Structural Inference With Long-run Recursive Empirical Models

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Abstract: This paper investigates conditions under which empirical models that use long-run recursive identifying assumptions will obtain structural impulse response functions. I present a class of structures defined as long-run partially recursive. If an economic structure falls into this class, then certain long-run recursive empirical models are able to identify some of the structural responses. This sufficient condition is first shown in a vector autoregression. A well-known example from the literature is used to illustrate this type of structure and some applications of the result. Then the result is shown in models of cointegrated time series. Necessary conditions for a long-run recursive model to identify structure are addressed in the conclusion.

Keywords: long-run multiplier, long-run partially recursive structure, moving average representation, vector autoregression, cointegration

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1. Introduction

Recursive models have a long history in empirical macroeconomic research.[1] More recently Sims (1980) began using these models to identify vector autoregressions. The coefficients for this short-run recursive system are obtained by the Cholesky decomposition of the covariance matrix for VAR innovations. A practical benefit of recursive models is that, in general, parameters are identifiable. However, Cooley and LeRoy (1985) and others have criticized Sims's approach for being "atheoretical". In response to this criticism, economists have devised various structural approaches to VAR modeling.

One such approach identifies economic structure using long-run restrictions derived from the steady-state properties of theoretical models. This method is particularly attractive to economists who believe theory describes long-run equilibrium phenomena better than short-run dynamics. Following Blanchard and Quah (1989), the empirical work based on long-run restrictions has frequently used recursive identification assumptions. A purely recursive long-run multiplier matrix is easily estimated by means of the Cholesky decomposition of a matrix constructed from the covariance matrix for VAR innovations and the sum of VAR coefficients. Since the set of recursive systems is quite limited compared to the set of all possible economic structures, some might be concerned that the criticisms leveled against short-run recursive orderings may also apply to long-run recursive orderings.[2]

Blanchard and Quah address a number of potential pitfalls with long-run identification restrictions in an appendix. Faust and Leeper (1997) extend Blanchard and Quah's investigation into the usefulness of long-run recursive orderings for economic analysis.[3] Other studies investigate potential econometric difficulties with models based on long-run restrictions. Such models are often be estimated by means of an instrumental variables method developed by Shapiro and Watson (1988) in which residuals from the structural equations serve as the instruments. Pagan and Robertson (1998) and Sarte (1997) investigate some of the problems that arise when the residuals are nearly uncorrelated with the variables that require instruments.

This paper is also concerned with the usefulness of empirical models that employ long-run identification restrictions. But, in contrast to other research, I investigate conditions under which long-run recursive empirical models are able to identify structural behavior. The frequent use of long-run recursive models in empirical research is a primary motivation for this study. I present a class of economic structures that permits long-run recursive empirical models to identify structure. If the economic structure is long-run block recursive, the equations in at least one of these blocks can be recursively ordered and the structural shocks are uncorrelated, then certain long-run recursive empirical models will obtain some structural impulse responses. Economies which satisfy these three conditions are called long-run partially recursive structures. If the chosen long-run ordering is consistent with this underlying structure, the empirical model will yield structural responses for each shock from the particular block of long-run recursive structural equations. The other shocks from this empirical model will not identify structural effects. The finding that the block of long-run recursive equations can occur anywhere within a block-recursive structure has not been shown before.[4]

The paper is composed as follows. Section 2 describes a popular method for constructing long-run recursive orderings in VARs with differenced data and shows the relationship between a long-run recursive ordering and a general economic structure. Section 3 uses linear projection arguments to prove that a long-run partially recursive structure will permit long-run recursive orderings to identify some structural responses in a VAR model with differenced data. Section 4 presents examples of long-run partially recursive structures, based on the economic theory developed in Amed, Ickes, Wang and Yoo (1993), to illustrate this class of structures and to clarify some ways that the result from Section 3 may be used. Section 5 extends the result to models with cointegrated data. Hence, the sufficient condition applies to some of the most popular multivariate time series models. Section 6 concludes the paper and briefly discusses necessary conditions for a long-run recursive model to identify structure.

2. Implementing Long-Run Recursive Orderings

This section examines recursive orderings under general structural assumptions, and shows that if the economy's structure is not recursive in the long run, a long-run recursive ordering will not, in general, be able to extract structural impulse response functions from the reduced form. Presenting this result is a useful starting point for determining conditions that permit long-run recursive orderings to produce structural responses.

The structural moving average representation (MAR) is a convenient tool for studying economic systems that are usually written in autoregressive form. This representation writes each endogenous variable as a function of current and past structural shocks. If y_t is an n-vector of difference-stationary time series and g_t is the vector of n structural shocks, the structural MAR is:

$$\Delta y_t = \theta(L)\varepsilon_t \tag{1}$$

where $\theta(L) = \sum_{j=0}^{\infty} \theta_j L^j$ and θ_j for j=0,1,2,...,4 is an n×n matrix of parameters. For the presentation of identification results, deterministic elements can be omitted without loss of generality. An econometrician would like to uncover equation (1) with impulse responses from a VAR model.

The multivariate Beveridge and Nelson (1981) decomposition for y_t is[5]

$$y_t = \overline{y} + \theta(1) \sum_{j=0}^{\infty} \varepsilon_{t-j} + \theta^*(L)\varepsilon_t \tag{2}$$

where $\theta^*(L) = \dfrac{\theta(L) - \theta(1)}{1 - L} = \sum_{i=0}^{\infty} \theta_i^* L^i$ with $\theta_i^* = -\sum_{k=i+1}^{\infty} \theta_k$ and $\overline{y}$ is the initial condition for y.

