UNIVERSITY OF KANSAS
WORKING PAPERS SERIES IN
THEORETICAL AND APPLIED ECONOMICS

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Paper no. 2001-04
September 2001

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Estimating Monetary Policy Effects When Interest Rates are Bounded at Zero

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First draft: December 2000
This version: September 2001

Abstract

Using a nonlinear structural VAR approach, we estimate the effects of exogenous monetary impulses in the presence of a zero lower bound constraint on nominal interest rates and examine the impact of such a constraint on the effectiveness of counter-cyclical monetary policies. We find that a binding zero bound on nominal interest rates can eliminate more than 50% of the effect of an exogenous monetary impulse on output based on the data from Japan. The conditional impulse response functions allow us to isolate the effect of monetary shocks operating through the interest rate channel when other possible channels of monetary transmission are present. Moreover, we also find that the zero bound on nominal interest rates could severely limit the ability of central banks to pursue a counter-cyclical monetary policy when facing adverse macroeconomic shocks.

JEL Classification: E52 (Monetary Policy), E55 (Central Banks)
Key Words: Zero lower bound, monetary transmission, nonlinear VAR.

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1 Introduction

According to the standard Keynesian textbook model, an expansionary monetary policy leads to a decline in the real interest rate which in turn decreases the cost of capital, thereby causes a rise in consumer and investment spending and hence, raises aggregate demand and output. Many economists seem to agree that such an interest rate channel is the key component of how monetary policy shocks are transmitted to the real economy [e.g. Taylor (1995)]. However, since there exists a zero lower bound on nominal interest rates, one implication of such a transmission mechanism is that a liquidity trap would eliminates the effect of monetary impulses on the real economy. Once the nominal interest rate hits the zero value, monetary policy would become impotent when it is mostly needed.

The possibility that such a lower bound on interest rates might interfere with the conduct of monetary policy is not just a purely theoretical concern. In fact, the recent experience of Japan, together with low inflation and a sharp decline in interest rates in many other countries in the past two decades, have brought the potential threat of deflation and a binding zero lower bound on nominal interest rates into focus. In the case of Japan, the overnight call rate, which is the policy instrument for the Bank of Japan, has been below 50 basis points since mid 1995, accompanied by economic stagnancy and deflationary pressure (see Figure 1). Such "zero interest rates" have not been observed in the United States and most other developed economies. However, with the recent phenomenon of low inflation, the proposition that the effectiveness of monetary policy could be severely reduced by a binding zero bound constraint on nominal interest rates no longer seems far-fetched.

These developments have given rise to a renewed interest in the implications of the zero lower bound for monetary policy. While most of the recent studies on this issue have relied on simulations of macroeconometric models, this paper will use the data on Japanese economy, which has experienced more than 5 years of "zero interest rates," to obtain empirical estimates of monetary policy effects when the zero bound constraint on nominal interest rates is binding and to investigate the extent to which such a zero bound might affect the ability of a central bank to conduct its policy. These empirical results will be useful in evaluating different policy options for the Japanese economy and allow us to draw lessons for other countries regarding the impact of the zero bound on monetary policy.

Estimating the effects of monetary policy under non-negativity constraints can also help us evaluate empirically the relative importance of different monetary transmission mechanisms. Many monetarist economists [e.g. Meltzer (1995)] have emphasized the importance of the monetary transmission mechanism operating through other asset prices. They argue that market interest rates are only one of the relative prices affected by monetary impulses. A monetary impulse that alters the nominal and real stocks of money changes actual and anticipated prices on a variety of domestic and foreign assets, which in turn may affect investment and consumer spending through Tobin's q theory of investment and the wealth effect on consumption.

Some economists also view frictions in the credit markets due to asymmetric information as playing an important role in the process of monetary transmission [e.g. Bernanke and Gertler (1995)]. They argue that either through the balance sheet channel or the bank lending channel, a change in monetary policy can have an additional (and significant) impact on investment and consumer spending, and hence affects the aggregate demand and output.

The responses of the economy to monetary impulses most likely reflect the joint effect of different monetary transmission mechanisms, whose individual impact might be difficult to identify empirically. Nevertheless, a binding zero bound constraint provides us with an excellent opportunity to isolate the impact of monetary policy operating through the channels other than interest rates. By comparing the responses of the economy to a monetary impulse under the zero interest rate with the responses when the interest rate is strictly positive, it is possible to obtain an assessment of the importance of the interest rate channel.

Our study is related to a recent literature on the zero lower bound on nominal interest rates, which has two strands. A first strand of the literature focuses on the theoretical issue of how to avoid the zero bound and a liquidity trap and how to escape from them if trapped, usually with specific references to Japan. While many agree on how to avoid them, a variety of ways to get out of a liquidity trap are proposed, emphasizing different channels of monetary transmission [e.g Buiter and Panigirtzoglou (1999), Christiano (1999), Krugman (1998), McCallum (2000a) and Svensson (2000) among others].

