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Abstract

This paper provides a new analysis of the effects of monetary policy on the term structure of interest rates using a tractable dynamic asset pricing model which incorporates a responsive monetary policy rule. The model shows how a change in the policy rule simultaneously affects inflation, interest rates, volatilities, co-movements between long and short rates, and is able to account for some empirical regularities of the term structure across different policy regimes in the United States.

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1 Introduction

It has been widely noted that there is a significant difference in the way monetary policy was conducted in the pre- and post-1979 periods in the United States [e.g. Clarida et al. (1998) and McCallum and Nelson (1999) among others]. Those studies also provide empirical evidence that the Federal Reserve was "accommodative" in the first period and has adopted a proactive stance toward controlling inflation since late 1979 and the early 1980s. While people have examined the implications of different monetary policies for macroeconomic performance [see papers in Taylor (1999) eds.], the objective of the present paper is to understand the impact of such a major shift in the policy regime on the term structure of interest rates.

In fact, many researchers have noticed changes in the behavior of the term structure and the influence of monetary policy. For example, Mankiw and Miron (1986) examined the expectation theory of the term structure using data at the short end of the maturity spectrum over different monetary policy regimes. Hamilton (1988) estimated an econometric regime-switching model for the term structure and found substantive evidence of a structural change in interest rates during the monetary experiment of October 1979. Cohen and Wenninger (1994) and Mehra (1996) both found that long rates seem to have become more sensitive to movements in monetary policy instruments in recent years. Rudebush (1995) estimated a daily model of Federal Reserve interest rate targeting behavior and explored the implications for the yield curve. Hsu and Kugler (1997) argued that the varying predictive power of the term spread for the future short-term rate can be attributed to the changes in the policy reaction function adopted by the U.S. monetary authority since the 1980s. Watson (1999) examined the different variability in long term interest rates during 1965-1978 and 1985-1998. Fleming and Remolona (1999) analyzed high-frequency responses of U.S. Treasury yields across the maturity spectrum to macroeconomic announcements. They found that the maturity pattern of announcement effects has changed substantially since the late 1970s/early 1980s. Piazzesi (2001) developed a factor model of the term structure which incorporates macroeconomic jump effects due to monetary policy actions in response to inflation pressure.

The present paper provides a new analysis of the relationship between monetary policy and the term structure of interest rates, using a tractable dynamic asset pricing model for an economy where monetary policy is characterized by a responsive policy rule that reflects actual central bank behavior. The rule calls for adjustments in money supply depending on the gap between the current inflation rate and a target rate set by the central bank. Different policy regimes are identified with different degrees of responsiveness of the policy instrument to the inflation pressure. It is a version of the policy rule that emerges in both positive and normative analysis of central bank behavior in recent literature.[1]

[1] See Clarida, Gali and Gertler (1999) for a review of that literature.

Closed form solution of the term structure is then obtained, where nominal interest rates are given by affine functions of exogenous state variables with the coefficients of the affine functions depending on the policy parameters. Comparative statics easily show how a change in the monetary policy, such as the one occurred in the late 1970s/early 1980s, simultaneously affects inflation, interest rates, volatilities and co-movements between long and short rates.

This paper draws from two strands of literatures. One is the recent research effort to incorporate a responsive monetary policy rule in a dynamic general equilibrium model of monetary economy [e.g. Leeper (1991), Rotemberg and Woodford (1999) among others]. The focus of those studies is usually on the evaluation of alternative policy rules for macroeconomic stability. The other is continuous time models of asset pricing in a general equilibrium setting. The prime general equilibrium model of the term structure is the Cox-Ingersoll-Ross (CIR) model (1985). Other general equilibrium models of the term structure include Vasicek (1977), Longstaff and Schwartz (1992) among others. Sun (1992) considered a CIR model in a monetary economy, assuming an exogenous inflation process that is correlated with consumption growth. Bakshi and Chen (1996) obtained a closed form solution for the term structure under an exogenous money supply process, where inflation is determined endogenously together with nominal interest rates. The present paper extends the approach of Bakshi and Chen (1996) by considering an endogenous money supply process under a responsive monetary policy rule.

In what follows, section 2 presents some descriptive statistics of interest rates in the United States, section 3 lays out the model, section 4 discusses the effects of monetary policy on the term structure, section 5 concludes.

2 Descriptive Statistics of the Term Structure

To further motivate our analysis, let's first look at some descriptive statistics of the term structure in the United States. The data include the yields on the Federal government pure discount bonds during the period of 1960 - 1995 at monthly frequency, constructed from the market prices of government coupon bonds.[2] The yields from 1960:01 - 1991:02 are in fact taken from Kwon and McCulloch (1993), and the yields from 1991:03 - 1995:12 are computed by Robert Bliss (1997) using the same McCulloch/Kwon procedure. The data include yields on eighteen nominal bonds of different maturities ranging from 1 month to 10 years. Bond yields are measured as continuously compounded annualized returns on these risk-free zero-coupon bonds.

[2] The data set is made available to me by Charles Evans at the Federal Reserve Bank of Chicago. It is the same data set used in Evans and Marshall (1997).

To see changes in the behavior of interest rates, I break the sample into two periods: 1960:01 - 1979:09 and 1979:10 - 1995:12.[3] I use the 1-month rate to approximate the instantaneous short-term interest rate. Table 1 reports the results from regressing long-term rates on the short rate. We can see that the regression coefficient on the short rate are much higher in the post-October 1979 period. The differences are not only statistically significant, but also quantitatively important. This seems to confirm the finding from other empirical studies that long-term rates have become more responsive to movements in the short-term rate since late 1979/early 1980s. Such a change in the relationship between the short-term rate and long-term rates can be more clearly seen in Figure 1 where the regression coefficients are plotted against time to maturity for different periods. While in general the responsiveness of long rates to changes in the short-term rate decline as maturity increases, the "response curve" shifts upward significantly after 1979. Moreover, this empirical regularity seems to be invariant with respect to choices of the break point. It can be seen from Figure 1 that in all three sub-sample periods following October 1979, the responsiveness of long-term rates to movements in the short rate are significantly higher compared with the pre-October 1979 period.

In fact, the structural change in the yield curve is not only reflected in the correlations between the short-term rate and long-term rates. Table 2 summarizes the average level of the yield curve and volatilities of the interest rates in the two periods. We can see that in the post-1979 period, the yield curve has a higher level on average than in the earlier period. This result holds regardless of whether one looks at the whole post-October 1979 period or the 1983:01-1995:12 period or the more recent 1985:01-1995:12 period (see Figure 3 for plots of the average yield curve in these different periods). As to the volatilities of the interest rates, the period between 1979:10 and 1995:12 seems to be characterized by much more volatile nominal interest rates than the earlier period (1960:01-1979:09). However, once we remove the period of 1979:10-1982:12 during which the Fed was thought to be in a transition between policy regimes and was experimenting with different policy instruments and operating procedures, the standard deviations of the interest rates become much lower. And if we look at the more recent period (1985-1995), the interest rates are in fact less volatile than those in the pre-October 1979 period. Figure 5 includes the plots of standard deviations of the interest rates in these different periods.

[3] One obvious reason to choose October 1979 as the break point is that Paul Volcker took over as Fed chairman in October 1979 and started an aggressive effort to reign in inflation in the U.S. Empirical studies of the term structure also confirm that there was a structural break in the yield curve around 1979, e.g. Wu (2001)

3 The Model

In this section, I use a tractable dynamic asset pricing model to analyze the underlying mechanisms by which shifts in the monetary policy can affect the term structure of interest rates. Two of the important components that determine nominal interest rates are the real interest rate and inflation. In the present paper, I focus on the impact of the second factor on the term structure of interest rates. I consider a simple representative-agent economy with exogenous endowment and flexible prices. Money is made completely neutral in the economy so that monetary policy affects nominal interest rates only through its impact on inflation. The caveats of these simplifying assumptions will be discussed later. Nevertheless, these simplifications lead us to a closed form solution of the term structure, with which the impact of a shift in the policy regime can easily be analyzed in the next section.