If Δy_t is a stationary vector process, then the last term in equation (2) represents a stationary multivariate moving average process. Consequently, this term has no effect on the level of y asymptotically. The second term in (2) sums the vector of structural shocks, indicating that each shock may have a permanent effect on y. The magnitude and direction of these permanent effects is given by the matrix of long-run structural multipliers $\theta(1)$, which is the matrix sum of the structural parameters in $\theta(L)$.

The VAR representation derived from equation (1) can be written as

$$b(L)\Delta y_t = v_t \tag{3}$$

where v_t an n-vector of VAR innovations and $b(L) = I - b_1 L - b_2 L^2 - \ldots - b_\ell L^\ell$ with b_j the n×n matrix of VAR coefficients on variables lagged j periods and I the n×n identity matrix. This VAR representation is derived from the underlying structural MAR by pre-multiplying (1) by the inverse[6] of $\theta(L)$ and then pre-multiplying the resulting expression by $\theta(0)$. Consequently, the VAR's coefficients are a function of the parameters in the structural MAR,[7]

$$b(L) = \theta(0)\theta(L)^{-1} \tag{4}$$

and the VAR innovations are given by $$v_t = \theta(0)\varepsilon_t\,. \tag{5}$$

In most examples from the literature, structural shocks are assumed to be uncorrelated white noise processes. Thus equation (5) shows that each innovation is serially uncorrelated because it is a linear combination of white noise structural shocks, and that these linear combinations are based on the contemporaneous structural parameters $\theta(0)$. The diagonal covariance matrix for ε_t is conveniently normalized to be an identity. If Σ_v is the covariance matrix for VAR innovations, then (5) implies that

$$\Sigma_v = \theta(0)\theta(0)'\,. \tag{6}$$

Economists who favor contemporaneous identification restrictions make use of this relationship. For example, Sims (1980) used Cholesky decompositions of Σ_v for identification purposes. A Cholesky decomposition is obtained by finding the unique[8] lower triangular matrix C that solves

$$\Sigma_v = CC'. \tag{7}$$

The first generation of structural VAR models applied contemporaneous restrictions on 2(0) derived from economic structures. Clearly if 2(0) is lower triangular, then C=2(0) because of the uniqueness of the recursive factorization. Methods other than the Cholesky decomposition may be required, however, if one needs to identify parameters from an economic structure that is not recursive.[9]

In contrast, long-run identification restrictions are based on the matrix of long-run multipliers. A relationship between structural long-run multipliers, contemporaneous structural parameters and the sum of VAR coefficients, given by b(1), is obtained by letting L=1 in equation (4):

$$b(1) = \theta(0)\theta(1)^{-1}. \tag{8}$$

One can solve (8) for 2(0), insert this result into (6) and simplify to obtain

$$b(1)^{-1}\Sigma_v[b(1)^{-1}]' = \theta(1)\theta(1)'. \tag{9}$$

This equation maps the long-run structural parameters into parameters from the reduced form.

A long-run recursive ordering is obtained by finding the representation

$$\Delta y_t = R(L)u_t, \tag{10}$$

such that R(1), the matrix of long-run multipliers for u_t, is triangular. The assumption that the u_t shocks are contemporaneously uncorrelated yields a diagonal covariance matrix which for convenience is normalized to the identity matrix. The MAR in (10) is mapped into the VAR representation following the same steps used with the structural system, yielding:

$$b(L) = R(0)R(L)^{-1} \tag{11}$$

and

$$v_t = R(0)u_t. \tag{12}$$

To construct R(1) from the VAR, first set L=1 in (11):

$$b(1) = R(0)R(1)^{-1} \tag{13}$$

and use (12) to find

$$\Sigma_v = R(0)R(0)' . \tag{14}$$

Then solve (13) for R(0), eliminate R(0) from (14) and simplify to obtain:

$$b(1)^{-1}\Sigma_v[b(1)^{-1}]' = R(1)R(1)' \tag{15}$$

A convenient way to calculate R(1) in (15) is by the Cholesky decomposition.

Coefficients from the recursive model are related to the structural parameters by equating (9) and (15):

$$R(1)R(1)' = \theta(1)\theta(1)' . \tag{16}$$

In the most general case, $\theta(1)$ is not lower triangular, and therefore each coefficient in R(1) is a nonlinear function of the $\theta(1)$ structural parameters. However, if the economic structure is recursive in the long run, $\theta(1)$ is lower triangular, and therefore

$$R(1) = \theta(1) \tag{17}$$

because the triangular factor is unique. In other words, if the structural system is long-run recursive and the economist chooses the correct ordering, the matrix of long-run structural multipliers is identified.

