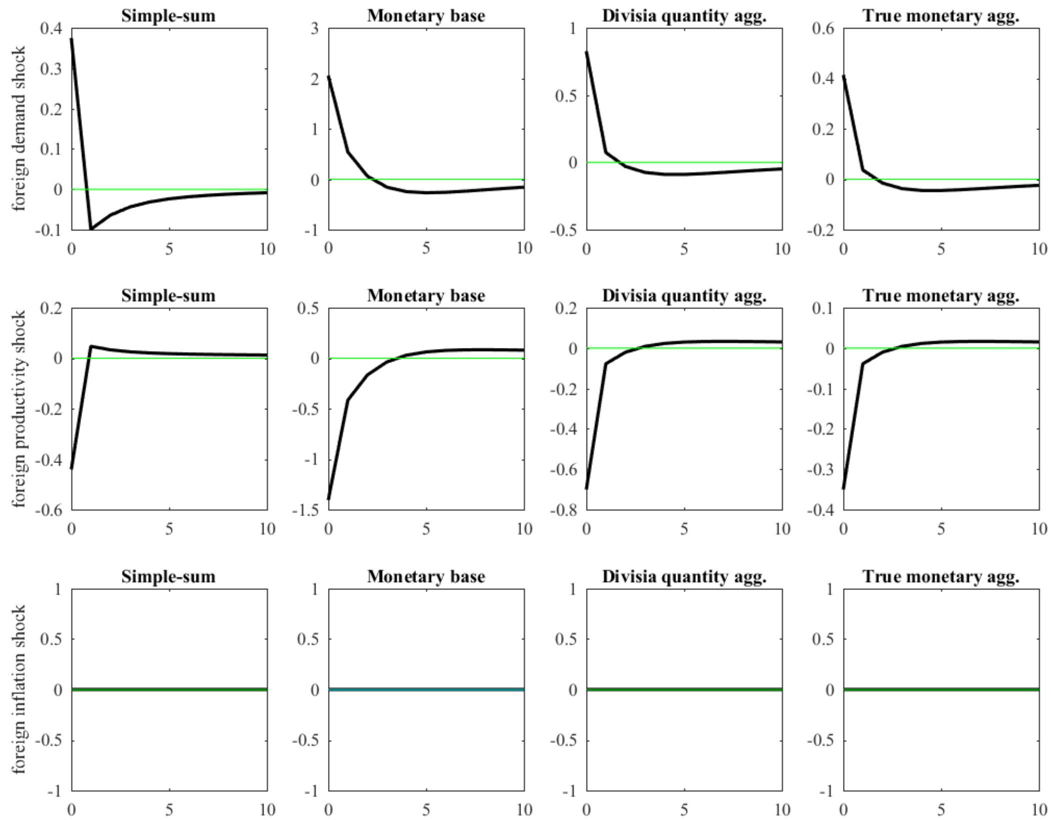

Figure 2. Impulse Responses of Growth Rate of Different Money Measures with Respect to Foreign Shocks, with Benchmark ω=2, η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