3.1 The Consumer's Problem

It is assumed that a representative consumer maximizes the following utility function subject to his intertemporal budget constraint:

$$Max\ E_0 \sum_{t=0,\triangle t,2\triangle t,\cdots}^{\infty} e^{-\rho t} u\left(c(t)\right) \triangle t \tag{1}$$

where $u(c) = log(c)$.[4] $\triangle t$ is the length of the time interval during which the consumer makes decisions about consumption, money and asset portfolio holdings. In the following discussion, I will consider the limiting case where $\triangle t \to 0$. $c(t)$ is the consumption flow between $[t, t+\triangle t]$. Money will also be demanded by the agent because consumption transactions are costly and increasing real balance holdings per unit of consumption decrease these transaction costs, which are represented by $f(\frac{m(t)}{c(t)})$, a function of the ratio between the real money balance $m(t)$ and consumption $c(t)$. It is assumed that $f(\cdot)$ is a continuously differentiable decreasing function which reaches its minimum level at some constant k. This implies that there is a satiation level of real cash balance per unit of consumption.[5] The consumer has an exogenous flow of endowment $y(t)$ during $[t, t+\triangle t]$. In equilibrium we have $c(t) = y(t) - f(\frac{m(t)}{c(t)})$.

[4] The results are not affected if the logarithm utility function is replaced with a more general CRRA utility function.

[5] Wolman (1997) estimated a "transactions technology" based money demand function and found evidence of the presence of a satiation level of cash balances per unit of consumption.

The budget constraint for the consumer can be written as:

$$
\begin{aligned}
&P(t)c(t)\triangle t + P(t)f\left(\frac{m(t)}{c(t)}\right)\triangle t + M(t) + e^{-R(t)\triangle t}B(t) + e^{-r(t)\triangle t}P(t)b(t) + \\
&\quad \sum_{i=1}^{N} Q_i(t)S_i(t) \leq P(t)y(t)\triangle t + e^{R^m(t-\triangle t)\triangle t}M(t-\triangle t) + B(t-\triangle t) + \\
&\quad P(t)b(t-\triangle t) + \sum_{i=1}^{N} Q_i(t)S_i(t-\triangle t) + \triangle G(t) \qquad (2)
\end{aligned}
$$

In the budget constraint, $P(t)$ is the price level at time t, $R(t)$ is the one-period nominal interest rate, $r(t)$ is the one-period real rate. $M(t)$ is nominal cash balances the consumer chooses to hold at time t and carries over to time $t+\triangle t$ and $m(t)$ is the real cash balances $\frac{M(t)}{P(t)}$. $B(t)$ is the number of the one-period nominal bond the consumer chooses at time t and holds to time $t+\triangle t$. Similarly, $b(t)$ is the one-period real bond. Other assets in the economy are N nominal long-term, zero-coupon, risk-less bonds represented by $S_i(t)$, for $i = 1$, $\cdots$, N. The prices of these long-term bonds are $Q_i(t)$. It is assumed that there are a net zero supply of all these assets. $\triangle G(t)$ are government transfers during $[t, t+\triangle t]$.

One of simplifying assumptions made in this paper is that the monetary authority pays interest on money balances. In particular, when the consumer chooses to hold $M(t-\triangle t)$ at time $t-\triangle t$ and carries it over to time t, he will get extra cash from the monetary authority at a rate $R^m(t-\triangle t)$, which is marginally below $R(t-\triangle)$, the rate on the one-period nominal bond. Hence I abstract from the negative dependence of money demand on positive nominal interest rates. This assumption, borrowed from King and Wolman (1999), results in a simple money demand function with constant velocity when $R^M(t)$ approaches $R(t)$ from below, which in turn leads to closed form solutions for the term structure of nominal interest rates as well as inflation. Empirically, transaction balances have become increasingly interest-rate bearing, which provides some justifications for this simplifying assumption. Moreover, as argued by King and Wolman (1999), the constant velocity money demand function can be thought of as the limiting case that applies when money is interest-bearing, when there is a satiation level of cash balances per unit of consumption, and the interest rate on money is close to the market rate. As we will see below, under this assumption money is completely neutral in the economy, I can therefore focus on the effects of different monetary policies on inflation and explore the implications for the term structure of nominal interest rates.

Finally, besides paying interest $R^M(t-\triangle t)$ on $M(t-\triangle t)$ at time t, new money balances are transfered to the consumer in a lump sum fashion. Hence $G(t)$ satisfies:

$$\triangle G(t) = M(t) - e^{R^M(t-\triangle t)\triangle t}M(t-\triangle t) \qquad (3)$$

Given (1) and (2), we can write the first order conditions for the consumer's problem as follows:

$$e^{-\rho t}u^{'}(c_t) = \lambda_t\left[1 - \frac{m_t}{c_t^2}f^{'}\left(\frac{m_t}{c_t}\right)\right]\cdot P_t \tag{4}$$

$$e^{-R_t^m\triangle t}\left(1 + c_t^{-1}f^{'}\left(\frac{m_t}{c_t}\right)\triangle t\right) = E_t\left(\frac{\lambda_{t+\triangle t}}{\lambda_t}\right) \tag{5}$$

$$e^{-R_t\triangle t} = E_t\left(\frac{\lambda_{t+\triangle t}}{\lambda_t}\right) \tag{6}$$

$$Q_{i,t} = E_t\left(\frac{\lambda_{t+\triangle t}}{\lambda_t}Q_{i,t+\triangle t}\right) \tag{7}$$

$$e^{-r_t\triangle t} = E_t\left(\frac{P_{t+\triangle t}\lambda_{t+\triangle t}}{P_t\lambda_t}\right) \tag{8}$$

From (5) and (6), we get an equation relating real cash balance to consumption and nominal interest rates:

$$e^{-R_t^m\triangle t}c_t^{-1}f^{'}(m_t/c_t)\triangle t = e^{-R_t\triangle t} - e^{-R_t^m\triangle t}$$

Note that it is assumed that $f^{'}(\cdot) \leq 0$ and $R_t^m \leq R_t$. The above equation can be simplified by letting $R_t^m \to R_t$ from below, i.e. we let the real cost of holding money goes to zero, then it follows that $f^{'}(\cdot)$ must be zero, which implies that:

$$M_t = kP_tc_t \tag{9}$$

where k is the constant such that $f^{'}(k) = 0$, which represents the satiation level of real cash balances per unit of consumption. We can think of the above money demand function with constant velocity as an approximation when the interest rate on the money balance is close to the market rate. For tractability, we will hence only consider the case where $R^m = R_-$ in the following discussions. Also note that when $R^m = R_-$, the transaction cost $f\left(\frac{m}{c}\right)$ is a constant because $\frac{m}{c}$ is constant. Without loss of generality, I simply assume $c(t) = y(t)$ in equilibrium.

3.2 Monetary Policy

Recently a great effort has been devoted to the study of monetary policy rules under which the central bank adjusts its policy instrument in response to developments in the economy. In this literature, policy makers seek to implement a particular equilibrium relationship between a policy instrument and some other endogenous variables by adopting an appropriate money supply process. Hence money supply is endogenous in the sense that the growth rate of money must respond to current and past exogenous monetary policy shocks, as well as other private economy shocks, in a way that is consistent with the policy rule.[6]

Following this literature, I assume that the central bank pursues a responsive monetary policy using money stock $M(t)$ as its instrument. Researchers have often used the short-term interest rate [Taylor (1993)] or money stock [McCallum (1988)] as the policy instrument in the literature. In either case, the policy rule calls for monetary tightening in the presence of inflation pressures.[7] In the same spirit, I postulate the following policy rule for the economy under consideration:[8]

$$\frac{\triangle M^s(t)}{M^s(t)} = \frac{\triangle P(t)}{P(t)} - \alpha \left(\frac{\triangle P(t)}{P(t)} - \pi^* \triangle t \right) + \frac{\triangle y(t)}{y(t)} + \frac{\triangle \xi(t)}{\xi(t)} \tag{10}$$

In the above equation, $\triangle X(t) \equiv X(t + \triangle t) - X(t)$, and $\alpha > \alpha^* > 0$ (α^* is some positive constant to be specified in the following), π^* is a target inflation rate set by the central bank. The central bank will seek to reduce the money growth rate if inflation exceeds this target level. Under this specification, the growth rate of money in the economy consists of two components. One is a systematic component which can be represented by a function of endogenous variables (the reaction function). Recent vector autoregression (VAR)-based literature on monetary policy have provided empirical evidence that most of the observed movements in the policy instrument can be explained by macroeconomic conditions [e.g. Bernanke et al. (1997), Christiano et al. (1998b) among others]. The other component is an exogenous policy shock $\frac{\triangle \xi(t)}{\xi(t)}$ whose property will be specified below. Possible sources of the policy shock include measurement errors in inflation and some discretionary actions by the central bank.