The relationship between the structural MAR in (1) and the MAR obtained by the long-run recursive model in (10) is of primary interest. Empirical researchers construct the MAR in (10) by premultiplying (3) by $b(L)^{-1}$ using (12) to eliminate VAR innovations:

$$\Delta y_t = b(L)^{-1}v_t = b(L)^{-1}R(0)u_t . \tag{18}$$

To relate R(L) to $\theta(L)$, use (4) to eliminate b(L) in (18):

$$\Delta y_t = \theta(L)\theta(0)^{-1}R(0)u_t = R(L)u_t \tag{19}$$

If $R(0)=\theta(0)$, then $R(L)=\theta(L)$. However, equation (19) should instead be put in terms of the long-run structural parameters and coefficients from the long-run recursive model. Solving (8) for $\theta(0)$ and (13) for R(0) yields

$$\theta(0)^{-1}R(0) = \theta(1)^{-1}b(1)^{-1}b(1)R(1) = \theta(1)^{-1}R(1)\,. \tag{20}$$

Substituting the result from (20) into (19) gives

$$R(L) = \theta(L)\theta(1)^{-1}R(1)\,. \tag{21}$$

Clearly if the long-run structure is recursive and the econometrician selects the correct recursive ordering, R(1)=2(1) and the empirical model will identify all the structural responses because R(L)=2(L). However, if the structure is not recursive in the long run, the MAR associated with u_t will generally be a complicated function of the structural MAR.

3. Structural Inference using VAR Models with Differenced Data

The objective is to determine general conditions under which long-run recursive orderings are able to identify structure. First I define a class of structures. I then show when VARs with differenced data are identified by certain long-run recursive orderings, some structural impulse response functions can be obtained for economies from this general class.

Definition 1: A **long-run partially recursive structure** consists of:

(i) a structural system with a block-recursive matrix of long-run multipliers;

(ii) equations in one of these blocks can be ordered recursively;

(iii) uncorrelated structural shocks.

The model of Section 2 is modified to consider a long-run partially recursive structure. Let the vector of n variables in the structural system be broken into three groups: The first n_1 variables are in y_{1t}, the next n_2 variables are in y_{2t}, the final n_3 variables are in y_{3t} and $n_1 + n_2 + n_3 = n$. Let the vector of structural shocks be similarly partitioned so that ε_{1t} has the first n_1 shocks, ε_{2t} holds the next n_2 shocks and ε_{3t} contains the final n_3 shocks. Once again the covariance matrix for uncorrelated structural shocks is normalized to the identity matrix.[10] Hence, the structural MAR is

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \end{bmatrix} = \begin{bmatrix} \theta_{11}(L) & \theta_{12}(L) & \theta_{13}(L) \\ \theta_{21}(L) & \theta_{22}(L) & \theta_{23}(L) \\ \theta_{31}(L) & \theta_{32}(L) & \theta_{33}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \tag{22}$$

where $\theta_{ij}(L) = \theta_{ij0} + \theta_{ij1}L + \theta_{ij2}L^2 + \ldots$, with Θ_{ijk} an $n_i \times n_j$ matrix for all non-negative integer k, i=1,2,3 and j=1,2,3. The long-run multiplier matrix for shocks from equation (22) can generally be written as

$$\theta(1) = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \tag{23}$$

where $\theta_{ij} = \theta_{ij}(1)$.[11]

Assumption 1: The structure is long-run partially recursive from the following restrictions:

$\theta_{12} = 0_{12}$, $\theta_{13} = 0_{13}$, $\theta_{23} = 0_{23}$ where 0_{ij} is an $n_i \times n_j$ matrix of zeros:

$$\theta(1) = \begin{bmatrix} \theta_{11} & 0_{12} & 0_{13} \\ \theta_{21} & \theta_{22} & 0_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix}, \tag{24}$$

and Θ_{22} is a lower triangular $n_2 \times n_2$ matrix.

Placing the block of recursive equations in the interior of this block-recursive system yields a fairly general long-run partially recursive form that can later be used to discuss interesting special cases. All remaining parameter matrices in equation (24) are unconstrained. If both 2_{11} and 2_{33} were also lower triangular, the analysis in Section 2 shows that the appropriate long-run recursive ordering would identify dynamic responses associated with each structural shock.

Proposition 1: If the economic structure is long-run partially recursive, each structural shock has a permanent effect on at least one variable and the data are not cointegrated, then VARs with differenced data are able to recover some structural impulse responses with long-run recursive identifying restrictions, as long as the recursive model is consistent with the underlying structure.

Consider the following linear combinations of structural disturbances:

$$\lambda_{1t} = \theta_{11}\varepsilon_{1t} \tag{25}$$

where each linear combination is written as:

$$\lambda_{1t}^{i} = \sum_{j=1}^{n_1} \theta_{11}^{ij}\varepsilon_{1t}^{j} \quad \text{for } \; i = 1,2,\ldots,n_1$$

where superscripts indicate particular elements in the vectors g_{1t} and 8_{1t} and in the matrix 2_{11}. Linear projection equations with 8_{1t} are used to map this structure into the long-run recursive empirical model. First project the second element in 8_{1t} onto the first element:

$$\lambda_{1t}^{2} = P_{1}^{21}\lambda_{1t}^{1} + \rho_{1t}^{2}$$

where P_1^{21} is the projection coefficient and ρ_{1t}^2 is the projection error. Continue projecting each variable in 8_{1t} onto all preceding variables:

$$\begin{aligned}
\lambda_{1t}^3 &= P_1^{31}\lambda_{1t}^1 + P_1^{32}\lambda_{1t}^2 + \rho_{1t}^3 \\
&. \\
&. \\
\lambda_{1t}^{n_1} &= P_1^{n_1 1}\lambda_{1t}^1 + \quad \ldots \quad + P_1^{n_1, n_1-1}\lambda_{1t}^{n_1-1} + \rho_{1t}^{n_1}
\end{aligned}$$