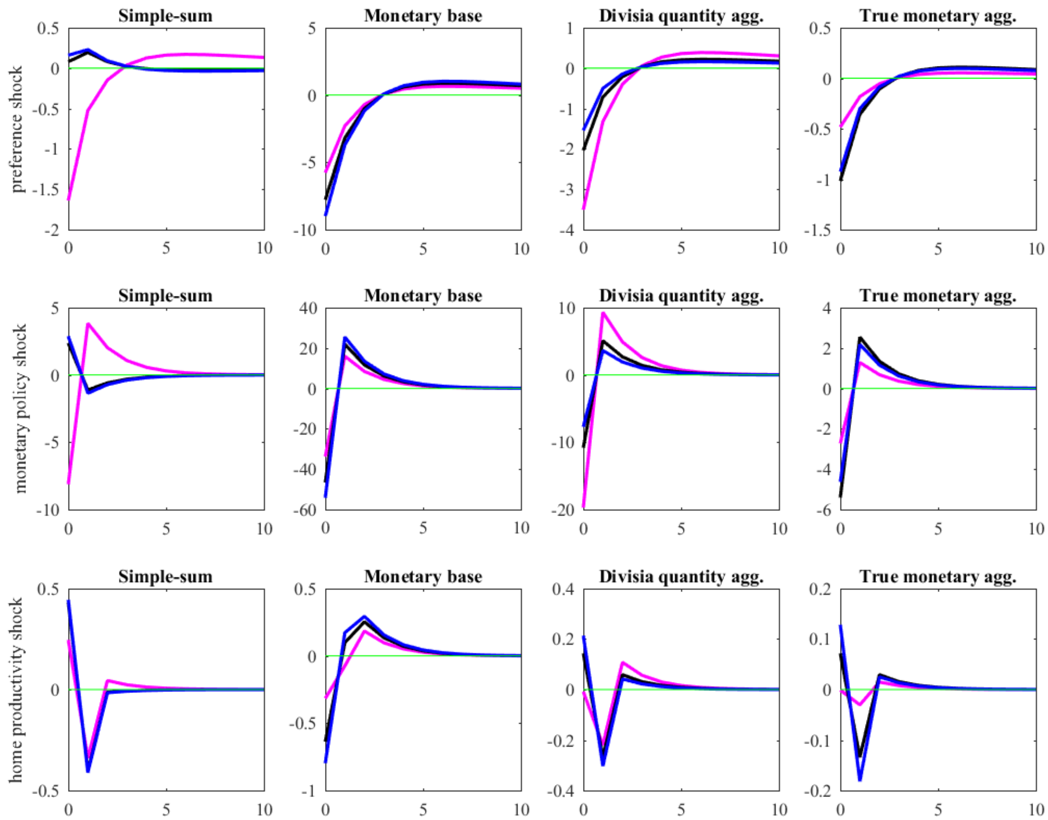

Figure 3. Impulse Responses of the Growth Rate of Different Money Measures with Respect To Domestic Shocks under Variation of the Parameter ω, pink for ω =1.16, black for ω =2, blue for ω =3, and benchmark η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