Note that since money can not affect output in this endownment economy, I have assumed that the central bank fully accommodates any fluctuations in output by fixing the coefficient on the growth rate of output at 1 in the policy rule. In the light of the money demand function (9), this means that the central bank tries to keep inflation constant when output fluctuates (recall that $c(t) = y(t)$ in equilibrium).

[6] See Christiano et al. (1998a) for a discussion of endogenous variable policy rules in general equilibrium context.

[7] See McCallum (1997) for a discussion of the issues in the design of monetary policy rules, including the choice of policy instrument.

[8] Using a money supply rule also allows me to avoid the issue of the zero lower bound constraint on nominal interest rates.

3.3 Exogenous State Variables

I assume that monetary policy shocks and supply shocks are the only sources of uncertainty in the economy, where $y(t)$ and $\xi(t)$ are driven by two independent state variables, $X(t)$ and $Z(t)$ respectively, in the following way when $\triangle t \to 0$:

$$\frac{dy(t)}{y(t)} = (hX(t) - \rho)dt + \epsilon\sqrt{X(t)}dW_1(t) \tag{11}$$

$$\frac{d\xi(t)}{\xi(t)} = (Z(t) - \bar{Z})dt + \omega\sqrt{Z(t)}dW_2(t) \tag{12}$$

where $X(t)$ and $Z(t)$ are characterized by the following stochastic differential equations respectively:

$$dX(t) = k_X(\bar{X} - X(t))dt + \sigma_X\sqrt{X(t)}dW_1(t) \tag{13}$$

$$dZ(t) = k_Z(\bar{Z} - Z(t))dt + \sigma_Z\sqrt{Z(t)}dW_2(t) \tag{14}$$

In the above equations, $W_1(t)$ and $W_2(t)$ are two independent standard Brownian motions, and all the coefficients in the stochastic differential equations are assumed to satisfy regularity conditions so that a unique solution exists for each of the stochastic differential equations. In particular, $k_i > 0$, $\sigma_i > 0$, $k_X\bar{X} > \frac{1}{2}\sigma_X^2$ and $k_Z\bar{Z} > \frac{1}{2}\sigma_Z^2$. These assumptions ensure that $X(t)$ and $Z(t)$ are both strictly positive, stationary, mean-reverting processes with $\bar{X}$ and $\bar{Z}$ being their respective steady state means.

3.4 Equilibrium

An equilibrium is a collection of $c(t)$, $M(t)$, $B(t)$, $b(t)$, $S_i(t)$, $G(t)$ and $P(t)$, $R(t)$, $r(t)$, $Q_i(t)$ with the following properties:

(i). $\{c(t), M(t), B(t), b(t), S_i(t)\}$ solves the consumer's problem given $\{P(t), R(t), r(t), Q_i(t), G(t)\}$;

(ii). Markets clearing: $c(t) = y(t) - f\left(\frac{m(t)}{c(t)}\right)$,[9] $M(t) = M^s(t)$, $B(t) = 0$, $b(t) = 0$ and $S_i(t) = 0$;

(iii). Government budget constraint (3) is satified.

(iv). Monetary authority implements the policy rule (10), i.e. as $\triangle t \to 0$,

$$\frac{dM(t)}{M(t)} = \frac{dP(t)}{P(t)} - \alpha\left(\frac{dP(t)}{P(t)} - \pi^* dt\right) + \frac{dy(t)}{y(t)} + \frac{d\xi(t)}{\xi(t)} \tag{15}$$

[9] In deriving the following results, I have assumed $f(k) = 0$.

4 The Results

4.1 Inflation

Consider the limiting case where $R^m(t) \to R(t)$ and $\triangle t \to 0$. We have the following result for inflation using (9) and (15) (see Appendix A for details):

Proposition 1 *At the equilibrium of the economy, inflation is given by:*

$$\begin{aligned} \frac{dP(t)}{P(t)} &= \mu_P(t)dt + \sigma_P(t)dW_2(t) \qquad (16) \\ \mu_P(t) &= \pi^* + \frac{1}{\alpha}(Z(t) - \bar{Z}) \\ \sigma_P(t) &= \frac{\omega}{\alpha}\sqrt{Z(t)} \end{aligned}$$

To see the implications of the responsive monetary policy rule (15), let's note that $\mu_P(t)$ has the interpretation of expected inflation at time t, and $\sigma_P^2(t)$ measures inflation volatility at time t. We can see from the above equations that even if monetary authority holds the inflation target constant, differences in the policy responsiveness to inflation have important implications for inflation volatility as well as expected inflation. In particular, the more responsive the monetary policy rule is with respective to inflation, as represented by a higher value of α, the lower the inflation volatility, and the smaller the gap between the expected inflation and the target level π^*. In other words, a responsive monetary policy helps stabilize inflation around the target level in this economy. The more aggressive the monetary authority acts against inflation, the more stable the inflation.

Moreover, the covariance between $\frac{dP(t)}{P(t)}$ and $\frac{d\xi(t)}{\xi(t)}$ is equal to $\frac{\omega}{\alpha}Z(t)$, which is greater than zero as long as α and ω are greater than 0. Hence inflation is positively correlated with exogenous monetary policy expansions. Also note that in the steady state distribution, the mean of $\mu_P(t)$ is π^* since the mean of $Z(t)$ equals $\bar{Z}$. We can therefore also interpret π^* as the "long-run" inflation level.

4.2 The Term Structure under the Monetary Policy Rule

4.2.1 The Short-Term Interest Rate

Note that when $f'(\cdot) = 0$, the first order condition (4) implies that $\lambda_t = e^{-\rho t}u'(c_t)/P_t$. It then follows from (6) that $e^{-R_t \triangle t} = e^{-\rho \triangle t} E_t \left(\frac{u'(c_{t+\triangle t})P_t}{u'(c_t)P_{t+\triangle t}} \right)$. Using the Taylor expansion and the above result for inflation (recall again $c(t) = y(t)$ in equilibrium), we have (see Appendix B for details):

Proposition 2 *As $\triangle t \to 0$, the instantaneous short term nominal interest rate is:*[10]

$$R(t) = \left(\pi^* - \frac{\omega^2 \bar{Z}}{\alpha^2} \right) + r(t) + (1 - \frac{\omega^2}{\alpha})(\mu_P(t) - \pi^*) \tag{17}$$

where $r(t)$ is the real short term interest rate, $\mu_P(t)$ is the expected inflation, and they are given respectively by:

$$r(t) = (h - \epsilon^2)X(t) \tag{18}$$

$$\mu_P(t) - \pi^* = \frac{1}{\alpha}(Z(t) - \bar{Z}) \tag{19}$$

First note that the standard Fisher relation, which states that the nominal interest rate equals the sum of the real interest rate and expected inflation, is a special case of the above result. If inflation volatility is zero, which is true when $\omega = 0$ (note that the conditional variance of inflation is given by $\frac{\omega^2 Z(t)}{\alpha^2}$), then equation (17) is reduced to $R(t) = r(t) + \mu_P(t)$, which is exactly the Fisher equation. The above result is more general because it takes into account the impact of inflation risk on nominal bond prices. It shows that not just the expected inflation, but also inflation volatility, affects the nominal interest rate in a very important way. In particular, a higher inflation volatility (due to either higher ω or lower α) will actually lead to a lower $R(t)$ holding other things constant, and vice versa. The economic intuitions will be discussed shortly (we will see that this is true for long-term nominal interest rates as well).