where projection errors are indexed by the dependent variable and projection coefficients are indexed by the dependent variable and the explanatory variable, respectively. Along with the identity which sets λ_{1t}^1 equal to itself, this system of projection equations can be written as:

$$\begin{bmatrix} 1 & 0 & 0 & . & 0 \\ -P_1^{21} & 1 & 0 & . & 0 \\ -P_1^{31} & -P_1^{32} & 1 & . & 0 \\ . & . & . & . & . \\ -P_1^{n_1 1} & . & . & -P_1^{n_1, n_1-1} & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1t}^1 \\ \lambda_{1t}^2 \\ \lambda_{1t}^3 \\ . \\ \lambda_{1t}^{n_1} \end{bmatrix} = \begin{bmatrix} \lambda_{1t}^1 \\ \rho_{1t}^2 \\ \rho_{1t}^3 \\ . \\ \rho_{1t}^{n_1} \end{bmatrix}$$

or more conveniently as:

$$P_1\lambda_{1t} = \rho_{1t}, \tag{26}$$

where P_1 is the lower triangular matrix of projection coefficients and D_{1t} is the vector of projection errors. The covariance matrix for D_{1t} is given by $E\rho_{1t}\rho_{1t}' = D_1$ where D_1 is a diagonal matrix by construction. Using $D_1^{1/2}$ as the square-root of the diagonal covariance matrix, the vector of projection errors can be written as:

$$\rho_{1t} = D_1^{1/2} u_{1t} \tag{27}$$

where u_{1t} has an identity covariance matrix, $Eu_{1t}u_{1t}' = I_1$, with I_j an $n_j \times n_j$ identity matrix. Equations (26) and (27) combine to yield the following expression for 8_{1t}:

$$\lambda_{1t} = P_1^{-1} D_1^{1/2} u_{1t} \tag{28}$$

where $P_1^{-1}D_1^{1/2}$, being the product of a lower triangular matrix and a diagonal matrix, is lower triangular.

Next define

$$\lambda_{3t} = \theta_{33}\varepsilon_{3t}, \tag{29}$$

and using a sequence of recursive linear projections similar to that which was used with 8_{1t}, obtain

$$P_3\lambda_{3t} = \rho_{3t}$$

where P_3 is lower triangular with ones along its main diagonal and the covariance matrix for D_{3t} is given by $E\rho_{3t}\rho'_{3t} = D_3$ where D_3 is a diagonal matrix. Projection errors can then be written as $\rho_{3t} = D_3^{1/2}u_{3t}$ where u_{3t} has an identity covariance matrix: $Eu_{3t}u'_{3t} = I_3$. Hence,

$$\lambda_{3t} = P_3^{-1}D_3^{1/2}u_{3t} \tag{30}$$

and $P_3^{-1}D_3^{1/2}$ is a lower triangular matrix.

From (25) and (28) solve for $\varepsilon_{1t} = \theta_{11}^{-1}P_1^{-1}D_1^{1/2}u_{1t}$, from (29) and (30) solve for $\varepsilon_{3t} = \theta_{33}^{-1}P_3^{-1}D_3^{1/2}u_{3t}$, then insert both of these expressions into (22):

$$\begin{bmatrix}\Delta y_{1t}\\ \Delta y_{2t}\\ \Delta y_{3t}\end{bmatrix} = \begin{bmatrix}\theta_{11}(L)\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{12}(L) & \theta_{13}(L)\theta_{33}^{-1}P_3^{-1}D_3^{1/2}\\ \theta_{21}(L)\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{22}(L) & \theta_{23}(L)\theta_{33}^{-1}P_3^{-1}D_3^{1/2}\\ \theta_{31}(L)\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{32}(L) & \theta_{33}(L)\theta_{33}^{-1}P_3^{-1}D_3^{1/2}\end{bmatrix}\begin{bmatrix}u_{1t}\\ \varepsilon_{2t}\\ u_{3t}\end{bmatrix}. \tag{31}$$

The covariance matrix for (u_{1t}, g_{2t}, u_{3t}) is by construction the identity matrix. Set L=1 in the matrix lag polynomial of equation (31), and use the restrictions from Assumption 1 to obtain the matrix of long-run multipliers:

$$\begin{bmatrix}P_1^{-1}D_1^{1/2} & 0_{12} & 0_{13}\\ \theta_{21}\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{22} & 0_{23}\\ \theta_{31}\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{32} & P_3^{-1}D_3^{1/2}\end{bmatrix}. \tag{32}$$

Since each block along the main diagonal is a lower triangular matrix and each block above the main diagonal consists of zeros, equation (32) is the matrix of long-run multipliers for a particular long-run recursive model, and therefore, (31) is the moving average representation for this long-run recursive model. Thus, when an economy has the long-run partially recursive structure given by Assumption 1, a long-run recursive empirical model with the (y_1', y_2', y_3') ordering will identify structural responses for g_{2t}. The other MARs from this recursive model are linear combinations of structural effects: The dynamic effects obtained for u_{1t} are a function of the structural responses to g_{1t} and the effects for u_{3t} are a function of the structural responses to g_{3t}.

4. Examples

Much empirical macroeconomic research identifies VAR models by using long-run restrictions.[12] I employ the economic structure developed in Amed et al. (1993) to illustrate the identification results from Section 3. One purpose of this section is to provide specific examples of long-run partially recursive structures. A second purpose is to present some of the ways the Section 3 result can be utilized by empirical researchers. This discussion raises an important point: Economists with different views about the appropriate economic theory, may nevertheless agree that a particular long-run recursive empirical model is informative about some structural issues.