A second strand, which is closest to our study, is based on simulations of macroeconometric models. Using structural models of the U.S. economy, several authors have numerically examined through simulations the extent to which the zero bound on nominal interest rates prevents real rates from falling and hence affects the central bank's ability to optimally respond to adverse macroeconomic shocks. Fuhrer and Madigan (1997) and Orphanides and Wieland (1998), Reifschneider and Williams (2000) find that monetary policy is significantly constrained by the zero bound in a policy regime with a low inflation target. Similar studies based on different models are conducted by Rotemberg and Woodford (1997) and Wolman (1998) with different conclusions about the importance of the zero bound as a constraint on monetary policy. The main objective of these studies is the normative implications for monetary policy of the zero lower bound on nominal interest rates. They try to evaluate "whether the zero bound introduce distortions that make low inflation undesirable". While Reifschneider and Williams (2000) consider together with the interest rate channel other channels of monetary transmission in their paper, most of the studies in this literature have focused mainly on the interest rate channel of monetary transmission.

In contrast, based on direct empirical evidence on macroeconomic performance in a "zero interest rates" environment using a structural VAR approach, we seek to understand to what extent the zero bound may affect a central bank's policy power if we allow for other channels of monetary transmission. Moreover, through state dependent impulse-response functions, we try to gain some insight into the relative importance of different monetary transmission channels. The paper also has a technical contribution to the structural VAR literature. It introduces a type of nonlinearity into a standard VAR model by incorporating a censored left hand variable and switching impact multipliers.

The rest of the paper is organized in the following way. Section 2 describes the data used in this study. We discuss our econometric strategy in section 3. Section 4 presents the main results and section 5 concludes.

2 The Data

Since the collapse of a speculative asset price bubble in early 1990, Japan has suffered a prolonged period of deflation and economic stagnancy. In Figure 1 we plot the monthly indices of industrial production and overall wholesale price together with the inter-bank call rates in Japan for the last 10 years. It can be seen clearly from the figure that output has failed to grow during the past decade while price level has been continuously declining (see Figure 1, both output and price are measured on the left scale). Even though output seems to pick up a little bit near the end of 2000, it is still well below the historical trend. Such economic distress has prompted the Bank of Japan to adopt an expansionary monetary policy by lowering nominal interest rates. By September 1995, the inter-bank call rate, which has been the policy instrument for the Bank of Japan, was pushed down to below 50 basis points and remained at that low level until the end of 2000 (Figure 1, measured on the right scale). The experience of Japanese monetary policy during this period therefore provides a good opportunity to study the impact on monetary policy of a zero bound constraint on nominal interest rates.

In this study, we use a monthly data set over the period from January 1991 to December 2000. The variables include the Japanese wholesale price index, an index of industrial production, the inter-bank overnight call rate, the Nikkei stock market index and the yen/dollar exchange rate. Data on all variables except industrial production are obtained from the Bank of Japan. Data on industrial production are obtained from OECD and the ministry of international trade and industry (MITI) of Japan. A reason for our focus on the time period between 1991 and 2000 is that there appear to be several structural changes in Japanese monetary policy during the past 30 years. For example, in the second half of 1980s, stabilizing the exchange rate seemed to be the main policy goal for the Bank of Japan due to the Louvre Accord. Moreover, the dramatic rise in asset prices starting in late 1980s made the Bank of Japan to target asset prices in conducting its monetary policy. See Hertzel (1999) for a discussion of Japanese monetary policy since 1970s. After the burst of asset price bubble in 1990, however, the main concern of the Bank of Japan is to deal with deflation and to revive domestic economic activity, and there doesn't seem to be any major structural change in its policy.

3 The Model

3.1 VAR with a censored variable

To examine the effect of monetary policy shocks on the economy when non-negative constraint on nominal interest rates is possibly binding, the standard VAR procedure is not appropriate. There are technical as well as conceptual issues in this problem. To address both issues, we will first consider the technical aspect of the problem and propose a solution. Next, we will discuss the conceptual implications of our proposed model.

From a technical point of view, the problem with a standard VAR model is the non-negativity constraint on one of the left hand side variables. A conventional solution is log transformation of the variable. In the case of a nominal interest rate, this treatment would be undesirable, however, for two reasons. First, the interest rate would become less sensitive to negative policy shocks than positive ones, which does not appear to be a realistic description of what we observe. Second, it would lose nice linear interpretation of monetary policy rules (as in the Taylor rule). A more natural way to handle this problem is to distinguish the level of the nominal interest rate implied by the model (or intended by the monetary authority) from the actual level of that rate. They are equal to each other only when the former is greater than zero. In other words, we treat the actually observed nominal rate as a censored variable on the left hand side of an equation representing the monetary reaction function. A visual inspection of the actual rates in Figure 1 also supports the view that the censoring started around 1995, when the trend apparently hits the lower bound. Formally, let R_t be the observed interest rate and R_t^* be the implied (or intended) rate that is not directly observable:

$$R_t = \begin{cases} R_t^* & \text{if } R_t^* \geq c \\ c & \text{otherwise} \end{cases} \quad (1)$$

where c is a small positive number, at which the the nominal interest rate is regarded as essentially zero. We will further discuss what value is the most appropriate for c below.