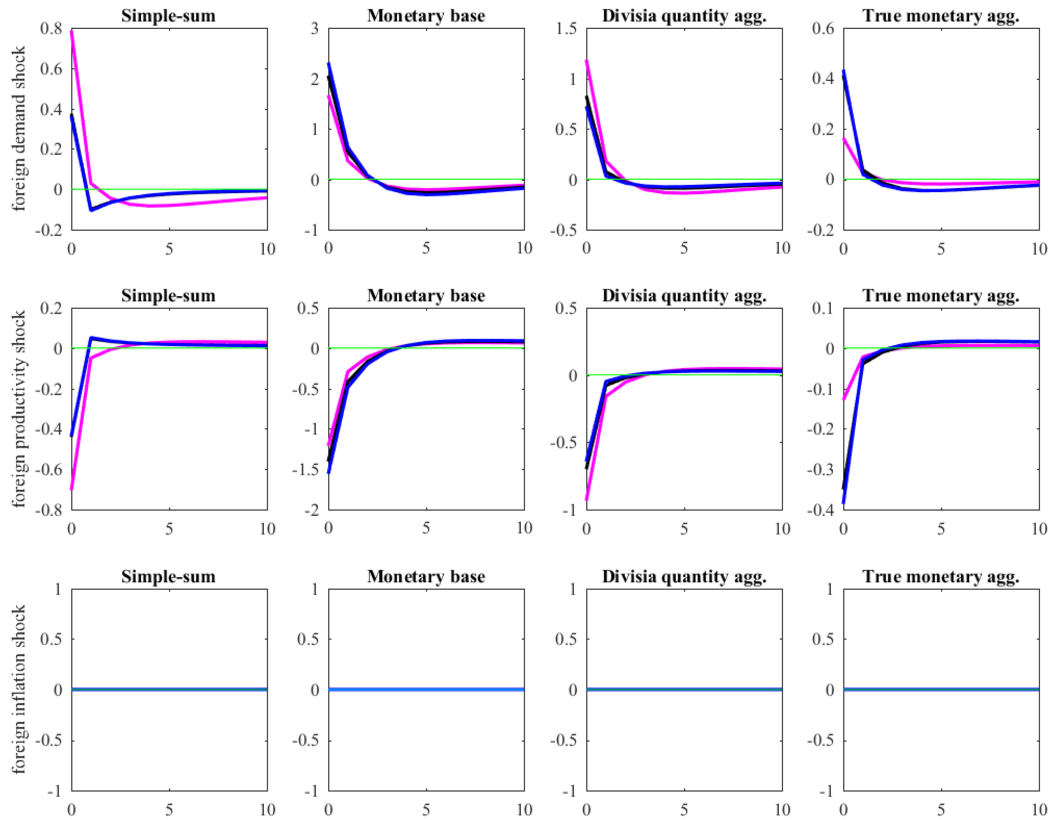

Figure 4. Impulse Responses of Growth Rate of Different Money Measures with Respect to Foreign Shocks under Variation of Parameter ω, with pink for ω =1.16, black for ω =2, and blue for ω =3, with benchmark η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

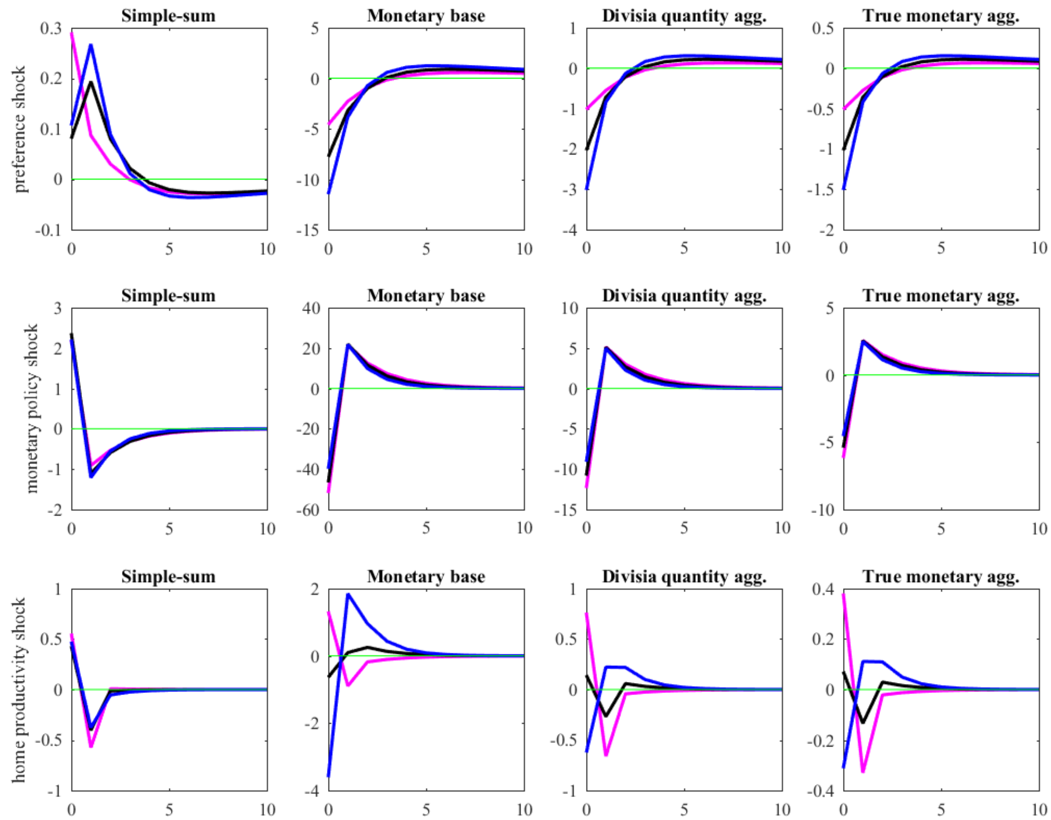

Figure 5. Impulse Responses of Growth Rate of Different Money Measures with Respect to Domestic Shocks under Variation of Parameter η, with pink for η =1.1, black for η =2, and blue for η =5, with benchmark ω=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