Amed et al. construct a 6 variable model that includes growth in labor hours for the home country (Δn_{ht}), the home country's output growth (Δy_{ht}), the foreign country's output growth (Δy_{ft}), the difference between private output growth rates between the two countries ($\Delta y^p_{ht} - \Delta y^p_{ft}$), the change in the log of the terms of trade (Δq_{ft}), and the difference in growth rates of nominal money between the two countries ($\Delta m_{ht} - \Delta m_{ft}$). These variables are driven by 6 structural disturbances with shocks arising from labor supply in the home country (τ_{ht}), world-wide technology (τ_t), labor supply in the foreign country (τ_{ft}), the cross-country difference in exogenous shocks to fiscal policy ($\eta^*_{ft} - \eta^*_{ht}$), the difference in preference shocks ($\varepsilon_{ft} - \varepsilon_{ht}$), and the cross-country difference in exogenous shocks to money supply ($\nu^*_{ht} - \nu^*_{ft}$).[13] Amed et al. identify their model by assuming structural shocks are uncorrelated and that the long-run multiplier matrix is completely recursive:[14]

$$\begin{bmatrix} \Delta n_{ht} \\ \Delta y_{ht} \\ \Delta y_{ft} \\ \Delta y^p_{ht} - \Delta y^p_{ft} \\ \Delta q_{ft} \\ \Delta m_{ht} - \Delta m_{ft} \end{bmatrix} = \begin{bmatrix} \Psi_{11} & 0 & 0 & 0 & 0 & 0 \\ \Psi_{21} & \Psi_{22} & 0 & 0 & 0 & 0 \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & 0 & 0 & 0 \\ \Psi_{41} & \Psi_{42} & \Psi_{43} & \Psi_{44} & 0 & 0 \\ \Psi_{51} & \Psi_{52} & \Psi_{53} & \Psi_{54} & \Psi_{55} & 0 \\ \Psi_{61} & \Psi_{62} & \Psi_{63} & \Psi_{64} & \Psi_{65} & \Psi_{66} \end{bmatrix} \begin{bmatrix} \tau_{ht} \\ \tau_t \\ \tau_{ft} \\ \eta^*_{ft} - \eta^*_{ht} \\ \varepsilon_{ft} - \varepsilon_{ht} \\ \nu^*_{ht} - \nu^*_{ft} \end{bmatrix}. \tag{33}$$

This model is motivated by a set of plausible structural assumptions. Amed et al. assume that long-run movements in hours worked for a particular country are caused exclusively by shocks to that country's labor supply and that a country's output is driven exclusively by domestic labor supply shocks and world-wide technology shocks in the long run. These restrictions yield the first three equations in (33).[15] Then they assume fiscal expenditures affect the composition of output between private and government spending, but can have no effect on aggregate output or hours worked in the long run. Their fourth equation allows shocks to home country labor supply, foreign labor supply, technology and fiscal policy to affect private spending. The fifth equation permits the terms of trade to respond in the long run to all the structural shocks from the first four equations and also to preference shocks. Preference shocks are not, however, allowed to affect private spending, aggregate spending, hours worked or fiscal policy in the long run. The sixth equation lets the relative growth of the money supply respond to all shocks in the model. This specification is based on the assumption that monetary policy may react to a wide variety of macroeconomic information. Money is assumed to be long-run neutral, and therefore money supply shocks have no long-run impact on any of the other variables in this system. Parameters for this purely recursive long-run structural model are estimated by means of Blanchard and Quah's (1989) technique.

Many alternative structural assumptions to those employed by Amed et al. exist that some economists might consider equally plausible. This point is not meant to detract from the importance of their research, but is instead a reflection on the field of macroeconomics which is currently without a unifying paradigm and undergoing rapid transformation. I illustrate the identification results from Section 3 by proposing two reasonable modifications to their assumptions:

Assumption A: Permanent technological improvement reduces labor supply by a wealth effect;

Assumption B: Fiscal policymakers react to the same information as central bankers.

Assumption A is quite plausible from economic theory. Assumption B is motivated by the fact that monetary and fiscal authorities have similar policy goals and frequently attempt to coordinate policies. If both assumptions are added to Amed et al., the long-run structure becomes:

$$\begin{bmatrix} \Delta n_{ht} \\ \Delta y_{ht} \\ \Delta y_{ft} \\ \Delta y^p_{ht} - \Delta y^p_{ft} \\ \Delta q_{ft} \\ \Delta m_{ht} - \Delta m_{ft} \end{bmatrix} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 & 0 & 0 & 0 \\ \Psi_{21} & \Psi_{22} & 0 & 0 & 0 & 0 \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & 0 & 0 & 0 \\ \Psi_{41} & \Psi_{42} & \Psi_{43} & \Psi_{44} & \Psi_{45} & \Psi_{46} \\ \Psi_{51} & \Psi_{52} & \Psi_{53} & \Psi_{54} & \Psi_{55} & 0 \\ \Psi_{61} & \Psi_{62} & \Psi_{63} & \Psi_{64} & \Psi_{65} & \Psi_{66} \end{bmatrix} \begin{bmatrix} \tau_{ht} \\ \tau_{t} \\ \tau_{ft} \\ \eta^*_{ft} - \eta^*_{ht} \\ \varepsilon_{ft} - \varepsilon_{ht} \\ \nu^*_{ht} - \nu^*_{ft} \end{bmatrix}. \qquad (34)$$

These additional assumptions yield a 3 block system in which the long-run structural multiplier matrix is block recursive. The first block contains the first two structural equations, the second block has the foreign output growth equation and the final block is composed of the last three equations. Since there is only one way to order a single item, the equation in the middle block has a unique recursive ordering.