We now consider the conceptual side of the problem. In a standard VAR analysis, monetary policy is usually described by a policy reaction function:

$$S_t = f(\Omega_t) + \varepsilon_t^s \qquad (2)$$

where S_t is a policy instrument or operating target [see McCallum (1997) for a discussion of related conceptual issues about monetary policy rules], f is a function representing the monetary authority's systematic feedback rule, Ω_t is the monetary authority's information set, and ε_t^s is an exogenous monetary shock due to, possibly, discretionary policy actions. This equation is interpreted as describing a mechanism through which the monetary authority takes actions to guide the policy instrument variable (or operating target) to the level desired by the feedback rule and any discretionary considerations. In many countries, including the United States and Japan, a short term nominal interest rate (the Federal Funds rate in the U.S. and the inter-bank call rate in Japan) is used as the policy instrument. The caveat, however, is that once the interest rate hits the zero bound, the monetary authority can no longer push it any lower even if a negative operating target is desired. Many economists have hence suggested that the monetary authority in such a case should use some other variables like the monetary base as a new policy instrument.

In the case of Japan, however, there is no indication that the central bank has adopted an alternative instrument in its conduct of monetary policy even when the nominal inter-bank call rate is constrained by the zero lower bound. Rather, it appears that the Bank of Japan (BOJ)has followed a policy rule described by (1) with R_t^* being set according to (2). This can be seen from various speeches made by the BOJ's officials: "we (the BOJ) will continue the zero interest rate policy until we reach a situation where deflationary concerns are dispelled." In other words, the BOJ not only continues to use the short-term nominal interest rate as its policy instrument in the presence of a binding zero bound constraint, but also continues to set its operating target in a feedback fashion. When the zero lower bound constraint is not binding, the central bank just set its policy instrument at the level determined by the systematic feedback rule and any discretionary (random) policy decision; otherwise, the central bank will guide the policy instrument to its lowest possible level.

Therefore in this kind of policy environment, we can interpret the exogenous variations in the intended operating target $\varepsilon_t^{s*} \equiv R_t^* - f(\Omega_t)$ as monetary policy shocks. When $R_t > c$, such policy shocks will be fully reflected in the exogenous variations in the policy instrument as in the standard case. However, when $R_t = c$, ε_t^{s*} will not generate the corresponding movement in the policy instrument because of the binding constraint. Nevertheless these shocks still lead to fluctuations in the intended operating target R_t^* and their impact on other macroeconomic variables can be estimated.

Of course, there is an issue about what value is the most appropriate for c. In general, the call rate cannot become exactly zero because of the existence of various transaction costs.[1] Those costs add up to 2-3 basis points. But the choice of the value for c should be made to best describe the BOJ's policy rule. The BOJ has set the uncollateralized overnight call rate guideline at 0.50% for 1995-1998 and at 0.25% after September 1998. Between February 1999 and July 2000, this lower bound was further pushed down to about 0.02-0.03%. It therefore appears to be a good approximation to the actual policy behavior to model the rate as being censored or equivalently $R_t^* \leq R_t$ as long as the actual rate R_t is less than 50 basis points. A glance at the plot of the actual rates in Figure 1 appears to suggest that a regime change had occured in 1995 when the nominal rate hit 50 basis points. Moreover, it is also supported by some economists including Krugman (1997), who believe that at a nominal rate of 0.43% "the economy is clearly in a very good approximation to liquidity trap conditions." Accordingly, throughout this paper, we use the terms such as 'zero interest rate' or 'zero bound' even when the actual lower bound is not necessarily equal to absolute zero. [2]

We now introduce a small VAR system. The system consists of three groups of variables: the first group includes standard macroeconomic variables such as output and price, the second group contains an indicator of monetary policy stance such as the overnight call rate, and the last group includes some broad financial market variables such as stock market indices and foreign exchange rates. These are the variables that can potentially play important roles in the monetary transmission process, particularly when the nominal interest rate gets stuck at its zero bound. Denote three groups of variables mentioned above by $\mathbf{Y}_t, R_t^*, \mathbf{X}_t$, respectively, where $\mathbf{Y}_t, R_t^*$ and $\mathbf{X}_t$ are $k \times 1, 1 \times 1$ and $n \times 1$ vectors respectively, and $m = k + 1 + n$. The VAR

[1] Okina and Oda (2000) discuss the details of various transaction costs for the overnight call rate in Japan.

[2] There is also a technical concern for the choice of the lower bound c. Throughout the sample covering the period of 1990 - 2000, there are only a few observations with the call rate being around 2 or 3 basis points, hence make it impossible to get sensible estimates of policy impact under such a circumstance.

system is then given by

$$\begin{bmatrix} \mathbf{Y}_t \\ R_t^* \\ \mathbf{X}_t \end{bmatrix} = \mathbf{B}(L) \begin{bmatrix} \mathbf{Y}_t \\ R_t \\ \mathbf{X}_t \end{bmatrix} + \mu + \mathbf{u}_t \tag{3}$$

where $\mathbf{B}(L) = \mathbf{B}_1 L - \cdots - \mathbf{B}_p L^p$, and μ is a vector of constants. The $\mathbf{u}_t$ stands for a vector of one-step-ahead forecast errors and we assume that $\mathbf{u_t} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ where Σ is a symmetric positive definite matrix. It is important to note that in model equation (3), the nominal rate on the left hand side of the equation is the implied rate R_t^* that is not always observable, while the rate on the right hand side is the actual rate R_t. This specific feature makes our model to exhibit interesting nonlinear dynamics.