The equations also clarify the impact of monetary policy on the short-term nominal interest rate. First, a temporary exogenous monetary policy shock (say an increase in $Z(t)$) affects $R(t)$ only through its effect on expected inflation $\mu_P(t)$. A higher than usual $Z(t)$ drives up $\mu_P(t)$, and hence $R(t)$. In

[10] In order to ensure that $R(t)$ is always positive, we need to impose that $\alpha > max(\omega^2, \frac{\bar{Z}}{\pi^*})$.

this frictionless economy, a monetary expansions immediately leads to higher inflation and has no effect on the real interest rate.

Secondly, if there is a permanent shift of the monetary policy rule, say an increase in the value of α, it will reduce the level of expected inflation $\mu_P(t)$ and hence decrease $R(t)$ through the Fisher relation. On the other hand, an increase in α also reduces inflation volatility and hence tends to increase the level of the nominal interest rate through the negative relationship between $R(t)$ and inflation volatility. Therefore, in the short run, the immediate effect of such a policy change is ambiguous, and the direction of interest rate movement depends on the relative magnitudes of these two effects. But in the long run, the average level of expected inflation $\mu_P(t)$ converges to the pre-specified target rate π^* (the unconditional mean of $\mu_P(t)$), the average level of $R(t)$ would therefore rise as inflation volatility is reduced by a higher value of α while holding π^* constant.

Moreover, as the policy rule shifts, for example α increases, volatility of the interest rate also changes. In fact, since $X(t)$ and $Z(t)$ are independent, substituting (18) and (19) into equation (17), we have that:

$$Var(R(t)) = (h - \epsilon^2)^2 Var(X(t)) + \left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)^2 Var(Z(t))$$

If we assume that the variance of $X(t)$ and $Z(t)$ remain constant over time, then $Var(R(t))$ will either increase or decrease depending on the values of α. When α is very small, a marginal increase in α will lead to an increase in the volatility of $R(t)$. But if α is large enough (specifically if $\alpha > 2\omega$), an increase in the value of α will reduce the volatility of the interest rate $R(t)$. This result is very intuitive. While a more responsive monetary policy helps stabilize expected inflation around the target level [see equation (15)] and hence reduces the interest rate volatility through this channel, a larger value of α also requires more aggressive movements in the policy instrument when inflation changes and hence tends to increase the interest rate variability. If the policy is not effective enough and inflation is still very volatile, then the "destabilizing" effect on $R(t)$ of a marginal increase in α would dominate the "stabilizing" effect, and we have higher interest rate volatility in equilibrium. But under a very proactive policy, inflation is effectively stabilized around π^* and hence there is little pressure for large movements in the policy instrument even if α is very large. In equilibrium we have lower interest rate volatility as a higher value of α further stabilizes inflation.

This result on interest rate volatility also sheds some light on the issue of interest rate "smoothing" that arises in many discussions of the optimal monetary policy rule. It is sometimes said that central banks should add a lagged interest rate term in the policy reaction function due to the concern that a responsive (with respect to inflation, employment) policy could result in a very volatile short term interest rate. However our model indicates that a proactive policy does not necessarily increase interest rate volatility in equilibrium because of its stabilizing effect on inflation.

4.2.2 Nominal Long-Term Interest Rates

Upon substituting (18) and (19) into (17), we have:

$$\begin{aligned} R(t) &= \theta_0 + \theta_1 X(t) + \theta_2 Z(t) \qquad (20) \\ \theta_0 &= \pi^* - \frac{\bar{Z}}{\alpha} \\ \theta_1 &= h - \epsilon^2 \\ \theta_2 &= \frac{1}{\alpha} - \frac{\omega^2}{\alpha^2} \end{aligned}$$

Given $X(t)$ and $Z(t)$ that are defined in (13) and (14) respectively, the above result suggests a two-factor affine model of the term structure of interest rates.[11] Hence the term structure has the following closed form solution:

Proposition 3 *At time t, the yield $R(t, \tau)$ on a nominal zero-coupon bond maturing at $t + \tau$ (for $\tau > 0$) is given by:*

$$R(t,\tau) = \theta_0 - \frac{\log H_{1,X}(\tau)}{\tau} - \frac{\log H_{1,Z}(\tau)}{\tau} + \frac{H_{2,X}(\tau)}{\tau}\theta_1 X(t) + \frac{H_{2,Z}(\tau)}{\tau}\theta_2 Z(t)$$

Where $H_{1,i}(\tau)$ and $H_{2,i}(\tau)$ for i = X, Z are given respectively by:

$$\begin{aligned} H_{1,X}(\tau) &= \left[\frac{2B_X e^{(B_X+A_X)\tau/2}}{(B_X+A_X)(e^{B_X\tau}-1)+2B_X}\right]^{2k_X\bar{X}/\sigma_X^2} \\ H_{2,X}(\tau) &= \frac{2(e^{B_X\tau}-1)}{(B_X+A_X)(e^{B_X\tau}-1)+2B_X} \\ A_X &= k_X + \epsilon\sigma_X \\ B_X &= (A_X^2 + 2(h-\epsilon^2)\sigma_X^2)^{1/2} \end{aligned}$$

[11] It can be show that the market prices of risk for the two factors are proportional to $\sqrt{X_t}$ and $\sqrt{Z_t}$ respectively from the first-order conditions of the consumer's problem. See Appendix C for details

and

$$
\begin{aligned}
H_{1,Z}(\tau) &= \left[\frac{2B_Z e^{(B_Z+A_Z)\tau/2}}{(B_Z+A_Z)(e^{B_Z\tau}-1)+2B_Z}\right]^{2k_Z\bar{Z}/\sigma_Z^2} \\
H_{2,Z}(\tau) &= \frac{2(e^{B_Z\tau}-1)}{(B_Z+A_Z)(e^{B_Z\tau}-1)+2B_Z} \\
A_Z &= k_Z + \frac{\omega\sigma_Z}{\alpha} \\
B_Z &= \left[\left(k_Z + \frac{\omega\sigma_Z}{\alpha}\right)^2 + 2\left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)\sigma_Z^2\right]^{1/2}
\end{aligned}
$$

With the above results, it is now very easy to examine changes in the term structure as the monetary policy shifts. Since $H_{i,X}$ (i=1, 2) does not depend on the policy parameter (neither α nor π^*), a shift in monetary policy affects $R(t,\tau)$ through θ_0 and $H_{i,Z}$ (i=1, 2). Hence for the purpose of exposition, let's for the moment ignore the terms associated with $X(t)$ and simply write $R(t,\tau)$ as:

$$
R(t,\tau) = \theta_0 - \frac{\log H_{1,Z}(\tau)}{\tau} + \frac{H_{2,Z}(\tau)}{\tau}\theta_2 Z(t) \tag{21}
$$

Or

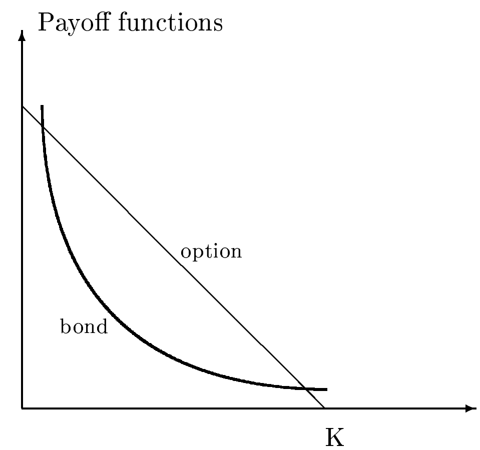
$$
\begin{aligned}
R(t,\tau) &= \left(\pi^* - \frac{H_{2,Z}(\tau)}{\tau}\frac{\omega^2\bar{Z}}{\alpha^2}\right) - \left(1 - \frac{H_{2,Z}(\tau)}{\tau}\right)\frac{\bar{Z}}{\alpha} - \frac{\log H_{1,Z}(\tau)}{\tau} + \\
&\quad \frac{H_{2,Z}(\tau)}{\tau}\left(1 - \frac{\omega^2}{\alpha}\right)(\mu_P(t) - \pi^*)
\end{aligned} \tag{22}
$$

We can then easily see from the above equation that there is a similar negative effect of inflation volatility on the level of long-term nominal interest rates $R(t,\tau)$ as that on the short rate $R(t)$ [see equation (17)]. Hence an increase in α tends to raise the entire yield curve because of reduced inflation volatility under a more responsive monetary policy, holding the expected inflation constant.