Given the structure in (34), consider what happens when Amed et al. estimate a model based on the long-run multiplier matrix in (33). While their empirical model would now be mispecified, results from Section 3 show that their long-run recursive ordering would still identify structural effects for the foreign labor supply shock.[16] The MAR for the first two shocks in their recursive ordering would, however, confound the effects of technology and home country labor supply shocks because of Assumption A. Similarly, the effects for the last three shocks in their empirical model will confound the dynamic effects of shocks to fiscal policy, preferences and money supply because of Assumption B.

Clearly, an appropriate long-run recursive ordering must be employed to identify structural effects. In the general case of Section 3, the middle n_2 variables must be ordered in a particular sequence to obtain structural results. However, the first n_1 variables can be arbitrarily ordered and so can the last n_3 variables. For example, given Assumptions A and B, Amed et al. could use a long-run recursive model in which they interchange the first 2 variables and/or select some other ordering of the last 3 variables to identify the effects of foreign labor supply shocks. In general, there are $n_1!n_3!$ different orderings for the system that will identify structural effects for the n_2 shocks in the interior block.

Special cases of the general specification of long-run partially recursive structure from Section 3 illustrate how the subset of recursive equations may occur in the first or last block of a system. Suppose only two sets of block-recursive structural equations exist and the second group consists of recursively ordered equations. This amounts to setting n_3=0 in the general case from Section 3. Under this assumption, an appropriate long-run recursive ordering will identify the structural MAR for shocks to the last n_2 equations. The only constraint placed on the initial n_1 equations is that they must be block recursive in the long run with respect to the remaining n_2 equations. For example, suppose Assumption A, but not Assumption B, is added to the structural assumptions in Amed et al. In this case, the block-recursive system consists of only two blocks. The first block contains the first two equations and the second block includes the last four. Adding only Assumption A to the structural model of Amed et al. implies that their recursive ordering will identify structural effects for the shocks to foreign labor supply, fiscal policy, preferences and the supply of money. The first two shocks from their recursive ordering, however, would not identify structural responses because technology shocks have a wealth effect.

A second special case is when the block recursive system has two blocks and the first block consists of recursively ordered structural equations. This example is handled by setting n_1=0 in the general case from Section 3. Hence, the MAR associated with each of the first n_2 shocks can be identified by an appropriate long-run recursive ordering. In this case, the remaining n_3 equations are left unconstrained. For example, suppose that only Assumption B is added to the set of assumptions in Amed et al. In this case, their first three equations would form one block and their last three would form the other block. These two blocks are block-recursive and the equations in the first block are equation-by-equation recursive in the long run. Therefore, the recursive ordering used by Amed et al. would identify the effects of shocks to home country labor supply, technology and foreign labor supply. The last three shocks from their empirical model would not identify structural responses because monetary and fiscal policymakers respond to a common set of information.

It is also worth noting that the basic result from Section 3 is easily extended to a long-run block-recursive system with more than three blocks in which multiple blocks consist of long-run recursive equations. Each block can be handled individually using the methods in Section 3. Hence, long-run recursive orderings consistent with this more complex block-recursive structure would identify structural responses for each block of long-run recursive equations.

5. Structural Inference using Models of Cointegration

While VARs with differenced data are quite common, even more economic research is conducted with models of cointegrated times series. It is, therefore, important to determine if the previous results extend to this class of models. To address this issue, I use the triangular representation[17] of Philips (1991) which writes the cointegrated system as a VAR with integrated and stationary variables. Let s_t be linear combinations of time series that are stationary to allow for all cointegrating relationships.[18] Assuming deterministic elements are removed from all variables, augment the model from Section 3 with variables s_t and shocks μ_t, with the number of these additional shocks matching the number of stationary variables:

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \\ s_t \end{bmatrix} = \begin{bmatrix} \theta_{11}(L) & \theta_{12}(L) & \theta_{13}(L) & \theta_{14}(L) \\ \theta_{21}(L) & \theta_{22}(L) & \theta_{23}(L) & \theta_{24}(L) \\ \theta_{31}(L) & \theta_{32}(L) & \theta_{33}(L) & \theta_{34}(L) \\ \theta_{41}(L) & \theta_{42}(L) & \theta_{43}(L) & \theta_{44}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \mu_t \end{bmatrix}. \tag{35}$$

These μ_t shocks are assumed to have no permanent effect on any variables. This is equivalent to having the number of independent permanent shocks match the number of differenced series in (35). The basis for this assumption is the common stochastic trends representation for cointegrated systems developed by Stock and Watson (1988). Transitory shocks are assumed uncorrelated with the permanent shocks, and this assumption is crucial for identifying permanent shocks. The transitory shocks may, however, be correlated with one another.

Since transitory shocks will have at most a temporary effect on integrated variables, the following restrictions must hold: $2_{14}(1)=0_{14}$, $2_{24}(1)=0_{24}$ and $2_{34}(1)=0_{34}$. These restrictions provide a structural basis for the triangular representation. The sum of parameters given by $2_{41}(1)$, $2_{42}(1)$, $2_{43}(1)$ and $2_{44}(1)$ are unconstrained because stationary variables, by definition, can not experience a permanent response to any shock.