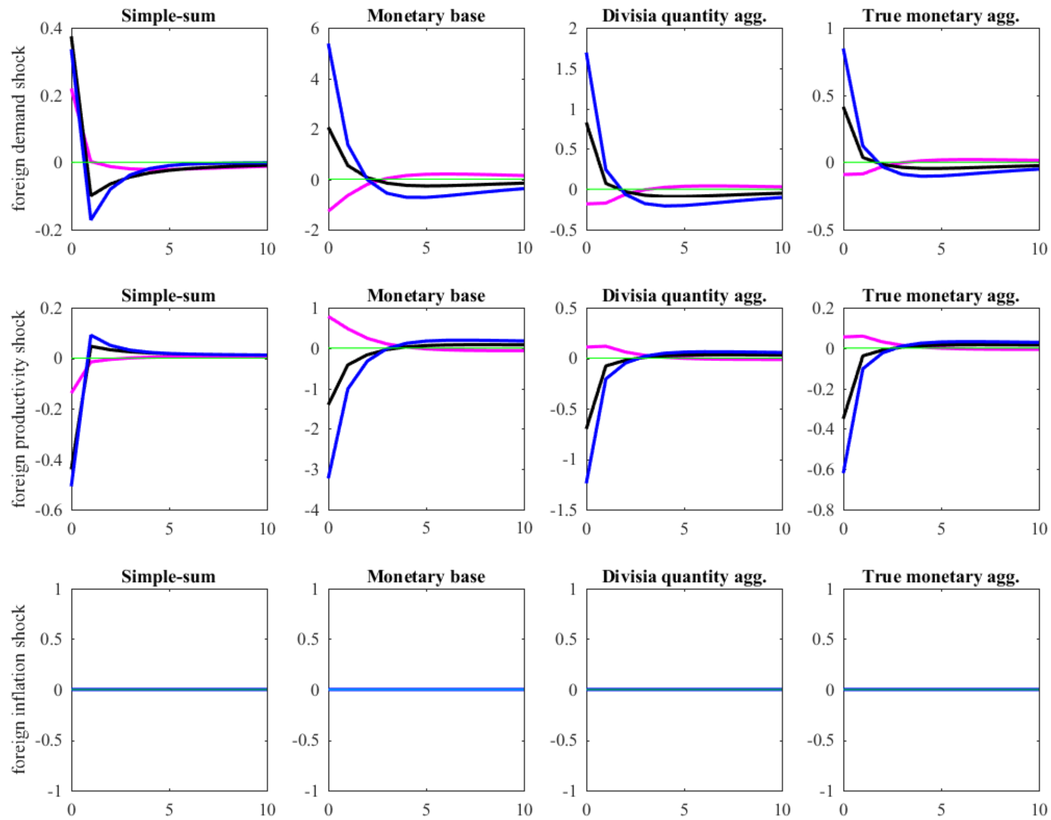

Figure 6. Impulse Responses of Growth Rate of Different Money Measures with Respect to Foreign Shocks under Variation of the Parameter η, with pink for η =1.1, black for η =2, blue for η =5, with benchmark ω=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