To understand this seemingly counter-intuitive result, namely that a higher inflation volatility leads to higher prices for nominal bonds and hence lower nominal interest rates and vice versa, let's recall that the time t price of a nominal risk-less bond maturing at $t+\tau$ is given by:

$$
\Lambda_{t,\tau} = E_t\left(\Phi_{t,t+\tau} \cdot f(\Pi_{t+\tau})\right)
$$

where Φ is some pricing kernel, the payoff function $f(\cdot)$ is given by $f(x) = \frac{1}{x}$, $\Pi_{t+\tau} = \frac{P_{t+\tau}}{P_t}$ and P is the general price level. Hence a nominal bond can be viewed as a "derivative asset" whose payoff is contingent upon the future inflation rate. In fact it mostly resembles an European put option maturing at $t+\tau$ with some strike price K if we compare their respective payoff functions:



An analogy can therefore be drawn between a put option and a nominal riskless bond. It is well known that higher volatility of the underlying stock price increases the value of the option, because the owner of the put option benefits from price decreases but has limited downside risk in the event of stock price increases. It is then not surprising that a higher inflation volatility increases the prices of nominal bonds and hence leads to lower nominal interest rates and vice versa, holding other things constant. This analogy shows that inflation volatility plays a critical role in the determination of nominal interest rates, in contrast to many previous studies on the relationship between inflation and nominal interest rates which have mainly focused on the role of expected inflation.

Moreover, note that in (21) and (22) as α increases, θ_0 increases. Since θ_0 affects $R(t)$ and $R(t,\tau)$ for any $\tau > 0$ the same way, therefore when the central bank raises the short-term interest rate $R(t)$ by permanently moving toward a more proactive policy (increasing α), long-term rates could rise with almost the same magnitude as the short rate if the effect of α on $H_{1,Z}$ and $H_{2,Z}$ is small. And depending on how α affects $H_{1,Z}$ and $H_{2,Z}$, it is possible for long-term rate $R(t,\tau)$ to increase even more than the short rate $R(t)$ does.

Changes in α not only shift the level of the yield curve, but also affect how long rates respond to movements in the short rate. From equation (20) and (21) we can easily see that if a monetary shock $Z(t)$ moves the short rate $R(t)$ by 1%, the impact on a long rate $R(t,\tau)$ will be given by the factor loading $\frac{H_{2,Z}(\tau)}{\tau}$. Or in other words, as the central bank takes actions to change the short-term rate, the factor loading $\frac{H_{2,Z}(\tau)}{\tau}$ determines the magnitude of the response of the long-term rate $R(t,\tau)$ to movements in the short rate $R(t)$. How changes in α

affect the sensitivities of long rates depends on how α affects $\frac{H_{2,Z}(\tau)}{\tau}$. Rewrite the definition of $H_{2,Z}$ here for convenience:

$$H_{2,Z}(\tau) = \frac{2(e^{B_Z\tau} - 1)}{(B_Z + A_Z)(e^{B_Z\tau} - 1) + 2B_Z}$$

Where:

$$\begin{aligned} A_Z &= k_Z + \frac{\omega\sigma_Z}{\alpha} \\ B_Z &= \left[\left(k_Z + \frac{\omega\sigma_Z}{\alpha}\right)^2 + 2\left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)\sigma_Z^2\right]^{1/2} \end{aligned}$$

Even though $H_{2,Z}$ is a complicated nonlinear function of α, large increases in α will usually lead to increases in $H_{2,Z}$, because both A_Z and B_Z are decreasing functions of α for $\alpha > 2\omega^2$, and for large τ (time to maturity) $H_{2,Z}$ can be approximated by $\frac{2}{B_Z+A_Z}$. This suggests that long-term rates tend to be more sensitive to movements in the short-term rate under a more proactive monetary policy rule.

The intuition of this result can be obtained from the derivation of the term structure. In particular, from (26), (30) and (33) in Appendix C, we can see that the short-term interest rate becomes more persistent under a risk neutral probability measure when a larger α reduces the volatility of inflation. But in the risk neutral world, a long term rate can be loosely thought of as the average of the expected future short term rate. Higher persistence in the short rate process therefore leads to larger response of long rates to the movements of the current short rate.

Furthermore, equation (21) implies that the standard deviation of $R(t, \tau)$ is given by:

$$std(R(t,\tau)) = \frac{H_{2,Z}(\tau)}{\tau}\left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right) \times std(Z(t)) \tag{23}$$

Since $H_{2,Z}$ tends to increase as we shift toward a more responsive policy rule, a change in α has an ambiguous effect on the volatility of the interest rates. On the one hand, a higher α stabilizes inflation and hence tends to reduce interest rate volatilities. On the other hand, such an increase in α leads to higher factor loadings and hence higher standard deviations of the interest rates. Nevertheless, following a large increase in α, the interest rate volatilities are more likely to decrease as $std(R(t, \tau))$ will be dominated by $\left(\frac{1}{\alpha} - \frac{\omega^2}{\alpha^2}\right)$ for large τ.

Also note that a change of π^*, the target rate of inflation, only affects the term structure through its impact on θ_0. Hence a change in π^* has uniform effect on the interest rates across the maturity spectrum. A higher target rate of inflation leads to higher yield curve as the expected inflation increases but leaves volatilities and correlations among the interest rates unchanged.

4.2.3 An Example

Figure 2, 4 and 6 further demonstrate the effects on the yield curve as we move from a less responsive to a highly responsive policy rule using some ad hoc parameter values. We set $\alpha = 0.2$ and $\alpha = 2$ respectively to represent two policy regimes, with the lower α representing a "passive" policy and the larger α representing an "active" policy. For the underlying state variable Z(t), I choose $k_Z = 0.085$, $\bar{Z} = .001$, $\sigma_Z = 0.08$. I also set ω at 0.3. The time to maturity τ is from 1 month to 30 years. These values are chosen so that they are consistent with the reduced form estimates of the term structure model [Wu (2001)].

Figure 2 is the plot of the factor loading $\frac{H_{2,Z}(\tau)}{\tau}$ against τ. As explained above, this term measures the responsiveness of long rates to movements in the short rate. We can see that when α increases from 0.2 to 2, the sensitivity of long rates to movements in the short rate increases significantly. The graph also shows that the impact is larger at longer maturities. A quick comparison between this figure and Figure 1, which is based on the government bond yields, suggests that the apparent structural change in the yield curve around 1979 can be accounted for to a large extent by the shift in the monetary policy.

Figure 4 includes the plot of $\theta_0 - \frac{\log H_{1,Z}}{\tau} + \frac{H_{2,Z}}{\tau}\theta_2 \bar{Z}$ for $\alpha = 0.2$ and $\alpha = 2.0$ respectively. It shows how a large increase in α affects the average level of the yield curve in the steady state distribution. In particular, our model suggests that as α increases from 0.2 to 2, in equilibrium we will tend to have a higher yield curve on average (note that the unconditional mean of Z(t) is $\bar{Z}$). We see a similar shift of the yield curve in the U.S. data as monetary policy became more responsive in the post-1979 period in Figure 3.

Figure 4 also shows that if an increase of the short rate R(t) by the central bank is due to a permanent shift of α to a higher value, then it is possible that the long rates also move in the same direction with the same or an even bigger magnitude. This result sheds some light on the puzzling behavior of long term interest rates in early 1994, where a moderate monetary policy tightening led to increases in all the long-term interest rates with similar magnitudes to that in the Federal funds rate[12].

Finally, Figure 6 plots the standard deviation of the interest rates as α increases from 0.2 to 2.0, which confirms that a more responsive monetary policy

[12] In the spring of 1994, a half percentage point increase in the federal funds rate driven by the Fed led to a half percentage to one percentage point increase in the long rates.

does not necessary lead to more volatile interest rates. Instead, as inflation stabilizes around the target level, We can have decreased interest rate volatility in equilibrium. Again, we see similar changes in interest rate volatilities following the policy shift in 1979 in Figure 5.