Assume the structure is long-run partially recursive with the form given in Assumption 1.

Precisely the same method from Section 3 can be used here to map the long-run partially recursive structure from equation (35) into the long-run recursive model:

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \\ s_t \end{bmatrix} = \begin{bmatrix} \theta_{11}(L)\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{12}(L) & \theta_{13}(L)\theta_{33}^{-1}P_3^{-1}D_3^{1/2} & \theta_{14}(L)\Omega \\ \theta_{21}(L)\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{22}(L) & \theta_{23}(L)\theta_{33}^{-1}P_3^{-1}D_3^{1/2} & \theta_{24}(L)\Omega \\ \theta_{31}(L)\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{32}(L) & \theta_{33}(L)\theta_{33}^{-1}P_3^{-1}D_3^{1/2} & \theta_{34}(L)\Omega \\ \theta_{41}(L)\theta_{11}^{-1}P_1^{-1}D_1^{1/2} & \theta_{42}(L) & \theta_{43}(L)\theta_{33}^{-1}P_3^{-1}D_3^{1/2} & \theta_{44}(L)\Omega \end{bmatrix} \begin{bmatrix} u_{1t} \\ \varepsilon_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}, \qquad (36)$$

along with an arbitrary set of identification restrictions, $\mu = \mathrm{S}u_{4t}$, which map the transitory structural shocks into a set of orthogonal shocks u_{4t}, each with unit variance and no correlation with the other shocks in the model. Setting L=1 in (36) shows that the long-run multiplier matrix for the effects of permanent shocks (u_{1t}, g_{2t}, u_{3t}) on the integrated variables $(y'_{1t}, y'_{2t}, y'_{3t})$ is given by the lower triangular matrix in equation (32). Since (36) is the MAR obtained from this particular ordering of long-run multipliers, the effects of g_{2t} are identified with this empirical model. Hence, all the results for long-run partially recursive structures found in Sections 3 and 4 naturally extend to models of cointegration. While the restrictions associated with u_{4t} would typically come from contemporaneous identification assumptions, $\mathrm{S}=I_4$ is also a possibility. In other words, except for the assumption that the transitory structural disturbances are uncorrelated with the permanent structural disturbances, assumptions about the transitory shocks are irrelevant.

6. Concluding Comments

This paper proves that a long-run partially recursive structure is sufficient for long-run recursive orderings to identify some structural effects, as long as the ordering is consistent with the underlying structure. One can also show that two-block versions of the long-run partially recursive structure are necessary for the initial block or the final block of shocks from a long-run recursive ordering to yield structural responses. These results follow from a simple extension of necessary conditions found in the proof from Section 5 of Keating (1996). To be specific, assume the series are integrated and replace the contemporaneous structural parameter matrix and the contemporaneous identification restrictions with the long-run partially recursive structural parameter matrix and the long-run recursive identification restrictions, respectively.[19]

A key implication of this paper is that long-run recursive empirical models yield structural responses for a class of economic systems much larger than the set of structures which are fully recursive in the long run. A typical application is when theory provides a particular long-run partially recursive structure, and an economist estimates an appropriate long-run recursive empirical model to address specific economic questions. In a second application, existing empirical studies that use long-run recursive models can be examined under alternative theoretical assumptions. If these alternative assumptions recast the structure as a long-run partially recursive system, then particular long-run recursive models are able to provide information about the economy. If the empirical model happens to be consistent with this alternative structure, it will still obtain specific structural information under this different set of assumptions. Researchers who want to attach structural interpretations to empirical models of integrated or cointegrated time series may find these applications useful and may develop additional ways to use this paper's results about identification.

Notes

1. See Strotz and Wold (1960) for classic references to this literature.

2. It should be noted, however, that economic theory often generates plausible long-run recursive structures. For example, following Blanchard and Quah (1989) much of this literature uses the assumption that aggregate demand has a long-run neutral effect on output, and this assumption typically yields a long-run recursive economic structure. Long-run recursive structures have also been developed by Bullard and Keating (1995), King and Watson (1997) and Roberts (1993) to address the superneutrality of money.

3. Their results are derived for long-run structures that are not necessarily recursive. Some of their results will also apply to contemporaneous structures.

4. The structure in Shapiro and Watson (1988) has a "lower block triangular" matrix of long-run multipliers. The results shown here hold for their system and for more complicated examples from the class of long-run partially recursive structures.

5. Equation (2) is constructed by integrating equation (1).

6. Lippi and Reichlin (1993) and Blanchard and Quah (1993) present alternative views on invertibility. Non-invertible structures can be constructed from the VAR's fundamental representation.

7. Equation (4) assumes R lags will adequately approximate the infinite order structural MAR in (1).

8. Hamilton (1994,p.91) proves that the triangular factorization is unique.

9. Bernanke (1986), Blanchard and Watson (1986) and Sims (1986) are foundational works on structural VAR identification with contemporaneous restrictions. Keating (1990) shows that exclusion restrictions on contemporaneous coefficients are generally invalid if agents are forward looking and have rational expectations. West (1990) also uses rational expectations in a structural VAR. Gali (1992) combines contemporaneous and long-run restrictions.