3.2 Identification

Equation (3) is a reduced form of the model, while the structural form is given by:

$$\mathbf{A}_0 \mathbf{Z}_t^* = \mathbf{A}(L)\mathbf{Z}_t + \mathbf{A}_0 \mu + \varepsilon_t \tag{4}$$

where $\mathbf{Z}_t^* = [\mathbf{Y}'_t, R_t^*, \mathbf{X}'_t]'$, $\mathbf{Z}_t = [\mathbf{Y}'_t, R_t, \mathbf{X}'_t]'$ and $\varepsilon_t = [\varepsilon_t^{Y}{}', \varepsilon_t^M, \varepsilon_t^{X}{}']'$ stands for the fundamental shocks. We assume that $\varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_m)$.

We impose the following block recursive restrictions to identify the model. First, we assume the call rate does not directly affect macroeconomic variables such as output and price, since the rate applies to only the overnight transactions among the commercial bank reserves. Second, we assume that other financial variables do not contemporaneously affect macroeconomic variables either. These two sets of identification restrictions are quite standard in the literature [e.g. Christiano et al (1999)] especially when monthly data are used. Third, we assume that the policy maker does not respond to current financial variables $\mathbf{X}_t$ when the instrument R_t^* is set, where $\mathbf{X}_t$ includes an aggregate stock price index and a foreign exchange rate. We believe that this is a reasonable assumption. Because since the burst of the asset price bubble in 1990, the focus of Japanese monetary policy has shifted to fighting deflation and the economic slump. Even if one believes that the monetary authority does not completely ignore asset prices when setting its policy instrument, at least it seems safe to say that it is no longer a systematic policy for the Bank of Japan to respond to stock prices or some other financial variables contemporaneously since 1990.

The above three assumptions imply that the matrix $\mathbf{A}_0$ is block triangular. Rewrite (4) as

$$\mathbf{Z}_{\mathbf{t}}^{*} = \mathbf{B}(L)\mathbf{Z}_t + \mu + \mathbf{C}_{\mathbf{0}}\varepsilon_{\mathbf{t}} \tag{5}$$

where $\mathbf{C}_0$ is the matrix of the impact multipliers. Since $\mathbf{C}_0 = \mathbf{A}_0^{-1}$, the matrix $\mathbf{C}_0$ is also block triangular

$$\mathbf{C}_0 = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_{21} & \mathbf{C}_{22} & \mathbf{0} \\ \mathbf{C}_{31} & \mathbf{C}_{32} & \mathbf{C}_{33} \end{bmatrix}. \tag{6}$$

We further allows the possibility that when the nominal interest rate is zero, the financial variables in $\mathbf{X}_t$ respond differently to a monetary policy shock. This is simply due to the fact when the nominal interest rate is zero, bonds and money become perfect substitutes, which may in turn lead to different reactions of the financial variables to monetary shocks. This situation is described by a threshold switching mechanism given by.

$$\mathbf{C}_{32} = \begin{cases} \mathbf{C}_{32}^{+} & \text{if } R_t^{*} \geq c \\ \mathbf{C}_{32}^{0} & \text{otherwise} \end{cases} \tag{7}$$

where $\mathbf{C}_{32}^{0} = \mathbf{C}_{32}^{+} + \mathbf{q}$. The $n \times 1$ dummy vector $\mathbf{q}$ measures potentially different reactions of $\mathbf{X}_t$ to monetary shocks when the zero bound constraint on the interest rate is binding.

Our primary interest is in the dynamic responses of $\mathbf{Z}_{t+h}$ to the monetary policy shock ε_t^M, for which the identifying restrictions assumed above are sufficient. In other words, $\mathbf{C}_{22}$ and $\mathbf{C}_{32}$ are identified without any additional restrictions. The model (5) subject to (1), (6) and (7) may be estimated by the maximum likelihood method applied to the whole system. Since the model is not fully identified, an additional zero restriction is needed on one element of $\mathbf{C}_{11}$. The $\mathbf{C}_{22}$ and $\mathbf{C}_{32}$ are invariant to this restriction. The derivation of the likelihood function is provided in the Appendix.

Unlike in the linear case, the impulse response function (IRF) will be history- and shock-dependent [Potter (2000), Koop, Pesaran, and Potter (1996)]. This feature of the model allows us to investigate the effect of a monetary policy shock under a zero or a positive interest rate, an issue we will discuss in more detail in the following sections.

4 Results

We estimate the nonlinear structural VAR model with the data on Japanese wholesale price index, industrial output, the inter-bank call rate, the Nikkei stock price index and the yen/dollar exchange rate (denoted by $\mathbf{Z} = (\mathbf{p}, \mathbf{y}, \mathbf{R}, \mathbf{s}, \mathbf{x})$) over the period between January 1991 to December 2000.[3] Since our primary interest is in the dynamic responses of variables, we do not report the direct estimates of the VAR parameters here but only mention some features of the estimated model. First, Table 1 shows that the signs of the estimates of $\mathbf{C_{21}}$ and $\mathbf{C_{22}}$ are consistent with the counter-cyclical monetary policy pursued by the Bank of Japan during that period. Namely, the Bank of Japan will take expansionary policy actions by cutting the interest rate when facing a deflationary shock or a negative shock to the output.