5 Concluding Remarks

The term structure of interest rates is influenced by many economic factors. A simple model like the one in the present paper probably is not able to account for all the empirical properties of the yield curve. The main purpose of this exercise is instead to understand how a major shift in the monetary policy, such as the one occurred in the late 1970s/early 1980s, can impact the term structure. One caveat of the model is that the monetary policy affects nominal interest rates only through inflation. Other channels, such as the real interest rate, are shut down in the model for tractability. To fully understand the relationship between the monetary policy and the term structure, it is necessary to take into account the real effect of money. From a long-run point of view, however, the neutrality assumption may still be appropriate.

A Proof of Proposition 1

First note that when $f^{'}(m_t/c_t) = 0$, the transaction cost is a constant. So without loss of generality, let's assume that $y(t) = c(t)$.

Consider the case where $\triangle t \to 0$. Assuming that $\frac{dP(t)}{P(t)} = \mu_P(t)dt + \sigma_1(t)dW_1(t) + \sigma_2(t)dW_2(t)$ and applying Ito's Lemma to (9):

$$\frac{dM(t)}{M(t)} = (\mu_P(t) + \mu_y(t) + \sigma_y(t)\sigma_1(t))dt + (\sigma_1(t) + \sigma_y(t))dW_1(t) + \sigma_2(t)dW_2(t) \quad (24)$$

where $\mu_y(t) = hX(t) - \rho$, $\sigma_y(t) = \epsilon\sqrt{X(t)}$.

From (15) we have the policy rule:

$$\frac{dM(t)}{M(t)} = \frac{dP(t)}{P(t)} - \alpha\left(\frac{dP(t)}{P(t)} - \pi^* dt\right) + \frac{dy(t)}{y(t)} + \frac{d\xi(t)}{\xi(t)} \quad (25)$$

where $\frac{d\xi(t)}{\xi(t)}$ is given in (12).

Using the above two equations, we have the following relations because of the unique representation of the diffusion process M(t):

$$
\begin{aligned}
&\sigma_1(t)+\sigma_y(t)=(1-\alpha)\sigma_1(t)+\sigma_y(t)\\
&\sigma_2(t)=(1-\alpha)\sigma_2(t)+\sigma_\xi(t)\\
&\mu_P(t)-\alpha(\mu_P(t)-\pi^*)+\mu_y+\mu_\xi(t)=\mu_P(t)+\mu_y(t)+\sigma_y(t)\sigma_1(t)
\end{aligned}
$$

where from (12) we have $\mu_\xi(t)=Z(t)-\bar{Z}$, $\sigma_\xi(t)=\omega\sqrt{Z(t)}$.

Solving the above three equations gives us Proposition 1.

B Proof of Proposition 2

Using the first order conditions (4) and (6) from the consumer's problem, when $f^{'}(m_t/c_t)=0$, we have:

$$
e^{-R(t)\triangle t} \quad = \quad e^{-\rho\triangle t}E_t\left[\frac{P(t)c(t)}{P(t+\triangle t)c(t+\triangle t)}\right]
$$

Using the Taylor expansion for the left-hand side,

$$
e^{-R(t)\triangle t}=1-R(t)\triangle t+\circ(\triangle t)^{3/2}
$$

Similarly for the right-hand side:

$$
\begin{aligned}
&e^{-\rho\triangle t}E_t\left[\frac{P(t)c(t)}{P(t+\triangle t)c(t+\triangle t)}\right]=(1-\rho\triangle t+\circ(\triangle t)^{3/2})\times\\
&E_t\left[\left(1-\frac{\triangle P_t}{P_t}+\left(\frac{\triangle P_t}{P_t}\right)^2+\circ(\triangle t)^{3/2}\right)\left(1-\frac{\triangle c_t}{c_t}+\left(\frac{\triangle c_t}{c_t}\right)^2+\circ(\triangle t)^{3/2}\right)\right]
\end{aligned}
$$

using the facts that $c_t=y_t$ and:

$$
\begin{aligned}
\frac{\triangle y_t}{y_t}&=\mu_{y,t}\triangle t+\sigma_{y,t}W_{1,t+\triangle t}\sqrt{\triangle t}\\
\frac{\triangle P_t}{P_t}&=\mu_{P,t}\triangle t+\sigma_{1,t}W_{1,t+\triangle t}\sqrt{\triangle t}+\sigma_{2,t}W_{2,t+\triangle t}\sqrt{\triangle t}
\end{aligned}
$$

where $W_{1,t}$ and $W_{2,t}$ are two independent standard normal variables. We hence have as $\triangle t\to 0$:

$$
R(t)=\rho+\mu_y(t)+\mu_P(t)-\sigma_y^2(t)-(\sigma_1^2(t)+\sigma_2^2(t))-\sigma_1(t)\sigma_y(t)
$$

Substituting in relevant terms in the above equation leads us to Proposition 2.

C Proof of Proposition 3

From the first order conditions we have that the state price deflator is given by $\pi(t) = \frac{e^{-\rho t}}{P(t)c(t)}$. Hence,

$$\begin{aligned}
\frac{d\pi(t)}{\pi(t)} &= -R(t)dt - (\epsilon, \frac{\omega}{\alpha}) \begin{pmatrix} \sqrt{X(t)} & 0 \\ 0 & \sqrt{Z(t)} \end{pmatrix} \begin{pmatrix} dW_1(t) \\ dW_2(t) \end{pmatrix} \\
&= -R(t)dt - (\lambda_r, \lambda_u) \begin{pmatrix} \sqrt{r(t)} & 0 \\ 0 & \sqrt{u(t)} \end{pmatrix} \begin{pmatrix} dW_1(t) \\ dW_2(t) \end{pmatrix}
\end{aligned}$$

Where $\lambda_r = \frac{\epsilon}{\sqrt{\theta_1}}$, $\lambda_u = \frac{\omega}{\alpha\sqrt{\theta_2}}$. See (20) for the definitions of θ_1 and θ_2.

Let $r(t) = \theta_1 X(t)$, $u(t) = \theta_2 Z(t)$, the instantaneous short-term interest rate in (20) can be written as:

$$R(t) = \theta_0 + r(t) + u(t) \tag{26}$$

$r(t)$ and $u(t)$ are given respectively by:

$$dr(t) = k_X(\bar{r} - r(t)) + \sigma_r \sqrt{r(t)} dW_1(t) \tag{27}$$
$$du(t) = k_Z(\bar{u} - u(t)) + \sigma_u \sqrt{u(t)} dW_2(t) \tag{28}$$

where $\bar{r} = \theta_1 \bar{X}$, $\sigma_r = \sqrt{\theta_1}\sigma_X$, $\bar{u} = \theta_2 \bar{Z}$ and $\sigma_u = \sqrt{\theta_2}\sigma_Z$.

Hence under the Equivalent Martingale Measure (EMM), we can rewrite (27) and (28) as:

$$dr(t) = \tilde{k}_r(\tilde{r} - r(t))dt + \sigma_r \sqrt{r(t)} d\tilde{W}_1(t) \tag{29}$$
$$du(t) = \tilde{k}_u(\tilde{u} - u(t))dt + \sigma_u \sqrt{u(t)} d\tilde{W}_2(t) \tag{30}$$

where $\tilde{W}_1(t)$ and $\tilde{W}_2(t)$ are two independent standard Brownian motions under EMM, and the coefficients are given in the following equations:

$$\tilde{k}_r = k_X + \lambda_r \sigma_r \tag{31}$$
$$\tilde{r} = \frac{k_X \bar{r}}{k_X + \lambda_r \sigma_r} \tag{32}$$
$$\tilde{k}_u = k_Z + \lambda_u \sigma_u \tag{33}$$
$$\tilde{u} = \frac{k_Z \bar{u}}{k_Z + \lambda_u \sigma_u} \tag{34}$$

Using the well-known results of the multi-factor Cox-Ingersoll-Ross Term Structure Model [e.g. Duffie (1996)], Proposition 3 follows.

References

[1] Bakshi, G. and Zhiwu Chen [1996], "Inflation, Asset Prices, and the Term Structure of Interest Rates in Monetary Economies", *The Review of Financial Studies*, Spring 1996, pp. 241-275.

[2] Bernanke, B. S., Mark Gertler and Mark Waston [1997], "Systematic Monetary Policy and the Effects of Oil Price Shocks", *New York University Working Paper*, June 1997.