10. Technically speaking, if the covariance matrix of structural shocks is block diagonal and if each shock in g_{2t} is uncorrelated with every other shock, then the results will still go through. This assumption allows shocks in the first block to be correlated with one another and shocks in the third block to be correlated with one another, but no correlation between shocks from different blocks.

11. In contrast, Keating (1996) uses 2_{ij} notation to denote contemporaneous structural parameters.

12. See Amed, Ickes, Wang and Yoo (1993), Bullard and Keating (1995), Gamber and Joutz (1993), Keating and Nye (1998) and Lastrapes and Selgin (1994) and their references to additional research.

13. The asterisks indicate that structural parameters multiply the monetary and fiscal shocks in their model. These parameters are irrelevant for the purposes of this paper, and therefore, to simplify notation they are not dealt with explicitly.

14. This way of representing a long-run structure comes from taking first differences of equation (2) and ignoring stationary effects.

15. These assumptions imply Q_{31}=0. Since Amed et al. do not impose any of the overidentifying restrictions generated by their assumptions, I will ignore overidentification issues.

16. Having Q_{56} = 0 in (34) does not imply anything else is structural from any recursive ordering.

17. The Appendix shows that the vector error correction model can also be used to show this result.

18. Inherently stationary variables yield a particularly simple cointegrating vector.

19. Equation (20) from Section 3 in this paper can be used to rationalize these substitutions.

Appendix: Structural Inference using Vector Error Corrections Models

The vector error corrections model (VECM) of Engle and Granger (1987) may be the most popular model for a system of cointegrated time series. In a VECM, dependent variables are differenced enough times to become stationary and the set of regressors includes lagged dependent variables along with stationary linear combinations of the series. Assume X_t is a vector of m variables that are integrated of order 1. This order of integration is arbitrary. Let $\beta' X_t$ be the stationary linear combinations of X. If there are m-n cointegrating vectors, then β is an m×(m-n) matrix where 0<n<m. Just like the triangular representation, the number of transitory shocks is equal to the number of cointegrating vectors, and thus the number of stochastic trends in the system must equal n.

King, Plosser, Stock and Watson (1991) developed a method to identify permanent shocks in a VECM based on a recursive ordering of permanent components. They write the moving average representation for a cointegrated system as $\Delta X_t = \Gamma(L)\eta_t$, where Γ(L) is obtained by inverting the VECM and $\eta_t = \begin{bmatrix} \eta_t^P \\ \eta_t^T \end{bmatrix}$, where η^P are the permanent shocks and η^T are the transitory shocks. Permanent shocks are assumed to be uncorrelated with one another. Transitory shocks are not necessarily identified, although the assumption that transitory shocks are uncorrelated with permanent shocks is crucial for identifying permanent shocks. The long-run effects of permanent shocks are constrained by the cointegrating relationships and by the econometrician's interpretation of the sources of the permanent components. King et al. show that the long-run multipliers can be written as $\Gamma(1) = \left[\tilde{A}\pi\ ,\quad 0_{m,m-n}\right]$ where $\tilde{A}$ is an m×n matrix of coefficients derived from the parameters in β and π is a full rank n×n matrix. The moving average representation can also be written as $\Delta X_t = \Gamma(1)\eta_t + \Gamma^*(L)\Delta\eta_t$ to separate permanent effects from transitory dynamics. From previous expressions, permanent effects are given by $\Gamma(1)\eta_t = \tilde{A}\pi\eta_t^P$. King et al. assume B is lower triangular, and show how to obtain B from the Cholesky decomposition of a matrix constructed from parameters of the VECM. The permanent components from their recursive model can be written as $e_t^P = \pi\eta_t^P$ where elements of e_t^P are associated with permanent shocks to variables. In their 6 variable model, King et al. find three cointegrating vectors which implies that the system has three independent permanent shocks. They associate the permanent components with real output (calling this a balanced growth shock), inflation and the real interest rate. Estimation of B allows them to decompose the permanent components for the three variables into three orthogonal permanent shocks 0^P. If the economic structure happens to be long-run recursive and the correct ordering of variables is used, then clearly each permanent shock will identify structural effects.

Let these permanent movements in variables be related to structural disturbances by the following system of equations: $e_t^P = \theta(1)\varepsilon_t$. Suppose that this structural system is partially recursive in the long run with the particular form given in Assumption 1, implying the structure is given as

$$\begin{bmatrix} e_{1t}^P \\ e_{2t}^P \\ e_{3t}^P \end{bmatrix} = \begin{bmatrix} \theta_{11} & 0_{12} & 0_{13} \\ \theta_{21} & \theta_{22} & 0_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

with 2_{22} a lower triangular matrix. To conform with these structural blocks, the long-run recursive empirical model can be written as:

$$\begin{bmatrix} e_{1t}^P \\ e_{2t}^P \\ e_{3t}^P \end{bmatrix} = \begin{bmatrix} \pi_{11} & 0_{12} & 0_{13} \\ \pi_{21} & \pi_{22} & 0_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \begin{bmatrix} \eta_{1t}^P \\ \eta_{2t}^P \\ \eta_{3t}^P \end{bmatrix}.$$

Using exactly the same transformations found in Section 3 it is easy to map the long-run partially recursive structure into this recursive model, and find that $B_{22} = 2_{22}$, $B_{32} = 2_{32}$ and $\eta_{2t}^P = g_{2t}$. Hence η_{2t}^P identifies the effects of g_{2t}.

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