Table 1: Estimates of Policy Reactions

$\mathbf{C_{21}^{(1)}}$	$\mathbf{C_{21}^{(2)}}$	$\mathbf{C_{22}}$
.0275	.0841	.2761
(.0348)	(.039)	(.0231)

(Note: $\mathbf{C_{21}}$ is a 2×1 vector, its first element $\mathbf{C_{21}^{(1)}}$ measures the response of R_t^* to an inflationary shock and its second element $\mathbf{C_{21}^{(2)}}$ measures the response of R_t^* to a positive shock to output. $\mathbf{C_{22}}$ measures the response of R_t^* to a monetary contraction shock. The numbers in parentheses are the standard errors.)

Second, nonlinearity is an important feature of the data because of the censorship and the different policy impacts when the zero bound constraint on the interest rate is binding. Figure 2 displays the intended operating targets based on our VAR estimates together with the actually observed call rates. We can see not only that all the intended operating targets lie below the actual interest rates for the whole period during which the zero bound constraint appears to be binding, but also that there are large fluctuations in the intended operating targets, indicating active policy movements during that period even the actual interest rate rarely moves. Note that it should not be surprising that the estimates of the intended operating targets are not far below the lower bound c of the call rate, given the BOJ's "zero interest rate policy". Moreover, the estimates of the coefficients on the threshold

[3] A 4-variable VAR excluding the yen/dollar exchange rate, that is $\mathbf{Z} = (\mathbf{p}, \mathbf{y}, \mathbf{R}, \mathbf{s})$, is also estimated. The results are very similar to those from the 5-variable VAR.

dummies in Table 2 clearly show that the financial variables react differently to a monetary shock when the zero bound constraint is binding. Following an expansionary policy shock, there is more substantial depreciation of the domestic currency when the zero bound constraint is binding than otherwise. The impact of the same policy shock on the aggregate stock price seems to be reduced by a little bit due to the binding constraint on the interest rate, though the difference is not significant. These results also seem to confirm the proposition that a monetary expansion or contraction changes the relative prices of a variety of assets, not just the short term nominal interest rate.

Table 2: Different Impacts on Financial Variables

$\mathbf{q^{(1)}}$	$\mathbf{q^{(2)}}$
.0007	-.0157
(.0135)	(.0062)

(Notes: The parameters $q^{(1)}$ and $q^{(2)}$ measure the additional impacts of a monetary shock on the aggregate stock price index and the yen/dollar exchange rate, respectively, when the zero bound constraint is binding. The numbers in parentheses are the standard errors.)

Based on the estimated VAR model, we now examine the dynamic responses of output, price, and other variables to an expansionary monetary policy shock when the zero bound constraint is binding and when it is not. Our interest centers around the following two questions: (i) How much of the effect of an expansionary monetary policy shock on output is actually eliminated by the zero bound constraint on nominal interest rates? (ii) How important is the interest rate channel compared with other channels of monetary transmission?

The impulse response function (IRF) is often obtained by the difference of the h-steps-ahead forecast of the series with a current shock of a unit size from that with a zero shock (the baseline case). In a linear time series, this difference reduces to the h-th order parameters in its moving-average (MA) representation. In a vector autoregression with a censored left hand variable, however, the MA representation is no longer linear in the shocks. As a result, the IRF for the nonlinear model is dependent upon the entire past history of the series as well as the size and direction of the shock. This state-dependent feature of the IRF allows us to analyze the policy effects conditional on the current state of the system.

We will follow the literature on nonlinear impulse response [Koop et al (1996), Gallant et al (1993), and Potter (2000)] and treat a nonlinear IRF as the difference between a pair of conditional expectations of the variables given a non-zero shock and a zero shock at the current period, i.e.

$$E(\mathbf{Z}_{t+h}|\Omega_{t-1}, \varepsilon_t) - E(\mathbf{Z}_{t+h}|\Omega_{t-1})$$

where Ω_{t-1} stands for the information set at $t-1$, and $h = 1, 2, \cdots$ is a time horizon. In other words, to calculate a nonlinear IRF, we have to specify the nature of the shock (its size and sign) and the initial condition, Ω_{t-1}. To calculate the conditional expectations, we simulate the model in the following manner. First, we randomly draw ε_{t+j} from $\mathcal{N}(\mathbf{0}, \mathbf{I_m})$ for $j = 1, 2, \cdots, h$ and then simulate the model conditional on an initial condition Ω_{t-1} and a particular shock ε_t. This process is repeated 500 times and the estimated conditional expectation is obtained as the average of the outcomes.

4.1 The effects of monetary policy shocks when the interest rate is zero

Figures 3(a) - 3(e) display the estimated IRFs of the variables included in our 5-variable nonlinear VAR. The solid and broken lines stand for, respectively, the IRFs of the variables to an expansionary monetary policy shock of size one standard deviation when the zero bound constraint is binding and when it is not, and the horizontal axis measures the number of months after the shock. The IRF in each regime is calculated as an average of all the IRFs corresponding to the historical dates belonging to each of the two regimes: the regime where the call rate is stuck at zero and the regime where the interest rate is positive and the zero bound constraint is not binding.