[3] Bliss, Robert, [1997], "Testing Term Structure Estimation Methods", *Advances in Futures and Options Research*, Vol.9 pp.197-231.

[4] Clarida, Richard, et al. [1999], "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature*, December 1999, pp. 1661-1707

[5] — [1998], "Monetary Policy Rules And Macroeconomic Stability: Evidence and Some Theory", *NBER Working Paper* No.6442, March 1998.

[6] Christiano, Lawrence., Martin Eichenbaum and Charles Evans [1998a], "Modeling Money", *NBER Working Paper* No.6371, January 1998.

[7] — [1998b], "Monetary Policy Shocks: What Have We Learned and to What End?", *NBER Working Paper* No.6440, February 1998.

[8] Cohen, G.D. and John Wenninger [1994], "Changing Relationship Between the Spread and the Funds Rate", *Federal Reserve Bank of New York Working Paper* No. 9408, New York, May 1994.

[9] Cox, John C. et al. [1985], "A Theory of The Term Structure Of Interest Rates", *Econometrica*, Vol. 53, No.2, March 1985, pp.385-407.

[10] Duffie, Darrell [1996], "Dynamic Asset Pricing Theory", 2nd ed. *Princeton University Press*, 1996.

[11] Evans, Charles L. and Davide A. Marshall [1997], "Monetary Policy and the Term Structure of Nominal Interest Rates: Evidence and Theory", *Federal Reserve Bank of Chicago Working Paper*.

[12] Flemming, Michael and Eli M. Remolona [1999], "The Term Structure of Announcement Effects", *Manuscript*.

[13] Hamilton, James [1988], "Rational-Expectations Econometric Analysis of Changes in Regimes: An Investigation of the Term Structural of Interest Rates", *Journal of Economic Dynamics and Control*, Vol.12, 1988, pp.385-423.

[14] Hsu, Chiente and Peter Kugler [1996], "The Revival of the Expectations Hypothesis of the US Term Structure of Interest Rates", *Economics Letters*, Vol.55, 1997, pp.115-120.

[15] King, Robert G. and Alexander Wolman [1999], "What Should the Monetary Authority Do When Prices are Sticky?", in John Taylor (Ed.) *Monetary Policy Rules*, Chicago: University of Chicago Press, 1999.

[16] Kwon, H. and J.H. McCulloch [1993], "U.S. Term Structure Data, 1947-1991", *Ohio State University Working Paper*, 93-6.

[17] Leeper, Eric M. [1991], "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies", *Journal of Monetary Economics* 27: 129-147.

[18] Mankiw, Gregory and Jeffrey Miron [1986], "The Changing Behavior of the Term Structure of Interest Rates", *The Quarterly Journal of Economics*, May 1986, Vol.101, pp211-228.

[19] McCallum, Bennett T. and Edward Nelson [1999], "Performance of Operational Policy Rules in an Estimated Semi-Classical Structural Model", in John B. Taylor (Ed.) *Monetary Policy Rules*, Chicago: University of Chicago Press, 1999.

[20] McCulloch, J.H. [1990], "U.S. Term Structure Data, 1946-87", *Handbook of Monetary Economics* Vol. 1, pp.627-715.

[21] Mehra, Yash P. [1996], "Monetary Policy and Long-Term Interest Rates", Federal Reserve Bank of Richmond *Economic Quarterly* Vol.82/3 Summer 1996, pp.27-49.

[22] Piazzesi, Monika [2001], "Monetary Policy and Macroeconomic Variables in a Model of the Term Structure of Interest Rates", *NBER working paper 8246*.

[23] Rudebusch, Glenn D. [1995], "Federal Reserve Interest Rate Targeting, Rational Expectations, and the Term Structure", *Journal of Monetary Economics* 35, pp.245-274.

[24] Sun, Tong Sheng [1992] "Real and Nominal Interest Rates: A Discrete-Time Model and Its Continuous-Time Limit", *The Review of Financial Studies*, Vol.5/4, pp.581-611

[25] Taylor, John B. [1993], "Discretion Versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy*, 39, pp195-214.

[26] — (Ed.) [1999], *Monetary Policy Rules*, Chicago: University of Chicago Press, 1999.

[27] Watson, Mark W. [1999], "Explaining the Increased Variability in Long Term Interest Rates", *Manuscript*, Princeton University.

[28] Wu, Shu [2001], "Inferring Changes in Monetary Policy From the Term Structure of Interest Rates", *Manuscript*, The University of Kansas.

Figure 1: OLS regression coefficients of long rates on the short rate

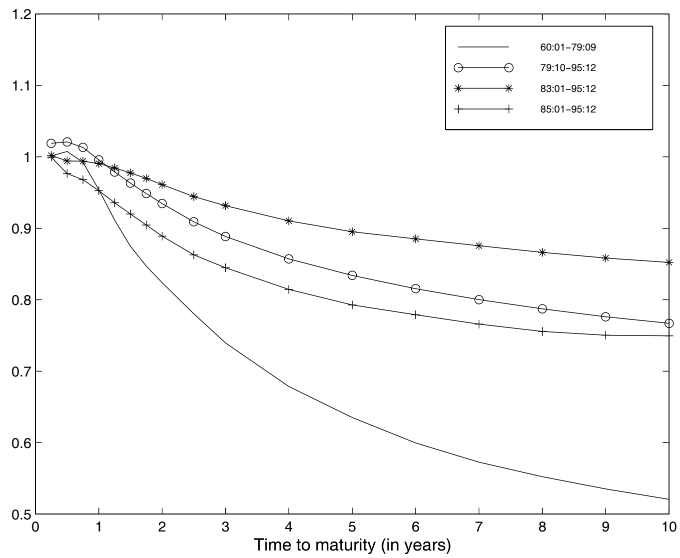
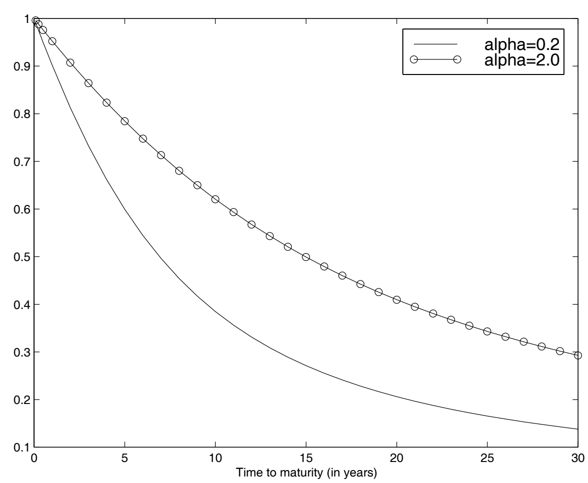


Figure 2: The impact of a policy change on the responsiveness of long rates



The curves are the plots of $\frac{H_{2,Z}(\tau)}{\tau}$ in equation (21) for $\alpha = 0.2$ and $\alpha = 2.0$ respectively.

They measure the responsiveness of long rates to movements in the short term rate.