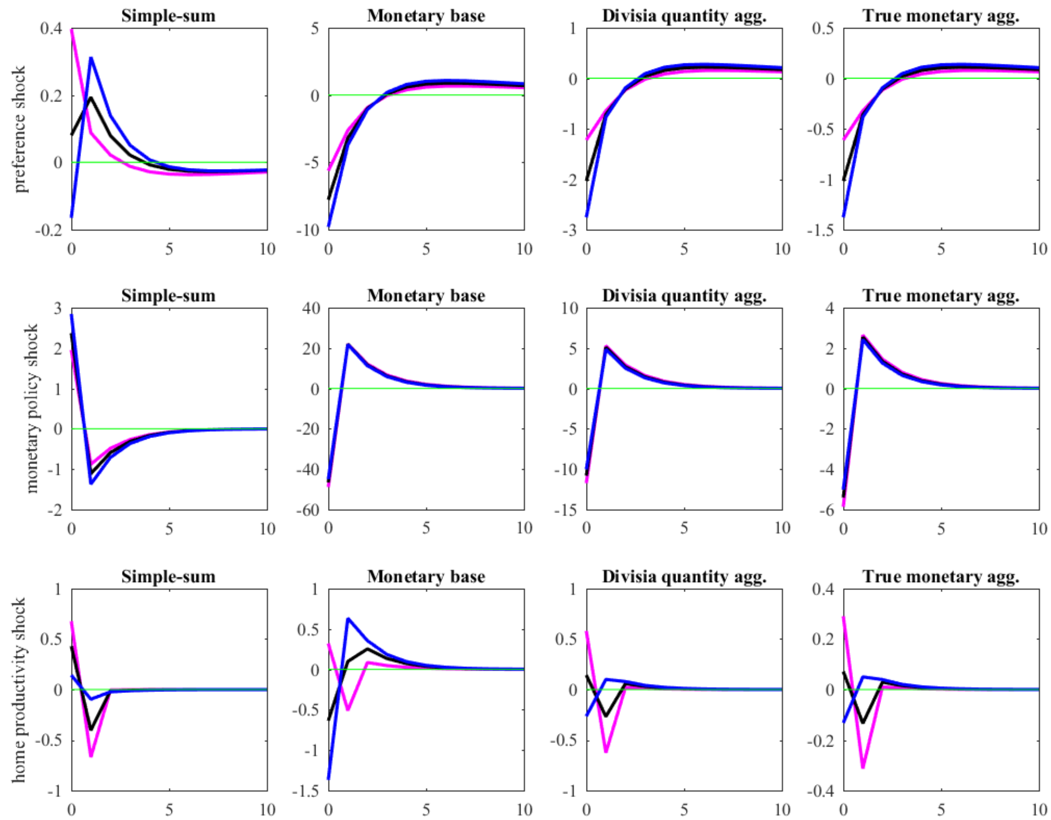

Figure 7. Impulse Responses of Growth Rate of Different Money Measures with Respect to Domestic Shocks under Variation of Parameter α, with pink for α =0.1, black for α =0.4, blue for α =0.8, with benchmark ω=2, η=2. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