The dynamic responses of output to a monetary shock under the two regimes show a striking difference. When the interest rate is above its lower bound, output rises sharply with little delay and reaches the peak in 2-3 months after an expansionary monetary policy shock. The impact of the initial shock disappears in about 18 months. In contrast, when the zero bound constraint is binding, we have a slower output response and much smaller impact of the shock. The increase in output is only about one half of the increase when the zero bound constraint is not binding. Output even initially declines and then rises to its peak in 4 months after the shock. The above pattern of the response of output is common in both 4-variable and

5-variable VARs (see Figure 4(b) for IRF of the 4-variable VAR). When we exclude the exchange rate from the system, output reaches its peak after about 10 months following a shock and the increase in output is only about one fourth when the zero bound constraint is binding.

The response of the price index to the monetary shock is also strikingly different under two regimes (Figure 3(a)). When the zero bound constraint is not binding, the price level rises moderately with some delay and shows a pattern similar to the "price puzzle"[4] frequently observed in the standard VAR studies using US data. When the zero bound constraint is binding, however, the price index responds more strongly and there is no such a puzzle.

In summary, an exogenous expansionary monetary shock results in a large increase in output level and a small price increase under the positive interest rate regime, while the same shock leads to a much smaller and delayed output increase together with a larger price increase under the zero interest rate regime. This sharp contrast of the output-price responses under the two regimes is consistent with the standard textbook explanation of the interest rate channel of monetary transmission. When the interest rate is positive and above its zero bound, it tends to fall sharply following an expansionary monetary shock as can be seen in Figure 3(c). Accordingly, even when the immediate price reaction is not large (or even negative), the real interest rate falls substantially, which leads to a strong rise in output. When the zero bound constraint is binding, however, the nominal interest rate cannot fall and is stuck at zero following the shock. A monetary expansion only leads to a higher price level initially. But as price level increases and the nominal interest rate remains unchanged, the real interest rate starts to decline and the resulting lower real interest rate may have a positive effect on output. Nevertheless, the impact of the original shock appears much smaller.

Some economists [e.g. Meltzer (1995)] argue that a monetary impulse not only changes a single short-term interest rate, but also alters the relative prices of a variety of assets. Indeed, Figure 3(d) shows an increase in the stock price index following the monetary shock even when the nominal interest rate remains at zero. Such an increase in asset prices can have positive influence on output either through the wealth effect on consumption or

[4] The "price puzzle" refers to the initial negative response of price level to an expansionary monetary policy shock in the monetary VAR literature.

through a mechanism involving Tobin's q theory of investment, or magnify the interest rate channel impact through a credit channel effect [Bernanke and Gertler (1995)]. The impact of a monetary impulse on asset prices is, however, found to be smaller when the interest rate is constrained at its zero bound.

Movements in foreign exchange rates is another channel through which a monetary shock can affect output in an open economy such as Japan. Figure 3(e)shows that the domestic currency depreciates substantially following a monetary expansion even when the nominal interest rate is constrained at its zero bound. The lower value of the domestic currency makes the domestic goods more competitive in the international market and hence tends to expand output. The sharp depreciation of the domestic currency may also explain the strong positive response of the price level following the shock. This is because the Japanese whole sale price index used in this study is a weighted average of domestic whole sale prices and import prices. A depreciation of the domestic currency would therefore put an inflationary pressure on the price index. It is interesting to notice that, when the interest rate is above its zero bound, the response of the exchange rate to a monetary shock exhibits a pattern similar to the "forward premium puzzle" observed in the international finance literature. When a monetary expansion lowers the domestic interest rate, we would normally expect the domestic currency to appreciate in the future according to the open interest rate parity. However, we observe here instead an instantaneous appreciation followed by persistent depreciations of the currency. Such deviations from uncovered interest rate parity are also reported in Eichenbaum and Evans (1995), in which they examined the response of the exchange rates to U.S. monetary policy shocks.

Even if we allow for additional channels through which a monetary impulse can affect the real economy, however, the zero bound constraint eliminates more than 50% of the impact of an exogenous monetary shock on real output, as is observed in Figure 3(a). A monetary expansion tends to be more inflationary and less effective in raising output when the zero bound constraint is binding than otherwise. Moreover, if we take the difference of the IRFs as a measure of the relative importance of the interest rate channel, our results appear to confirm that the interest rate channel is the most important mechanism of monetary transmission.

4.2 The impact of the lower bound constraint on the effectiveness of monetary policy

Some recent studies have investigated, in a slightly different way, the effect of the zero bound constraint on the ability of monetary authority to conduct effective policy [e.g Fuhrer and Madigan (1996), Orphanides and Wieland (1998) among others]. Based on structural models of the U.S. economy, these studies try to find, through numerical simulation, the extent to which a zero bound on nominal interest rates prevents the monetary authority from pursuing a counter cyclical interest-rate policy in response to negative macroeconomic shocks. An analogous exercise in the current context would be to subject the VAR to some adverse macroeconomic shocks and see how the system would respond conditional on whether or not the zero bound constraint on the nominal interest rate is binding.