Figure 3: Average level of the yield curve

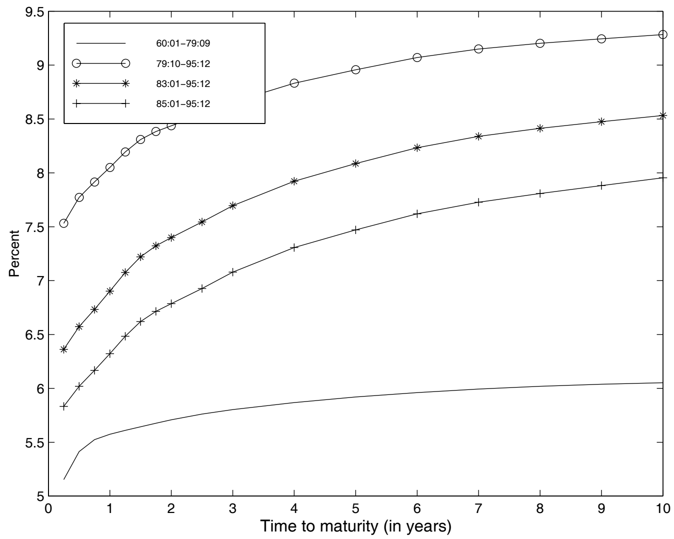
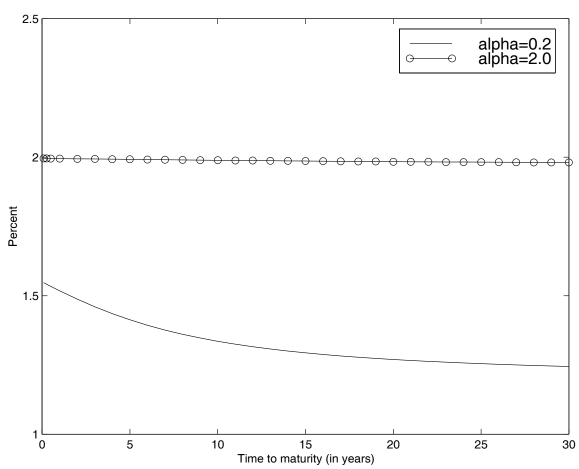


Figure 4: The impact of a policy change on the average level of the yield curve



The curves are the plots of $\theta_0 - \frac{\log H_{1,Z}(\tau)}{\tau} + \frac{H_{2,Z}(\tau)}{\tau}\theta_2 Z(t)$ in equation (21) for $\alpha = 0.2$ and $\alpha = 2.0$ respectively, replacing $Z(t)$ with its unconditional mean $\bar{Z}$. They measure the impact of changes in α on the average level of the yield curve in the steady state distribution, holding everything else constant.

Figure 5: Interest rate volatility

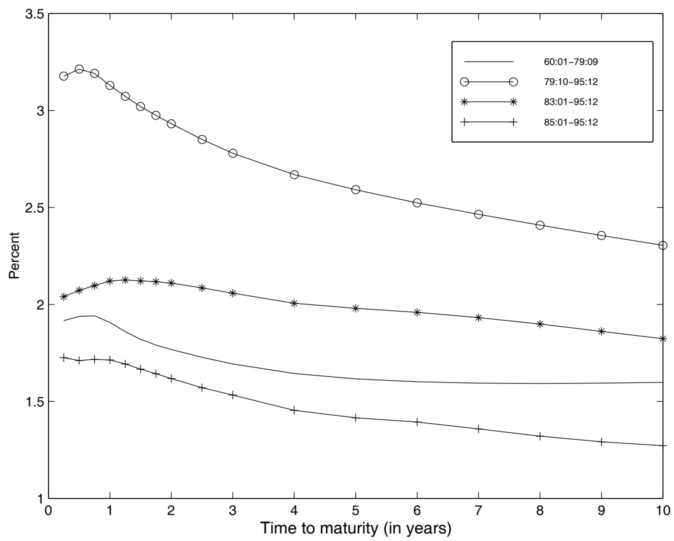
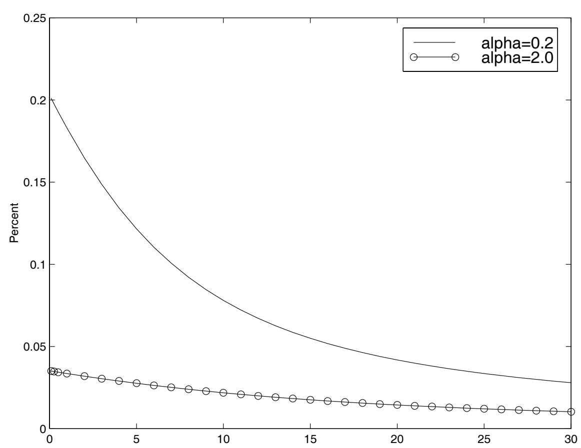


Figure 6: The impact of a policy change on the volatilities of long rates



The curves are the plots of $\frac{H_{2,Z}(\tau)}{\tau}\theta_2 \times std(Z(t))$ in equation (23) for $\alpha = 0.2$ and $\alpha = 2.0$ respectively. They measure the impact of changes in α on the standard deviations of the nominal interest rates of different maturities.

Table 1: **OLS regression of long rates on the short rate**: $R(t)$ is the 1-month rate, $D(t) = 1$ if $t > 1979:09$ and 0 otherwise. The standard errors are computed using the Newy-West procedure.

$R(t,\tau)$	Constant	std. error	$R(t)$	std. error	$D(t) \cdot R(t)$	std. error	R-square
3-month	0.2626	0.0024	1.0014	8.783E05	0.0176	3.712E05	0.9978
6-month	0.4922	0.0068	1.0074	0.0003	0.0135	0.0001	0.9956
9-month	0.6828	0.0105	0.9924	0.0004	0.0208	0.0002	0.9938
1-year	0.9299	0.0137	0.9541	0.0005	0.0416	0.0002	0.9921
1.25-year	1.1824	0.0166	0.9113	0.0006	0.0675	0.0003	0.9908
1.5-year	1.4006	0.0188	0.8748	0.0007	0.0885	0.0003	0.9898
1.75-year	1.5742	0.0207	0.8471	0.0008	0.1017	0.0003	0.9888
2-year	1.7231	0.0222	0.8239	0.0008	0.1107	0.0004	0.9880
2.5-year	1.9983	0.0247	0.7803	0.0009	0.1288	0.0004	0.9865
3-year	2.2487	0.0274	0.7396	0.0010	0.1489	0.0005	0.9853
4-year	2.6275	0.0312	0.6786	0.0010	0.1787	0.0005	0.9837
5-year	2.9030	0.0347	0.6353	0.0011	0.1987	0.0005	0.9823
6-year	3.1318	0.0385	0.5996	0.0012	0.2159	0.0006	0.9810
7-year	3.3064	0.0416	0.5728	0.0013	0.2275	0.0006	0.9800
8-year	3.4408	0.0440	0.5523	0.0015	0.2350	0.0006	0.9794
9-year	3.5515	0.0461	0.5354	0.0014	0.2409	0.0006	0.9790
10-year	3.6456	0.0480	0.5207	0.0015	0.2465	0.0007	0.9787

Table 2: **Mean and std. dev. of the interest rates in different periods**

	1960:01 - 1979:09		1979:10 - 1995:12		1983:01 - 1995:12		1985:01 - 1995:12	
$R(t,\tau)$	Mean	Std	Mean	Std	Mean	Std	Mean	Std
3-month	5.1536	1.9162	7.5314	3.1769	6.3622	2.0394	5.8327	1.7253
6-month	5.4137	1.9379	7.7728	3.2128	6.5736	2.0722	6.0191	1.7109
9-month	5.5244	1.9413	7.9164	3.1912	6.7333	2.0972	6.1677	1.7167
1-year	5.5744	1.9071	8.0508	3.1298	6.9021	2.1204	6.3215	1.7138
1.25-year	5.6098	1.8600	8.1937	3.0726	7.0760	2.1262	6.4832	1.6931
1.5-year	5.6433	1.8200	8.3091	3.0203	7.2209	2.1216	6.6199	1.6662
1.75-year	5.6768	1.7907	8.3840	2.9749	7.3229	2.1175	6.7149	1.6426
2-year	5.7083	1.7676	8.4368	2.9320	7.3996	2.1098	6.7862	1.6176
2.5-year	5.7615	1.7273	8.5417	2.8510	7.5448	2.0854	6.9269	1.5705
3-year	5.8030	1.6927	8.6578	2.7796	7.6957	2.0579	7.0788	1.5326
4-year	5.8681	1.6437	8.8326	2.6692	7.9229	2.0061	7.3070	1.4539
5-year	5.9197	1.6158	8.9570	2.5914	8.0862	1.9798	7.4711	1.4156
6-year	5.9611	1.6011	9.0705	2.5235	8.2335	1.9593	7.6206	1.3933
7-year	5.9938	1.5942	9.1497	2.4644	8.3387	1.9322	7.7279	1.3579
8-year	6.0193	1.5925	9.2024	2.4092	8.4134	1.8986	7.8088	1.3207
9-year	6.0387	1.5943	9.2444	2.3554	8.4753	1.8612	7.8825	1.2919
10-year	6.0525	1.5983	9.2839	2.3048	8.5329	1.8230	7.9550	1.2718