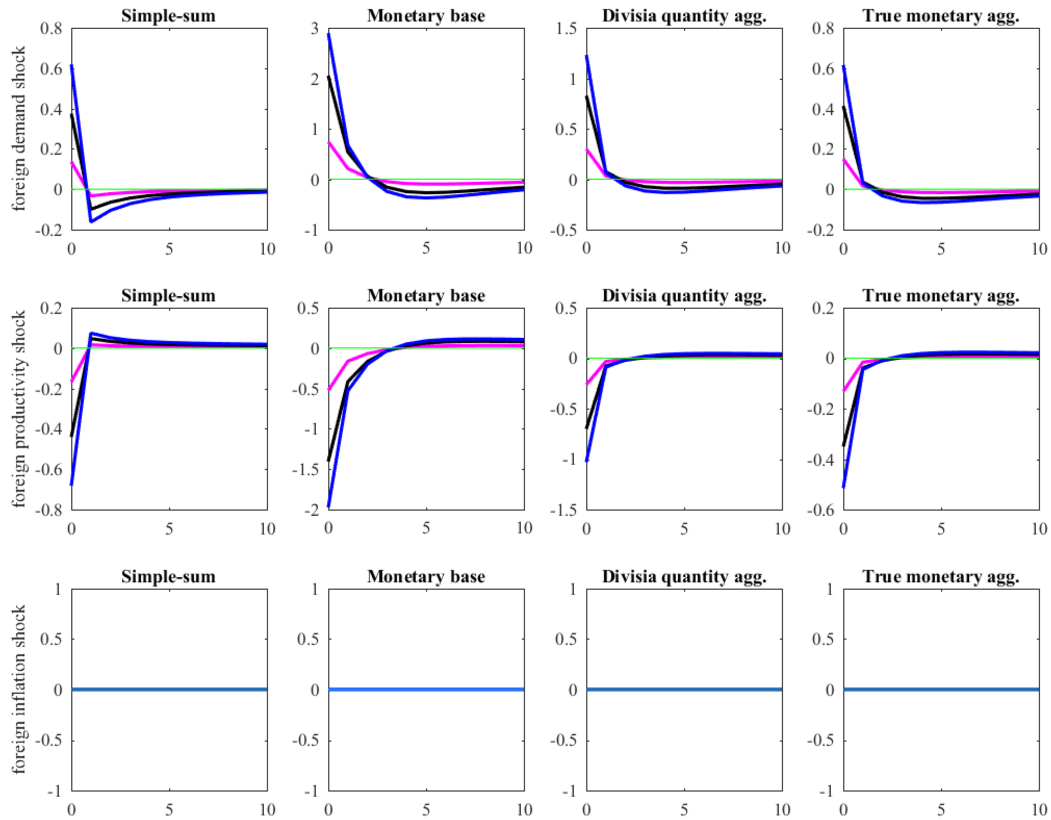

Figure 8. Impulse Responses of Growth Rate of Different Money Measures w.r.t Foreign Shocks under Variation of Parameter α, pink for α =0.1, black for α =0.4, blue for α =0.8, benchmark ω=2, η=2. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

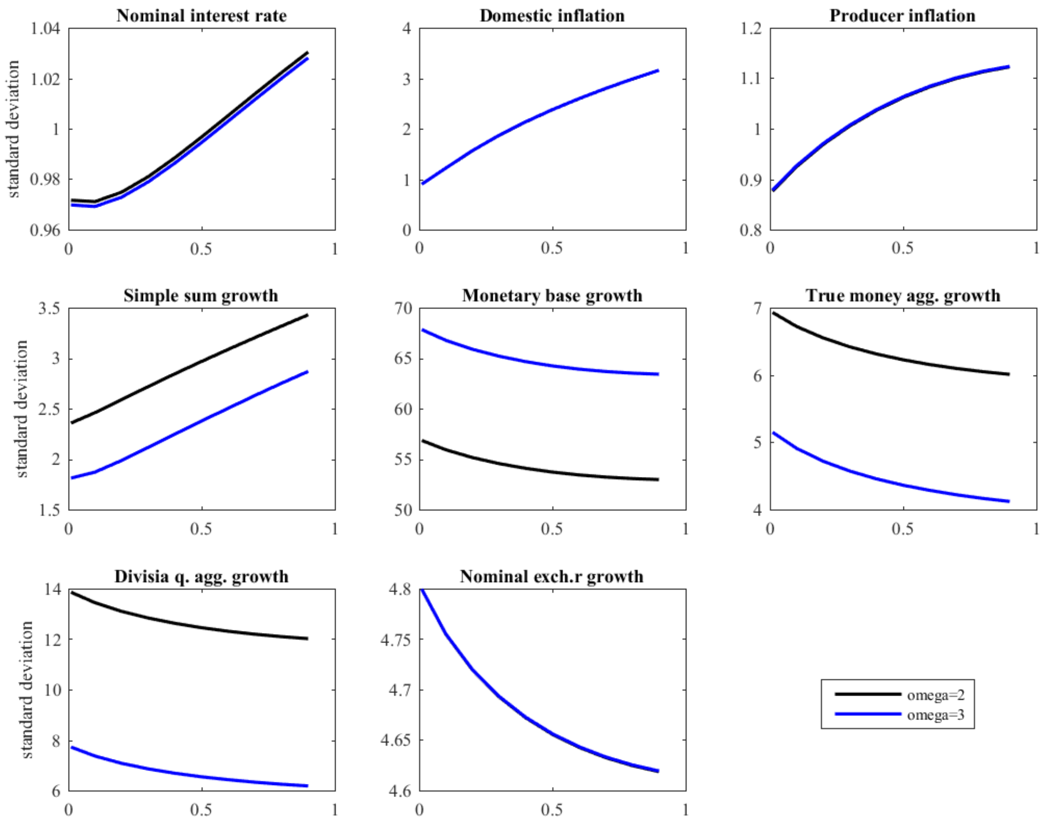

Figure 9. Volatility of Macroeconomic Variables under Variation of Parameter α. Each panel shows the standard deviation in percentage-point under different values of α from 0 to 1.