To conduct this type of exercise, we draw a macroeconomic shock that, when hitting the economy, would generate an output decline for two consecutive quarters (the standard definition of a recession) if the zero bound constraint is not binding. Such a shock can be any mixture of inflation, output, and monetary shocks. We then subject the VAR to the same shock conditional on a binding zero bound on the nominal interest rate. Figures 5(a) - 5(b) display the IRFs of the interest rate and the output respectively. In a normal situation where the interest rate is not constrained at its zero bound, such a negative shock will drive down output as well as interest rates as the monetary authority pursues a counter cyclical policy. The resulting lower interest rate would eventually push the economy out of the recession. In contrast, if the zero bound constraint on the interest rate is binding, the monetary policy would lose its leverage against such a negative shock. Figures 5(a) and 5(b) show that the interest rate cannot move and is stuck at zero, while the output decline is about 50% deeper and it takes longer for the economy to get out of the recession. If the interest rate is above its zero bound when the negative shock hits the economy, output will go back to its original level in about 12 months. But if the zero bound is binding when the shock hits the economy, it would take about 18 months for the economy to be fully recovered.

We may also look at the influence of the zero bound constraint in a different way by postulating a hypothetical situation in which the zero bound constraint on the interest rate is entirely removed. How differently would the economy evolve after 2001 under stochastic macroeconomic shocks with and without the zero bound constraint? We simulates the estimated VAR starting from the last sample period, and compare the dynamics of output and the interest rate. Figures 6(a) - 6(b) display the results from such an exercise. We can see that if there were no zero lower bound constraint, the interest rate would become significantly negative, which in turn stimulates the economy and the output starts to grow. In contrast, in the presence of the zero bound constraint, the interest rate would be stuck at zero and output remains stagnant. Of course, one major caveat of such a comparison is that the model parameters are estimated while imposing the zero bound constraint. Nevertheless, the large discrepancy of the two output series found in Figure 6(b) confirms that the zero bound could have a significant impact on the macro economic performance of the economy.

5 Concluding remarks

In this paper we estimated the effect of an exogenous monetary shock in the presence of the zero lower bound constraint on nominal interest rates and examined the impacts of the zero bound constraint on the effectiveness of a counter-cyclical monetary policy. Applying a nonlinear VAR model to Japanese data, we found that when the zero bound constraint on nominal interest rates is binding, more than 50% of the impact of an exogenous monetary shock on output is eliminated while more inflationary pressure is posed. The conditional IRFs allow us to isolate the impact of monetary shocks operating through the interest rate channel when other possible channels of monetary transmission are present. It is found that (i) an exogenous monetary shock may still have a significant effect on the real economy through the channels other than the interest rate channel when nominal interest rates are constrained at the zero bound, (ii) it is the interest rate channel that appears to be the most important mechanism of monetary transmission. Moreover, consistent with our previous results, we also found that the presence of the zero bound on nominal interest rates could severely limit the ability of central banks to pursue a counter-cyclical interest rate policy when facing adverse macroeconomic shocks.

While there are many recent studies trying to evaluate the extent to which the zero bound on nominal interest rates interferes with the conduct of monetary policy by simulating structural models of the U.S. economy, those quantitative results are inevitably model specific and often lack direct empirical support. The low interest rates and the apparent presence of the liquidity trap in Japan during the past decade make it possible to address such issues empirically using a nonlinear structural VAR. This approach also allows us to separate the effects of monetary policy shocks operating through the interest rate channel from the effects from other channels and give a quantitative evaluation of the importance of the interest rate channel relative to other channels of monetary transmission.

This paper also bears some direct policy implications with regard to the situation in Japan. It is often debated that whether or not the Bank of Japan should conduct further monetary easing given the stagnant domestic economy and a binding zero bound constraint on its policy instrument. Our results suggest that further monetary easing either through normal money market operations or outright purchases of government bonds would probably have a limited impact on the real economy.

Appendix

Derivation of the likelihood function:

We first rearrange the order of the variables in $\mathbf{Z}_t^*$. Define

$$\mathbf{J} = \begin{bmatrix} \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{I}_k & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_n \end{bmatrix}$$

so that $\mathbf{J}'\mathbf{J} = \mathbf{I}_m$. Rewrite (1) as

$$\mathbf{J}\mathbf{B}(L)\mathbf{J}'\mathbf{J}\mathbf{Z}_t^* = \mathbf{J}\mu + \mathbf{J}\mathbf{C}_0\mathbf{J}'\mathbf{J}\varepsilon_t$$

or

$$\widetilde{\mathbf{B}}(L)\widetilde{\mathbf{Z}}_t^* = \widetilde{\mu} + \widetilde{\mathbf{C}}_0\widetilde{\varepsilon}_t$$

where $\widetilde{\mathbf{B}}(L) = \mathbf{J}\mathbf{B}(L)\mathbf{J}'$, $\widetilde{\mathbf{C}}_0 = \mathbf{J}\mathbf{C}_0\mathbf{J}'$, $\widetilde{\mu} = \mathbf{J}\mu$, $\widetilde{\mathbf{Z}}_t^* = [R_t^*, [\mathbf{Y}'_t, \mathbf{X}'_t]]' = [Z_{1t}^*, \mathbf{Z}_{2t}^{*\prime}]'$ and $\widetilde{\varepsilon}_t = [\varepsilon_t^M, \varepsilon_t^{Y\prime}, \varepsilon_t^{X\prime}]' \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_m)$.

Write $\widetilde{\mathbf{B}} = [\widetilde{\mathbf{B}}_1, \cdots, \widetilde{\mathbf{B}}_p, \widetilde{\mu}]$ and $\widetilde{\mathbf{Z}}^{*t} = [\widetilde{\mathbf{Z}}_{t-1}^{*\prime}, \cdots, \widetilde{\mathbf{Z}}_{t-p}^{*\prime}, 1]'$. Then we have

$$\widetilde{\mathbf{Z}}_t^* = \widetilde{\mathbf{B}}\widetilde{\mathbf{Z}}^{*t} + \widetilde{\mathbf{u}}_t$$

with $E(\widetilde{\mathbf{u}}_t\widetilde{\mathbf{u}}_t') = \mathbf{\Sigma} = \begin{bmatrix} \widetilde{\sigma}_{11} & \widetilde{\mathbf{\Sigma}}_{12} \\ \widetilde{\mathbf{\Sigma}}_{21} & \widetilde{\mathbf{\Sigma}}_{22} \end{bmatrix} = \widetilde{\mathbf{C}}_0\widetilde{\mathbf{C}}_0' = \mathbf{J}\mathbf{C}_0\mathbf{C}_0'\mathbf{J}'$.

The likelihood function conditional on $(\widetilde{\mathbf{Z}}_0, \cdots, \widetilde{\mathbf{Z}}_{1-p})$ is given by

$$\begin{aligned} L &= \prod_{R_t^* \geq c} f(\widetilde{Z}_{1t}, \widetilde{\mathbf{Z}}_{2t}) \prod_{R_t^* < c} \int_{-\infty}^{c} f(\widetilde{Z}_{1t}, \widetilde{\mathbf{Z}}_{2t}) d\widetilde{Z}_{1t} \\ &= \prod_{R_t^* \geq c} f(\widetilde{Z}_{1t}, \widetilde{\mathbf{Z}}_{2t}) \prod_{R_t^* < c} f(\widetilde{\mathbf{Z}}_{2t}) \int_{-\infty}^{c} f(\widetilde{Z}_{1t} | \widetilde{\mathbf{Z}}_{2t}) d\widetilde{Z}_{1t} \end{aligned}$$

Noting that $\widetilde{u}_{1t} = \widetilde{\mathbf{\Sigma}}_{12}\widetilde{\mathbf{\Sigma}}_{22}^{-1}\widetilde{\mathbf{u}}_{2t} + e_t$ where $e_t \sim \mathcal{N}(0, \widetilde{\sigma}_{11\cdot 2}^2)$ with $\widetilde{\sigma}_{11\cdot 2}^2 = \widetilde{\sigma}_{11}^2 - \widetilde{\mathbf{\Sigma}}_{12}\widetilde{\mathbf{\Sigma}}_{22}^{-1}\widetilde{\mathbf{\Sigma}}_{21}$, we find

$$\widetilde{Z}_{1t} | \widetilde{\mathbf{Z}}_{2t} \sim \mathcal{N}(\mu_{1\cdot 2}, \widetilde{\sigma}_{11\cdot 2}^2)$$

where $\mu_{1\cdot 2} = (\widetilde{\mathbf{B}}_1 - \widetilde{\mathbf{\Sigma}}_{12}\widetilde{\mathbf{\Sigma}}_{22}^{-1}\widetilde{\mathbf{B}}_2)\widetilde{\mathbf{Z}}^t + \widetilde{\mathbf{\Sigma}}_{12}\widetilde{\mathbf{\Sigma}}_{22}^{-1}\widetilde{\mathbf{Z}}_{2t}$ and $\widetilde{\mathbf{B}} = [\widetilde{\mathbf{B}}_1', \widetilde{\mathbf{B}}_2']'$. Hence, the log likelihood function takes the form as

$$\begin{aligned}\ln L \propto & -(T_1/2)\ln|\widetilde{\mathbf{C}}_0\widetilde{\mathbf{C}}_0'| \\ & -(1/2)\sum_{R_t>c}(\widetilde{\mathbf{Z}}_t - \widetilde{\mathbf{B}}\widetilde{\mathbf{Z}}^t)'(\widetilde{\mathbf{C}}_0\widetilde{\mathbf{C}}_0')^{-1}(\widetilde{\mathbf{Z}}_t - \widetilde{\mathbf{B}}\widetilde{\mathbf{Z}}^t) - (T/2)\ln|\widetilde{\mathbf{\Sigma}}_{22}| \\ & -(1/2)\sum_{R_t=c}(\widetilde{\mathbf{Z}}_{2t} - \widetilde{\mathbf{B}}_2\widetilde{\mathbf{Z}}^t)'(\widetilde{\mathbf{\Sigma}}_{22}^0)^{-1}(\widetilde{\mathbf{Z}}_{2t} - \widetilde{\mathbf{B}}_2\widetilde{\mathbf{Z}}^t) + \sum_{R_t=c}\ln\Phi\left(\frac{c-\mu_{1\cdot 2}^0}{\widetilde{\sigma}_{11\cdot 2}^0}\right)\end{aligned}$$

where T_1 stands for the number of observations for which $R_t > c$.

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